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# Multiuser Parameter Estimation using Divided Difference Filter in CDMA Systems

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***Abstract***

*In this paper we investigates time delay and channel gain estimation for multipath fading Code Division Multiple Access (CDMA) signals using the second order Divided Difference Filter (DDF). Given the nonlinear dependency of the channel parameters on the received signals in multiuser/multipath scenarios, we show that the DDF achieves better performance than its linear counterparts. The DDF, which is a derivative-free Kalman filtering approach, avoids the errors associated with linearization in the conventional Extended Kalman Filter (EKF). The Numerical results also show that the proposed DDF is simpler to implement, and more resilient to near-far interference in CDMA networks.*

## 1. Introduction

Multiuser parameter estimation has been an active area of research and has led to the development of a plethora of techniques over the years. Time delay estimates are used in numerous applications such a radiolocation, radar, sonar, seismology, geophysics, ultrasonic, to mention a few. In most applications, the estimated parameters are fed into subsequent processing blocks of communication systems to detect, identify, and locate radiating sources.

Direct-sequence code-division multiple-access technology includes higher bandwidth efficiency which translates into capacity increases, speech privacy, immunity to multipath fading and interference, and universal frequency reuse [1, 2], over existing and other proposed technologies make it a popular choice. As with all cellular systems, CDMA suffers from multiple-access interference (MAI). In CDMA, however, the effects of the MAI are more considerable since the frequency band is being shared by all the users who are separated by the use of the distinct pseudo noise (PN) spreading codes. These PN codes of the different users are non-orthogonal giving rise to the interference, which is considered to be the main factor limiting the capacity in DS-CDMA systems.

Accurate channel parameter estimation for CDMA signals impaired by multipath fading and multiple access interference (MAI) is an active research field that continues to draw attention in the CDMA literature. In particular, the joint estimation of the arriving multi-path time delays and corresponding channel tap gains for closely-spaced (within a chip interval) delay profiles is quite challenging, and has led the development of several joint multiuser parameter estimators, e.g., [3,4]. These have been extended to the case of multipath channels with constant channel taps and constant or slowly varying time delays [7]. An attempt at extending subspace methods to tracking time delays was given in [6], On the other hand, time delay trackers based on the Delay Lock Loop (DLL) combined with interference cancellation techniques have also been developed for multi-user cases [10]. Near–far resistant time delay estimators are not only critical for accurate multi-user data detection, but also as a supporting technology for time-of-arrival based radiolocation applications in CDMA cellular networks

[5, 8-10]. The maximum-likelihood-based technique has been employed in [11], and [12] for single-user channel and/or multiuser channel estimation with training symbols or pilots.

The Kalman filter framework based methods were considered in [13-15], where Extended Kalman Filter (EKF) has been applied to parameter estimations. Unscented Kalman Filters (UKF) and Divided Difference Filters (DDF) and their variants, also termed as derivative free filters, have been proposed as viable and more accurate alternatives to the Extended Kalman Filters (EKF) for nonlinear estimations. UKF was proposed by Julier who approached the problem of accurately capturing the relevant prior statistics of a random variable using the second order terms in the Taylor series expansion of the true quantities by choosing a set of weighted sigma points. Under this scheme, errors are only introduced in the higher (> 2) order terms [10,16,17].

The DDF is described as a sigma point filter (SPF) in a unified way where the filter linearizes the nonlinear dynamic and measurement functions by using an interpolation formula through systematically chosen sigma points. The linearization is based on polynomial approximations of the nonlinear transformations that are obtained by Stirling's interpolation formula, rather than the derivative-based Taylor series approximation [18, 19]. Conceptually, the implementation principle resembles that of the EKF, the implementation, however, is significantly simpler because uses a finite number of functional evaluations instead of analytical derivatives. It is not necessary to formulate the Jacobian and/or Hessian matrices of partial derivatives of the nonlinear dynamic and measurement equations. Thus, the new nonlinear state filter, divided difference filter (DDF), can also replace the extended Kalman filter (EKF) and its higher-order estimators in practical real-time applications that require accurate estimation, but less computational cost.

Many of the algorithms presented in previous work have focused on single-user and/or single-path propagation models. However, in practice, the arriving signal typically consists of several epochs from different users, and it becomes therefore necessary to consider multi-user/multi-path channel models. In this paper, we present a joint estimation algorithm for channel coefficients and time delays in a multipath CDMA environment using a non-linear filtering approach based on the second order Divided Difference Filter (DDF) with a particular emphasis on closely spaced paths in a multipath fading channel.

The rest of the article is organized as follows. In Section 2, the signal and channel models are presented. Section 3 provides a description of the nonlinear filtering method used for multiuser parameter estimation that utilizes Divided Difference Filter. Section 4 describes computer simulation and performance discussion followed by the conclusion.

## 2. Divided Difference Filter

Consider a nonlinear function, $\mathbf{y} = \mathbf{h}(\mathbf{x})$ with $\bar{\mathbf{x}}$ and covariance $\mathbf{P}_{\mathbf{xx}}$. If the function $\mathbf{h}$ is analytic, then the multi-dimensional Taylor series expansion of a random variable $x$ about the mean $\bar{\mathbf{x}}$ is given by the following [6, 7]

$$\mathbf{y} \simeq \mathbf{h}(\bar{\mathbf{x}} + \Delta\mathbf{x}) = \mathbf{h}(\bar{\mathbf{x}}) + \mathrm{D}_{\Delta x}\,\mathbf{h} + \frac{1}{2!}\mathrm{D}^2_{\Delta x}\,\mathbf{h} + \frac{1}{3!}\mathrm{D}^3_{\Delta x}\,\mathbf{h} + \frac{1}{4!}\mathrm{D}^4_{\Delta x}\,\mathbf{h} + \ldots$$

where $\mathrm{D}^{\mathrm{i}}_{\Delta\mathbf{x}}\,\mathbf{h}$ is the total derivative of $\mathbf{h}(\mathbf{x})$ given by

$$D_{\Delta\mathbf{x}}^{i}\mathbf{h} = \left( \Delta x_1 \frac{\partial}{\partial x_1} + \Delta x_2 \frac{\partial}{\partial x_2} + \ldots + \Delta x_n \frac{\partial}{\partial x_n} \right)^i \mathbf{h}(\mathbf{x}) \Bigg|_{\mathbf{x}=\bar{\mathbf{x}}}$$

The first and second order operators can be written as

$$D_{\Delta\mathbf{x}}\mathbf{h} = \left( \sum_{p=1}^{n} \Delta x_p \frac{\partial}{\partial x_p} \right) \mathbf{h}(\mathbf{x}) \Bigg|_{\mathbf{x}=\bar{\mathbf{x}}}$$

$$D_{\Delta\mathbf{x}}^{2}\mathbf{h} = \left( \sum_{p=1}^{n} \sum_{q=1}^{n} \Delta x_p \Delta x_q \frac{\partial}{\partial x_p \partial x_q} \right) \mathbf{h}(\mathbf{x}) \Bigg|_{\mathbf{x}=\bar{\mathbf{x}}}$$

The second order divided difference approximation of the function is formulated by using the vector form of Stirling's interpolation formula, which is similar to the extension of the Taylor series approximation

$$\mathbf{y} \approx \mathbf{h}(\bar{\mathbf{x}}) + \tilde{D}_{\Delta\mathbf{x}}\mathbf{h} + \frac{1}{2!}\tilde{D}_{\Delta\mathbf{x}}^{2}\mathbf{h}$$

Where the operators $\tilde{D}_{\Delta\mathbf{x}}$ and $\tilde{D}_{\Delta\mathbf{x}}^{2}$ are given by

$$\tilde{D}_{\Delta\mathbf{x}}\mathbf{h} = \frac{1}{h}\left( \sum_{p=1}^{n} \Delta x_p \mu_p \delta_p \right) \mathbf{h}(\bar{\mathbf{x}})$$

$$\tilde{D}_{\Delta\mathbf{x}}^{2}\mathbf{h} = \frac{1}{h^2}\left( \sum_{p=1}^{n} \Delta x_p^2 \delta_p^2 + \sum_{p=1}^{n} \sum_{q=1,\, p\neq q}^{n} \Delta x_p \Delta x_q (\mu_p \delta_p)(\mu_q \delta_q) \right) \mathbf{h}(\bar{\mathbf{x}})$$

where $h$ is an interval of length, taken as $h = \sqrt{3}$ for a Gaussian distribution and $\delta_p$ and $\mu_p$ denote the partial difference operator and the partial average operator respectively

$$\delta_p \mathbf{h}(\bar{\mathbf{x}}) = \mathbf{h}\left( \bar{\mathbf{x}} + \frac{h}{2}\mathbf{e}_p \right) - \mathbf{h}\left( \bar{\mathbf{x}} - \frac{h}{2}\mathbf{e}_p \right)$$

$$\mu_p \mathbf{h}(\bar{\mathbf{x}}) = \frac{1}{2}\left\{ \mathbf{h}\left( \bar{\mathbf{x}} + \frac{h}{2}\mathbf{e}_p \right) - \mathbf{h}\left( \bar{\mathbf{x}} - \frac{h}{2}\mathbf{e}_p \right) \right\}$$

and $\mathbf{e}$ is the $p$th unit vector along the coordinate axis in the space spanned by $\mathbf{x}$. The following linear transformation of $\mathbf{x}$ is introduced to illustrate how others can be derived.

$$\mathbf{z} = \mathbf{S}_{\mathbf{x}}^{-1}\mathbf{x}$$

where $\mathbf{S}_{\mathbf{x}}$ is the Cholesky factor of the covariance matrix $\mathbf{P}_{\mathbf{xx}}$. A new function $\tilde{\mathbf{h}}$ is defined by

$$\tilde{\mathbf{h}}(\mathbf{z}) \equiv \mathbf{h}(\mathbf{S}_{\mathbf{x}}\mathbf{z}) = \mathbf{h}(\mathbf{x})$$

The Taylor series approximation of $\tilde{\mathbf{h}}$ is identical to that of $\mathbf{h}$, while the interpolation formula does not yield the same results for $\tilde{\mathbf{h}}$ and $\mathbf{h}$ due to the following

$$2\mu_p \delta_p \tilde{\mathbf{h}}(\bar{\mathbf{z}}) = \tilde{\mathbf{h}}(\bar{\mathbf{z}} + h\mathbf{e}_p) - \tilde{\mathbf{h}}(\bar{\mathbf{z}} - h\mathbf{e}_p) = \mathbf{h}(\bar{\mathbf{x}} + \mathbf{s}_p) - \mathbf{h}(\bar{\mathbf{x}} - h\mathbf{s}_p)$$

where $\mathbf{s}_p$ denotes the $p$th column of $\mathbf{S}_{\mathbf{x}}$. Thus, $\tilde{D}_{\Delta\mathbf{x}}\mathbf{h}$ and $\tilde{D}_{\Delta x}^{2}\mathbf{h}$ will be different from $\tilde{D}_{\Delta\mathbf{z}}\tilde{\mathbf{h}}$ and $\tilde{D}_{\Delta\mathbf{z}}^{2}\tilde{\mathbf{h}}$.

### 2.1 First-Order Divided Difference Filter (DDF1)

Consider the nonlinear equations

$$\mathbf{x}_{k+1} = \mathbf{f}(\mathbf{x}_k, \mathbf{w}_k, k)$$

$$\mathbf{y}_k = \mathbf{h}(\mathbf{x}_k, \mathbf{v}_k, k)$$

where $\mathbf{x}_k$ is the $n\times 1$ state vector, $\mathbf{y}_k$ is the $m\times 1$ observation vector, $\mathbf{w}_k$ is the state noise process vector and $\mathbf{v}_k$ is the $r\times 1$ measurement noise vector. It is assumed that the noise vectors are uncorrelated white Gaussian processes with expected means and covariances

$$E\{\mathbf{w}_k\} = \overline{\mathbf{w}},\ E\left\{[\mathbf{w}_k - \overline{\mathbf{w}}_k][\mathbf{w}_j - \overline{\mathbf{w}}_k]^T\right\} = \mathbf{Q}_k$$

$$E\{\mathbf{v}_k\} = \overline{\mathbf{v}},\ E\left\{[\mathbf{v}_k - \overline{\mathbf{v}}_k][\mathbf{v}_j - \overline{\mathbf{v}}_k]^T\right\} = \mathbf{R}_k$$

Let the square Cholesky factorizations

$$\mathbf{P}_0 = \mathbf{S}_\mathbf{x}\mathbf{S}_\mathbf{x}^T$$

$$\mathbf{Q} = \mathbf{S}_\mathbf{w}\mathbf{S}_\mathbf{w}^T$$

The predicted state vector is

$$\hat{\mathbf{x}}_{k+1}^- = \mathbf{f}(\hat{\mathbf{x}}_k, \overline{\mathbf{w}}_k, k)$$

The predicted state covariance is determined by the symmetric matrix product

$$\mathbf{P}_{k+1}^- = \mathbf{S}_\mathbf{x}^-(k+1)(\mathbf{S}_\mathbf{x}^-(k+1))^T$$

where

$$\mathbf{S}_\mathbf{x}^-(k+1) = \left[\mathbf{S}_{x\hat{x}}^{(1)}(k+1)\ \ \mathbf{S}_{xw}^{(1)}(k+1)\right]$$

with

$$\mathbf{S}_{x\hat{x}}^{(1)}(k+1) = \frac{1}{2h}\left\{\mathbf{f}_i(\hat{\mathbf{x}}_k + h\mathbf{s}_{x,j}, \overline{\mathbf{w}}_k) - \mathbf{f}_i(\hat{\mathbf{x}}_k - h\mathbf{s}_{x,j}, \overline{\mathbf{w}}_k)\right\}$$

$$\mathbf{S}_{xw}^{(1)}(k+1) = \frac{1}{2h}\left\{\mathbf{f}_i(\hat{\mathbf{x}}_k, \overline{\mathbf{w}}_k + h\mathbf{s}_{w,j}) - \mathbf{f}_i(\hat{\mathbf{x}}_k, \overline{\mathbf{w}}_k - h\mathbf{s}_{w,j})\right\}$$

where $\mathbf{s}_{x,j}$ is the column of $\mathbf{S}_\mathbf{x}$ and $\mathbf{s}_{w,j}$ is the column of $\mathbf{S}_\mathbf{w}$.

the square Cholesky factorizations are performed

$$\mathbf{P}_{k+1}^- = \mathbf{S}_\mathbf{x}^-\mathbf{S}_\mathbf{x}^{-T}$$

$$\mathbf{R} = \mathbf{S}_\mathbf{v}\mathbf{S}_\mathbf{v}^T$$

The predicted observation vector $\hat{\mathbf{y}}_{k+1}^-$ and its predicted covariance are

$$\hat{\mathbf{y}}_{k+1}^- = \mathbf{h}(\hat{\mathbf{x}}_{k+1}^-, \overline{\mathbf{v}}_{k+1}, k+1)$$

$$\mathbf{P}_{k+1}^{vv} = \mathbf{S}_v(k+1)\mathbf{S}_v^T(k+1)$$

where

$$\mathbf{S}_v(k+1) = \left[\mathbf{S}_{y\hat{x}}^{(1)}(k+1)\ \ \mathbf{S}_{yv}^{(1)}(k+1)\right]$$

$$\mathbf{S}_{y\hat{x}}^{(1)}(k+1) = \frac{1}{2h}\left\{\mathbf{h}_i(\hat{\mathbf{x}}_{k+1} + h\mathbf{s}_{x,j}^-, \overline{\mathbf{v}}_{k+1}) - \mathbf{h}_i(\hat{\mathbf{x}}_{k+1}^- + h\mathbf{s}_{x,j}^-, \overline{\mathbf{v}}_{k+1})\right\}$$

$$\mathbf{S}_{yv}^{(1)}(k+1) = \frac{1}{2h}\left\{\mathbf{h}_i(\hat{\mathbf{x}}_{k+1}^-, \overline{\mathbf{v}}_{k+1} + h\mathbf{s}_{v,j}) - \mathbf{h}_i(\hat{\mathbf{x}}_{k+1}^-, \overline{\mathbf{v}}_{k+1} + h\mathbf{s}_{v,j})\right\}$$

where $\mathbf{s}_{x,j}^{-}$ is the column of $\mathbf{S}_{\mathbf{x}}^{-}$ and $\mathbf{s}_{v,j}$ is the column of $\mathbf{S}_{\mathbf{v}}$. The innovation covariance $\mathbf{P}_{k+1}^{vv}$ is computed as

$$\mathbf{P}_{k+1}^{vv} = \mathbf{P}_{k+1}^{yy} + \mathbf{R}_{k+1}$$

with

$$\mathbf{P}_{k+1}^{yy} = \mathbf{S}_{y\hat{x}}^{(1)}(k+1)\left(\mathbf{S}_{y\hat{x}}^{(1)}(k+1)\right)^{T}$$

Finally the cross covariance matrix is determined by

$$\mathbf{P}_{k+1}^{xy} = \mathbf{S}_{x}^{-}(k+1)\left(\mathbf{S}_{y\hat{x}}^{(1)}(k+1)\right)^{T}$$

The filter gain $\mathrm{K}_{k+1}$, the updated estimated state vector $\hat{\mathbf{x}}_{k+1}^{+}$ and the updated covariance $\mathbf{P}_{k+1}^{+}$ are computed using

$$\mathrm{K}_{k+1} = \mathbf{P}_{k+1}^{xy}(\mathbf{P}_{k+1}^{vv})^{-1}$$
$$\hat{\mathbf{x}}_{k+1}^{+} = \hat{\mathbf{x}}_{k+1}^{-} + \mathrm{K}_{k+1}\left(\mathbf{y}_{k+1} - \hat{\mathbf{y}}_{k+1}\right)$$
$$\mathbf{P}_{k+1}^{+} = \mathbf{P}_{k+1}^{-} - \mathrm{K}_{k+1}\mathbf{P}_{k+1}^{vv}\mathrm{K}_{k+1}^{T}$$

The DDF1 has been outlined in table.1

### 2.2 Second-Order Divided Difference Filter (DDF2)

The second-order divided difference filter (DDF2) is obtained by using the calculation of the mean and covariance in the second-order polynomial approximation section. First, the following additional matrices containing divided difference are defined [6,7],

$$\mathbf{S}_{x\hat{x}}^{(2)}(k+1) = \frac{\sqrt{\gamma-1}}{2\gamma}\left\{\mathbf{f}_i(\hat{\mathbf{x}}_k + h\mathbf{s}_{x,j}, \bar{\mathbf{w}}_k) + \mathbf{f}_i(\hat{\mathbf{x}}_k - h\mathbf{s}_{x,j}, \bar{\mathbf{w}}_k) - 2\mathbf{f}_i(\hat{\mathbf{x}}_k, \bar{\mathbf{w}}_k)\right\}$$

$$\mathbf{S}_{xw}^{(2)}(k+1) = \frac{\sqrt{\gamma-1}}{2\gamma}\left\{\mathbf{f}_i(\hat{\mathbf{x}}_k, \bar{\mathbf{w}}_k + h\mathbf{s}_{w,j}) + \mathbf{f}_i(\hat{\mathbf{x}}_k, \bar{\mathbf{w}}_k - h\mathbf{s}_{w,j}) - 2\mathbf{f}_i(\hat{\mathbf{x}}_k, \bar{\mathbf{w}}_k)\right\}$$

where $\mathbf{s}_{x,j}$ is the jth column of $\mathbf{S}_{\mathbf{x}}$, $\mathbf{s}_{w,j}$ is the jth column of $\mathbf{S}_{\mathbf{w}}$ and $\gamma = h^2$ is a constant parameter. The predicted state equation is

$$\begin{aligned}\hat{\mathbf{x}}_{k+1}^{-} &= \frac{\gamma-(n_x+n_w)}{\gamma}\mathbf{f}(\hat{\mathbf{x}}_k, \bar{\mathbf{w}}_k) \\ &+ \frac{1}{2\gamma}\sum_{p=1}^{n_x}\left\{\mathbf{f}(\hat{\mathbf{x}}_k + h\mathbf{s}_{s,p}, \bar{\mathbf{w}}_k) + \mathbf{f}_i(\hat{\mathbf{x}}_k - h\mathbf{s}_{s,j}, \bar{\mathbf{w}}_k)\right\} \\ &+ \frac{1}{2\gamma}\sum_{p=1}^{n_x}\left\{\mathbf{f}(\hat{\mathbf{x}}_k, \bar{\mathbf{w}}_k + h\mathbf{s}_{w,p}) + \mathbf{f}_i(\hat{\mathbf{x}}_k, \bar{\mathbf{w}}_k - h\mathbf{s}_{s,p})\right\}\end{aligned}$$

In Table 1. $n_x$ denotes the dimension of the state vector, and $n_w$ is the dimension of process noise vector. The Cholesky factorization of the predicted covariance is computed as

$$\mathbf{S}_{\mathbf{x}}^{-}(k+1) = \left[\mathbf{S}_{x\hat{x}}^{(1)}(k+1)\ \ \mathbf{S}_{xw}^{(1)}(k+1)\ \ \mathbf{S}_{x\hat{x}}^{(2)}(k+1)\ \ \mathbf{S}_{xw}^{(2)}(k+1)\right]$$

The predicted covariance is computed using

$$\mathbf{P}_{k+1}^{-} = \mathbf{S}_{\mathbf{x}}^{-}(k+1)(\mathbf{S}_{\mathbf{x}}^{-}(k+1))^{T}$$

the predicted observation vector

Table 1. First Order Divided Difference Filter Algorithm

Initialization Step:

$$\hat{\mathbf{x}}_k = E\left[\mathbf{x}_k\right], \mathbf{P}_k = E\left[(\mathbf{x}_k - \hat{\mathbf{x}}_k)(\mathbf{x}_k - \hat{\mathbf{x}}_k)^T\right]$$

Square Cholesky factorizations:

$$\mathbf{P}_0 = \mathbf{S}_{\mathbf{x}}\mathbf{S}_{\mathbf{x}}^T \qquad \mathbf{Q}_k = \mathbf{S}_{\mathbf{w}}\mathbf{S}_{\mathbf{w}}^T \qquad \mathbf{R} = \mathbf{S}_{\mathbf{v}}\mathbf{S}_{\mathbf{v}}^T$$

$$\mathbf{S}_{x\hat{x}}^{(1)}(k+1) = \frac{1}{2h}\left\{\mathbf{f}_i(\hat{\mathbf{x}}_k + h\mathbf{s}_{x,j}, \overline{\mathbf{w}}_k) - \mathbf{f}_i(\hat{\mathbf{x}}_k - h\mathbf{s}_{x,j}, \overline{\mathbf{w}}_k)\right\}$$

$$\mathbf{S}_{xw}^{(1)}(k+1) = \frac{1}{2h}\left\{\mathbf{f}_i(\hat{\mathbf{x}}_k, \overline{\mathbf{w}}_k + h\mathbf{s}_{w,j}) - \mathbf{f}_i(\hat{\mathbf{x}}_k, \overline{\mathbf{w}}_k - h\mathbf{s}_{w,j})\right\}$$

$$\mathbf{S}_{\mathbf{x}}^{-}(k+1) = \left[\mathbf{S}_{x\hat{x}}^{(1)}(k+1)\mathbf{S}_{xw}^{(1)}(k+1)\right]$$

State and covariance Propagation:

$$\hat{\mathbf{x}}_{k+1}^{-} = \mathbf{f}(\hat{\mathbf{x}}_k, \overline{\mathbf{w}}_k, k)$$

$$\mathbf{P}_{k+1}^{-} = \mathbf{S}_{\mathbf{x}}^{-}(k+1)(\mathbf{S}_{\mathbf{x}}^{-}(k+1))^T$$

$$\mathbf{S}_{y\hat{x}}^{(1)}(k+1) = \frac{1}{2h}\left\{\mathbf{h}_i(\hat{\mathbf{x}}_{k+1} + h\mathbf{s}_{x,j}^{-}, \overline{\mathbf{v}}_{k+1}) - \mathbf{h}_i(\hat{\mathbf{x}}_{k+1}^{-} + h\mathbf{s}_{x,j}^{-}, \overline{\mathbf{v}}_{k+1})\right\}$$

$$\mathbf{S}_{yv}^{(1)}(k+1) = \frac{1}{2h}\left\{\mathbf{h}_i(\hat{\mathbf{x}}_{k+1}^{-}, \overline{\mathbf{v}}_{k+1} + h\mathbf{s}_{v,j}) - \mathbf{h}_i(\hat{\mathbf{x}}_{k+1}^{-}, \overline{\mathbf{v}}_{k+1} + h\mathbf{s}_{v,j})\right\}$$

$$\mathbf{S}_v(k+1) = \left[\mathbf{S}_{y\hat{x}}^{(1)}(k+1)\;\; \mathbf{S}_{yv}^{(1)}(k+1)\right]:$$

Observation and Innovation Covariance Propagation:

$$\hat{\mathbf{y}}_{k+1}^{-} = \mathbf{h}(\hat{\mathbf{x}}_{k+1}^{-}, \overline{\mathbf{v}}_{k+1}, k+1)$$

$$\mathbf{P}_{k+1}^{vv} = \mathbf{S}_v(k+1)\mathbf{S}_v^T(k+1)$$

$$\mathbf{P}_{k+1}^{xy} = \mathbf{S}_{\hat{x}}^{(1)}(k+1)\left(\mathbf{S}_{y\hat{x}}^{(1)}(k+1)\right)^T$$

Update:

$$\mathrm{K}_{k+1} = \mathbf{P}_{k+1}^{xy}(\mathbf{P}_{k+1}^{vv})^{-1}$$

$$\hat{\mathbf{x}}_{k+1}^{+} = \hat{\mathbf{x}}_{k+1}^{-} + \mathrm{K}_{k+1}\left(\mathbf{y}_{k+1} - \hat{\mathbf{y}}_{k+1}\right)$$

$$\mathbf{P}_{k+1}^{+} = \mathbf{P}_{k+1}^{-} - \mathrm{K}_{k+1}\mathbf{P}_{k+1}^{vv}\mathrm{K}_{k+1}^{T}$$

$$\hat{\mathbf{y}}_{k+1}^{-} = \frac{\gamma-(n_x+n_v)}{\gamma}\mathbf{h}(\hat{\mathbf{x}}_{k+1}^{-},\bar{\mathbf{v}}_{k+1})$$
$$+\frac{1}{2\gamma}\sum_{p=1}^{n_x}\left\{\mathbf{h}(\hat{\mathbf{x}}_{k+1}^{-}+h\mathbf{s}_{x,p}^{-},\bar{\mathbf{v}}_{k+1})+\mathbf{h}(\hat{\mathbf{x}}_{k+1}^{-}-h\mathbf{s}_{x,p}^{-},\bar{\mathbf{v}}_{k+1})\right\}$$
$$+\frac{1}{2\gamma}\sum_{p=1}^{n_x}\left\{\mathbf{h}(\hat{\mathbf{x}}_{k+1}^{-},\bar{\mathbf{v}}_{k+1}+h\mathbf{s}_{v,p})+\mathbf{h}(\hat{\mathbf{x}}_{k+1}^{-},\bar{\mathbf{v}}_{k+1}-h\mathbf{s}_{v,p})\right\}$$

where $n_v$ is the dimension of the measurement noise, $\mathbf{s}_{x,p}^{-}$ is the pth column of $\mathbf{S}_{\mathbf{x}}^{-}$, and $\mathbf{s}_{v,p}$ is the pth column of $\mathbf{S}_{\mathbf{v}}$. The innovation covariance matrix is given by

$$\mathbf{P}_{k+1}^{vv} = \mathbf{S}_v(k+1)\mathbf{S}_v^T(k+1)$$

with

$$\mathbf{S}_v(k+1) = \left[\mathbf{S}_{x\hat{x}}^{(1)}(k+1)\ \ \mathbf{S}_{xw}^{(1)}(k+1)\ \ \mathbf{S}_{x\hat{x}}^{(2)}(k+1)\ \ \mathbf{S}_{xw}^{(2)}(k+1)\right]$$
$$\mathbf{S}_{y\hat{x}}^{(2)}(k+1) = \frac{\sqrt{\gamma-1}}{2\gamma}\left\{\mathbf{h}_i(\hat{\mathbf{x}}_{k+1}^{-}+h\mathbf{s}_{x,j}^{-},\bar{\mathbf{v}}_{k+1})+\mathbf{h}_i(\hat{\mathbf{x}}_{k+1}^{-}-h\mathbf{s}_{x,i}^{-},\bar{\mathbf{v}}_{k+1})-2\mathbf{h}_i(\hat{\mathbf{x}}_{k+1}^{-},\bar{\mathbf{v}}_{k+1})\right\}$$
$$\mathbf{S}_{yv}^{(2)}(k+1) = \frac{\sqrt{\gamma-1}}{2\gamma}\left\{\mathbf{h}_i(\hat{\mathbf{x}}_{k+1}^{-},\bar{\mathbf{v}}_{k+1}+h\mathbf{s}_{x,j}^{-})+\mathbf{h}_i(\hat{\mathbf{x}}_{k+1}^{-},\bar{\mathbf{v}}_{k+1}-h\mathbf{s}_{x,j}^{-})-2\mathbf{h}_i(\hat{\mathbf{x}}_{k+1}^{-},\bar{\mathbf{v}}_{k+1})\right\}$$

The cross correlation matrix is

$$\mathbf{P}_{k+1}^{xy} = \mathbf{S}_{\hat{x}}^{(1)}(k+1)\left(\mathbf{S}_{y\hat{x}}^{(1)}(k+1)\right)^T$$

The filter gain $\mathrm{K}_{k+1}$, the updated estimated state vector $\hat{\mathbf{x}}_{k+1}^{+}$ and the updated covariance $\mathbf{P}_{k+1}^{+}$ are computed using

$$\mathrm{K}_{k+1} = \mathbf{P}_{k+1}^{xy}(\mathbf{P}_{k+1}^{vv})^{-1}$$
$$\hat{\mathbf{x}}_{k+1}^{+} = \hat{\mathbf{x}}_{k+1}^{-} + \mathrm{K}_{k+1}\left(\mathbf{y}_{k+1}-\hat{\mathbf{y}}_{k+1}\right)$$
$$\mathbf{P}_{k+1}^{+} = \mathbf{P}_{k+1}^{-} - \mathrm{K}_{k+1}\mathbf{P}_{k+1}^{vv}\mathrm{K}_{k+1}^{T}$$

The DDF2 algorithm has been outlined in table 2.

## 3. Channel and Signal Model

We consider a typical asynchronous CDMA system model where *K* users transmit over an *M*-path fading channel. The received baseband signal sampled at $t = lT_s$ is given by

$$r(l) = \sum_{k=1}^{K}\sum_{i=1}^{M} c_{k,i}(l)d_{k,m_l}a_k(l-m_lT_b-\tau_{k,i}(l))+n(l) \tag{1}$$

where $c_{k,i}(l)$ represents the complex channel coefficients, $d_{k,m_l}$ is the $m^{\text{th}}$ symbol transmitted by the $k^{\text{th}}$ user, $m_l = \left[(l-\tau_k(l)/T_b\right]$, $T_b$ is the symbol interval, $a_k(l)$ is the spreading waveform used by the $k^{\text{th}}$ user, $\tau_{k,i}(l)$ is the time delay associated with the $i^{\text{th}}$ path of the $k^{\text{th}}$ user, and $n(l)$ represents Additive White Gaussian Noise (AWGN) assumed to have a zero mean and variance $\sigma^2 = E[|n(l)|^2] = N_0/T_s$ where $T_s$ is the sampling time.

Table 2. Second Order Divided Difference Filter Algorithm

**Initialization Step:**

$$\hat{\mathbf{x}}_k = E\left[\mathbf{x}_k\right], \mathbf{P}_k = E\left[(\mathbf{x}_k - \hat{\mathbf{x}}_k)(\mathbf{x}_k - \hat{\mathbf{x}}_k)^T\right]$$

**Square Cholesky factorizations:**

$$\mathbf{P}_0 = \mathbf{S_x S_x^T} \qquad \mathbf{Q}_k = \mathbf{S_w S_w^T} \qquad \mathbf{R} = \mathbf{S_v S_v^T}$$

$$\mathbf{S}_{x\hat{x}}^{(2)}(k+1) = \frac{\sqrt{\gamma-1}}{2\gamma}\left\{\mathbf{f}_i(\hat{\mathbf{x}}_k + h\mathbf{s}_{x,j}, \bar{\mathbf{w}}_k) + \mathbf{f}_i(\hat{\mathbf{x}}_k - h\mathbf{s}_{x,j}, \bar{\mathbf{w}}_k) - 2\mathbf{f}_i(\hat{\mathbf{x}}_k, \bar{\mathbf{w}}_k)\right\}$$

$$\mathbf{S}_{xw}^{(2)}(k+1) = \frac{\sqrt{\gamma-1}}{2\gamma}\left\{\mathbf{f}_i(\hat{\mathbf{x}}_k, \bar{\mathbf{w}}_k + h\mathbf{s}_{w,j}) + \mathbf{f}_i(\hat{\mathbf{x}}_k, \bar{\mathbf{w}}_k - h\mathbf{s}_{w,j}) - 2\mathbf{f}_i(\hat{\mathbf{x}}_k, \bar{\mathbf{w}}_k)\right\}$$

**State and covariance Propagation:**

$$\hat{\mathbf{x}}_{k+1}^- = \frac{\gamma - (n_x + n_w)}{\gamma}\mathbf{f}(\hat{\mathbf{x}}_k, \bar{\mathbf{w}}_k)$$

$$+\frac{1}{2\gamma}\sum_{p=1}^{n_x}\left\{\mathbf{f}(\hat{\mathbf{x}}_k + h\mathbf{s}_{s,p}, \bar{\mathbf{w}}_k) + \mathbf{f}_i(\hat{\mathbf{x}}_k - h\mathbf{s}_{s,j}, \bar{\mathbf{w}}_k)\right\}$$

$$+\frac{1}{2\gamma}\sum_{p=1}^{n_x}\left\{\mathbf{f}(\hat{\mathbf{x}}_k, \bar{\mathbf{w}}_k + h\mathbf{s}_{w,p}) + \mathbf{f}_i(\hat{\mathbf{x}}_k, \bar{\mathbf{w}}_k - h\mathbf{s}_{s,p})\right\}$$

$$\mathbf{S}_{\mathbf{x}}^{-}(k+1) = \left[\mathbf{S}_{x\hat{x}}^{(1)}(k+1) \;\; \mathbf{S}_{xw}^{(1)}(k+1) \;\; \mathbf{S}_{x\hat{x}}^{(2)}(k+1) \;\; \mathbf{S}_{xw}^{(2)}(k+1)\right]$$

$$\mathbf{S}_{\mathbf{x}}^{-}(k+1) = \left[\mathbf{S}_{x\hat{x}}^{(1)}(k+1) \;\; \mathbf{S}_{xw}^{(1)}(k+1) \;\; \mathbf{S}_{x\hat{x}}^{(2)}(k+1) \;\; \mathbf{S}_{xw}^{(2)}(k+1)\right]^T$$

$$\mathbf{P}_{k+1}^- = \mathbf{S}_{\mathbf{x}}^{-}(k+1)(\mathbf{S}_{\mathbf{x}}^{-}(k+1))^T$$

**Observation and Innovation Covariance Propagation:**

$$\hat{\mathbf{y}}_{k+1}^- = \frac{\gamma - (n_x + n_v)}{\gamma}\mathbf{h}(\hat{\mathbf{x}}_{k+1}^-, \bar{\mathbf{v}}_{k+1})$$

$$+\frac{1}{2\gamma}\sum_{p=1}^{n_x}\left\{\mathbf{h}(\hat{\mathbf{x}}_{k+1}^- + h\mathbf{s}_{x,p}^-, \bar{\mathbf{v}}_{k+1}) + \mathbf{h}(\hat{\mathbf{x}}_{k+1}^- - h\mathbf{s}_{x,p}^-, \bar{\mathbf{v}}_{k+1})\right\}$$

$$+\frac{1}{2\gamma}\sum_{p=1}^{n_x}\left\{\mathbf{h}(\hat{\mathbf{x}}_{k+1}^-, \bar{\mathbf{v}}_{k+1} + h\mathbf{s}_{v,p}) + \mathbf{h}(\hat{\mathbf{x}}_{k+1}^-, \bar{\mathbf{v}}_{k+1} - h\mathbf{s}_{v,p})\right\}$$

$$\mathbf{P}_{k+1}^{vv} = \mathbf{S}_v(k+1)\mathbf{S}_v^T(k+1)$$

$$\mathbf{P}_{k+1}^{xy} = \mathbf{S}_{\hat{x}}^{(1)}(k+1)\left(\mathbf{S}_{y\hat{x}}^{(1)}(k+1)\right)^T$$

**Update:**

$$\mathrm{K}_{k+1} = \mathbf{P}_{k+1}^{xy}(\mathbf{P}_{k+1}^{vv})^{-1}$$

$$\hat{\mathbf{x}}_{k+1}^+ = \hat{\mathbf{x}}_{k+1}^- + \mathrm{K}_{k+1}\left(\mathbf{y}_{k+1} - \hat{\mathbf{y}}_{k+1}\right)$$

$$\mathbf{P}_{k+1}^+ = \mathbf{P}_{k+1}^- - \mathrm{K}_{k+1}\mathbf{P}_{k+1}^{vv}\mathrm{K}_{k+1}^T$$

In order to use a Kalman filtering approach, we adopt a state-space model representation where the unknown channel parameters (path delays and gains) to be estimated are given by the following $2KM \times 1$ vector,

$$\mathbf{x} = [\mathbf{c}\,;\boldsymbol{\tau}] \tag{2}$$

with $c = [c_{11}, c_{12}, ..., c_{1M}, c_{21}, ..., c_{2M}, ..., c_{K1}, ..., c_{KM}]^T$

and $\tau = [\tau_{11}, \tau_{12}, ..., \tau_{1M}, \tau_{21}, ..., \tau_{2M}, ..., \tau_{K1}, ..., \tau_{KM}]^T$

The complex-valued channel amplitudes and real-valued time delays of the K users are assumed to obey a Gauss- Markov dynamic channel model, i.e.

$$c(l+1) = \mathbf{F}_c c(l) + \mathbf{v}_c(l)$$

$$\tau(l+1) = \mathbf{F}_\tau \tau(l) + \mathbf{v}_\tau(l)$$

where $\mathbf{F}_c$ and $\mathbf{F}_\tau$ are $KM \times KM$ state transition matrices for the amplitudes and time delays respectively whereas $v_c(l)$ and $v_\tau(l)$ are $K \times 1$ mutually independent Gaussian random vectors with zero mean and covariance given by $\mathrm{E}\{v_c(i)v_c^T(j)\} = \delta_{ij}\mathbf{Q}_c$ , $\mathrm{E}\{v_\tau(i)v_\tau^T(j)\} = \delta_{ij}\mathbf{Q}_\tau$ , $\mathrm{E}\{v_c(i)v_\tau^T(j)\} = 0\ \forall i, j$ with $\mathbf{Q}_c = \sigma_c^2\mathbf{I}$ and $\mathbf{Q}_\tau = \sigma_\tau^2\mathbf{I}$ are the covariance matrices of the process noise $v_c$ and $v_\tau$ respectively, and $\delta_{ij}$ is the two-dimensional Kronecker delta function equal to 1 for $i = j$, and 0 otherwise.

Using (2-4), the state model can be written as

$$\mathbf{x}(l+1) = \mathbf{F}\mathbf{x}(l) + \mathbf{v}(l) \tag{3}$$

where

$\mathbf{F} = \begin{bmatrix} \mathbf{F}_c & 0 \\ 0 & \mathbf{F}_\tau \end{bmatrix}, \mathbf{v} = \begin{bmatrix} \mathbf{v}_c^T & \mathbf{v}_\tau^T \end{bmatrix}, \mathbf{Q} = \begin{bmatrix} \mathbf{Q}_c & 0 \\ 0 & \mathbf{Q}_\tau \end{bmatrix}$ are $2KM \times 2KM$ state transition matrix, $2KM \times 1$ process noise vector with mean of zero and covariance matrix respectively. The scalar measurement model follows from the received signal of (1) by

$$z(l) = h(\mathbf{x}(l)) + \eta(l) \tag{4}$$

where the measurement $z(l) = r(l)$, and

$$h(\mathbf{x}(l)) = \sum_{k=1}^{K}\sum_{i=1}^{M} c_{k,i}(l) d_{k,m_l} a_k(l - m_l T_b - \tau_{k,i}(l)) \cdot$$

The scalar measurement $z(l)$ is a nonlinear function of the state $\mathbf{x}(l)$. Given the state-space and measurement models, we may find the optimal estimate of $\hat{\mathbf{x}}(l)$ denoted as $\hat{\mathbf{x}}(l\,|\,l) = E\{\mathbf{x}(l)\,|\,z^l\}$, with the estimation error covariance

$$\mathrm{P} = \mathrm{E}\left\{\left[\mathbf{x}(l) - \hat{\mathbf{x}}(l\,|\,l)\right]\left[\mathbf{x}(l) - \hat{\mathbf{x}}(l\,|\,l)\right]^T \,|\, z^l\right\}$$

where $z^l$ denotes the set of received samples up to time $l$.

## 3. Parameter Estimation using the Divided Difference Filter

For the nonlinear dynamic system model such as above, the conventional Kalman algorithm can be invoked to obtain the parameter estimates [17, 18]. The most well known application of the Kalman filter framework to nonlinear systems is the Extended Kalman filter (EKF). Even though the EKF is one of the most widely used approximate solutions for nonlinear estimation and filtering, it has some limitations [17]. Firstly, the EKF only uses the first order terms of the Taylor series expansion of the nonlinear functions which often introduces large errors in the estimated statistics of the posterior distributions especially when the effects of the higher order terms of the Taylor series expansion becomes significant. Secondly, linearized transformations are only reliable if the error propagation can be well approximated by a linear function. If this condition does not hold, the linearized approximation can be extremely poor. At best, this undermines the performance of the filter. At worst, it causes its estimates to diverge altogether. And also linearization can be applied only if the Jacobian matrix exists. However, this is not always the case. Some systems contain discontinuities, others have singularities. Calculating Jacobian matrices can be very difficult.

DDF, unlike EKF, is a sigma point filter (SPF) where the filter linearizes the nonlinear dynamic and measurement functions by using an interpolation formula through systematically chosen sigma points. The linearization is based on polynomial approximations of the nonlinear transformations that are obtained by Stirling's interpolation formula, rather than the derivative-based Taylor series approximation [18]. Conceptually, the implementation principle resembles that of the EKF. However, it is significantly simpler because uses a finite number of functional evaluations instead of analytical derivatives. It is not necessary to formulate the Jacobian and/or Hessian matrices of partial derivatives of the nonlinear dynamic and measurement equations. Thus, the new nonlinear state filter, Divided Difference Filter (DDF), can also replace the Extended Kalman Filter (EKF) and its higher-order estimators in practical real-time applications that require accurate estimation, but less computational cost. The derivative free, deterministic sampling based DDF outperform the EKF in terms of estimation accuracy, filter robustness and ease of implementation.

### 3.1. Application to Channel Estimation with Multipath/ Multiuser model

We have simulated a CDMA system with varying number of users and with multipaths using DDF. The delays are assumed to be constant during one measurement. For the state space model we assumed $\mathbf{F} = 0.999\mathbf{I}$ and $\mathbf{Q} = 0.001\mathbf{I}$ where $\mathbf{I}$ is the identity matrix. We will be considering fading multipaths and multiuser environment with 2, 5 and 10 user scenario. The SNR at the receiver of the weaker user is taken to be of 10 dB. The near far ratio of 20 dB has been assumed with the power of the strong user is $P_1 = 1$ and that of the weak user is $P_1/10$. We note that the data bits, $d_{k,m}$, are not included in the estimation process, but are assumed unknown a priori. In the simulations, we assume that the data bits are available from decision-directed adaptation, where the symbols $d_{k,m}$ are replaced by the $\hat{d}_{k,m}$ decisions shown in Figure 1.We also considered the special case of closely spaced multipaths.

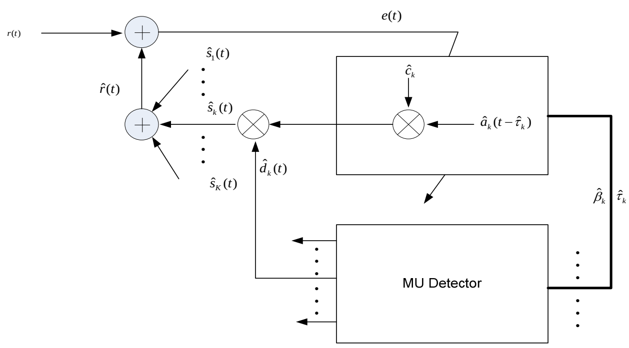


Figure 1. Multiuser parameter estimation receiver

Figures 2, 3 and 4 show the timing epoch in a multiuser scenario with three multipaths with the path separation of 1/2 chip. We have considered the case of the weaker user and have compared it with the stronger user. Proposed estimator converges to the close to the true value approximately in 6-8 symbols even in the presence of MAI and is able to track desired user delay even when the paths are closely spaced. Figures (5) show the mean square error for channel coefficients for first arriving path with a ten-user/ two-path channel model. The estimator/tracker is able to accurately track the time-varying channel coefficients of each user, even for fast fading rates. It is seen that a user is capable to accurately converge to the correct delays and channel coefficient for both estimator. The strong as well as the weker user is successfully tracked thus demonstrating the near far resistant nature of the proposed estimator.

Now if we compare the UKF algorithm [10] with the DDF algorithm, we see that the performance of the two is nearly same. This has been demonstrated in Figure (6). It is due to the fact that DDF is based on the derivative approximation on Stirling formula whereas UKF is based on Taylor series approximation for the nonlinear function. It is interesting to compare the DDF with EKF for the closely spaced multipaths. Figure (7) shows the timing epoch estimation of the first arriving path in a five user three path model whereas Figure (8) shows the tracking of the timing epoch of the first arriving path of the weaker user in a fifteen user two path model.. It clearly demonstrates that DDF outperforms the linearized EKF when the paths are closely spaced in a near far environment.

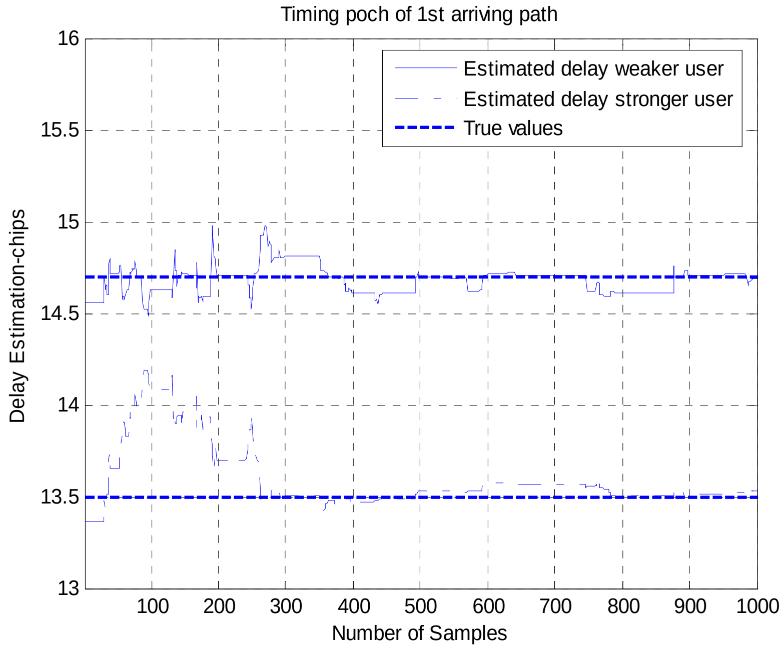


Figure 2. Timing epoch estimation for first arriving path with a five-user/ three-path channel model (with 1/2-chip path separation)

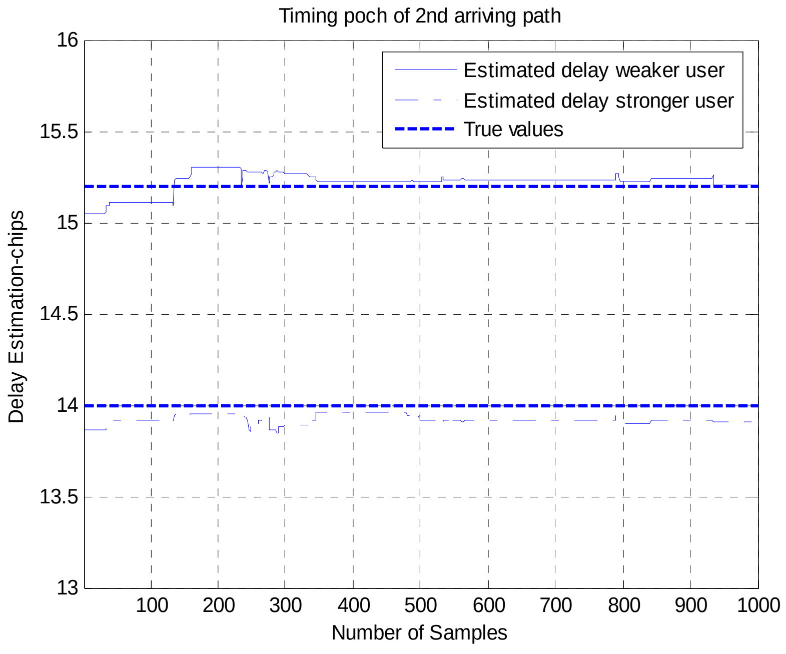


Figure 3. Timing epoch estimation for second arriving path with a five-user/ three-path channel model (with 1/2-chip path separation)

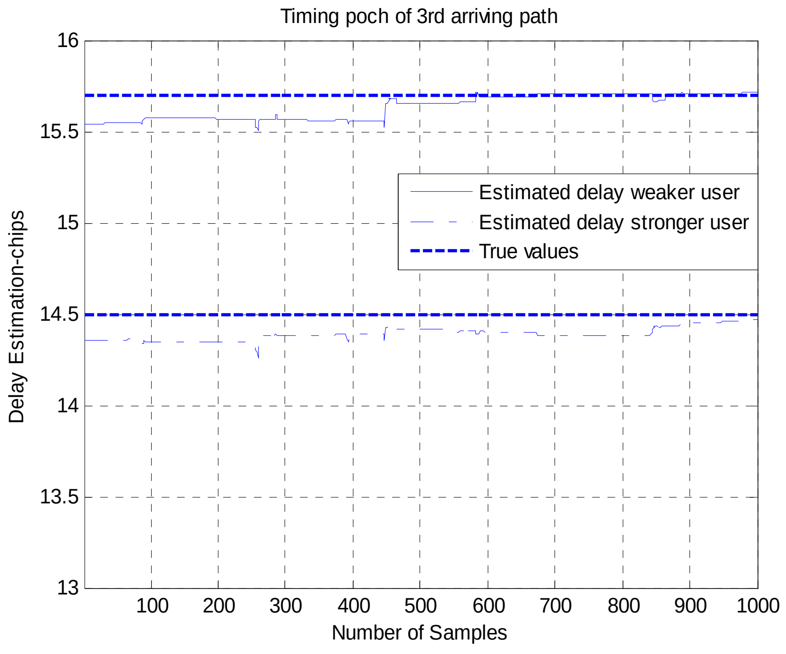


Figure 4. Timing epoch estimation for third arriving path with a five-user/ three-path channel model (with 1/2-chip path separation)

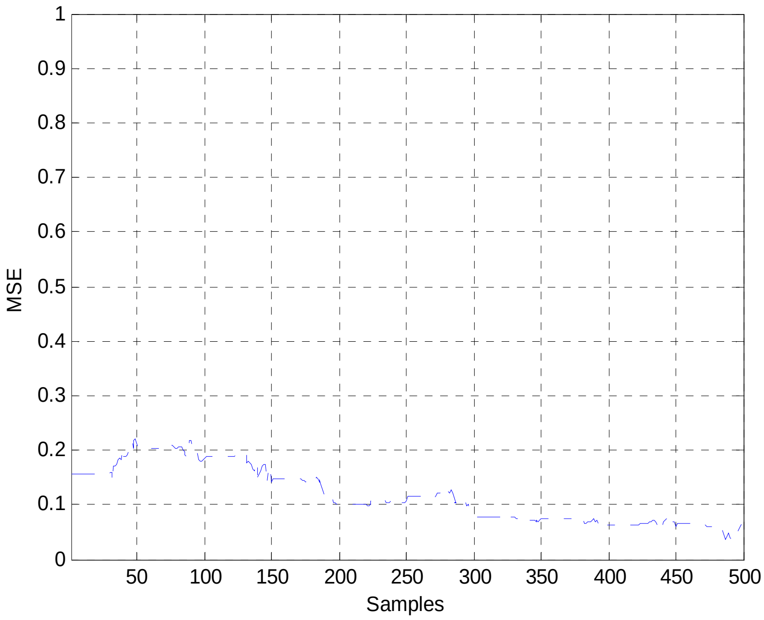


Figure 5. MSE of the channel coefficients for first arriving path with a ten-user/ two-path channel model

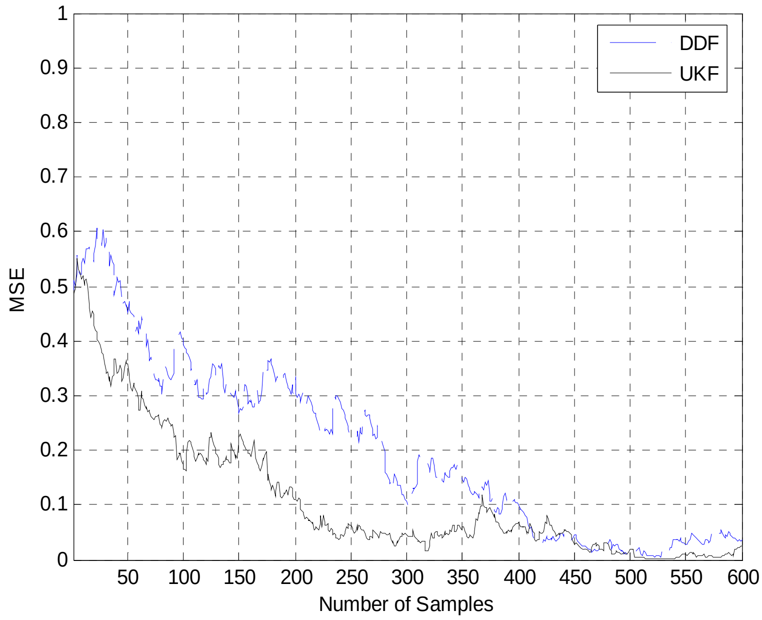


Figure 6. Comparison of the DDF with UKF in terms of MSE of the first arriving path in a ten user/two path

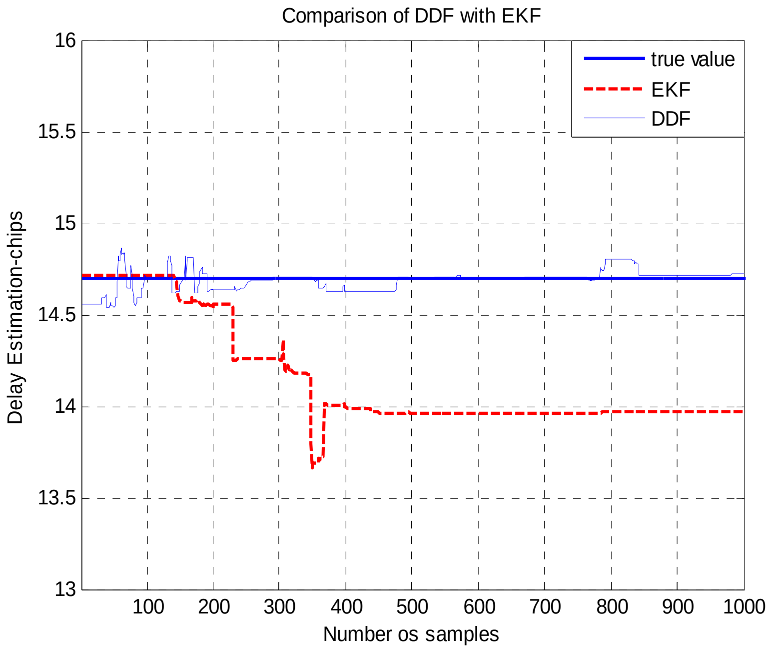


Figure 7. Comparison of the DDF with EKF in terms of timing epoch estimation for first arriving path with a five-user/ three-path channel model (with 1/2-chip path separation)

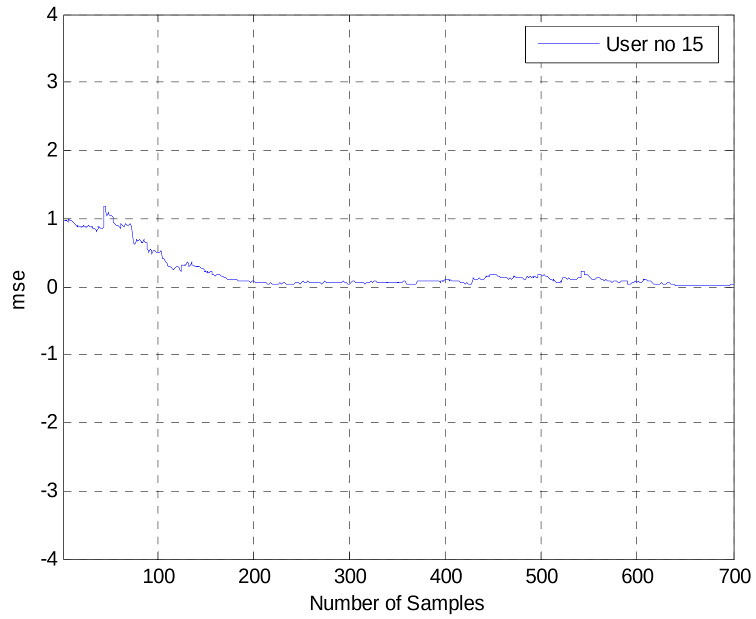


Figure 8. MSE of the timing epoch for first arriving path with a fifteen-user/ two-path channel model

# References

[1] H. F. Gilhousen et al., “On the capacity of a cellular CDMA system,” IEEE Trans. Veh. Tech., vol. 40, no.2, pp. 303–312, May 1991.
[2] R. L. Pickholtz, L. B. Milstein, and D. L. Schilling, “Spread spectrum for mobile communications,” IEEE Trans. Veh. Tech., vol. 40, Issue 2, pp.313–322, May 1991.
[3] R. A. Iltis and L. Mailaender, “An adaptive multiuser detector with joint amplitude and delay estimation,” IEEE J. Select. Areas Comm., vol. 12, no.5,pp. 774–785, June 1994.
[4] A. Radovic, “An iterative near–far resistant algorithm for joint parameter estimation in asynchronous CDMA systems,” in Proc. of 5th IEEE Int. Symp. on Personal, Indoor, Mobile Radio Comm., vol.1, pp. 199–203, Sep. 1994.
[5] E. G. Ström, S. Parkvall, S. L. Miller, and B. E. Ottersten, “Propagation delay estimation in asynchronous direct-sequence code-division multiple access systems,” IEEE Trans. Comm., vol. 44, no.1, pp. 84–93, Jan. 1996.
[6] S. E. Bensley and B. Aazhang, “Subspace-based channel estimation for code division multiple access communication systems,” IEEE Trans. Comm., vol. 44, no.8, pp. 1009–1020, Aug. 1996.
[7] E. G. Ström, S. Parkvall, S. L. Miller, and B. E. Ottersten, “DS-CDMA synchronization in time-varying fading channels,” IEEE J. Select. Areas Comm., vol. 14, no.8, pp. 1636–1642, Oct. 1996.
[8] M. Latva-aho and J. Lilleberg, “Delay trackers for multiuser CDMA receivers,” in Proc. IEEE Int. Conf. Universal Personal Comm., pp. 326–330, 1996.
[9] J. J. Caffery Jr. and G. L. Stüber, “Overview of radiolocation in CDMA cellular systems,” IEEE Comm. Mag., vol. 36, pp. 38–45, Apr. 1998.
[10] J. J. Caffery Jr. and G. L. Stüber, “Nonlinear Multiuser Parameter Estimation and Tracking in CDMA Systems,” IEEE Transactions on Comm., vol. 48, no. 12, pp.2053-2063, Dec. 2000,
[11] E. G. Strom and F. Malmsten, “A maximum likelihood approach for estimating DS-CDMA multipath fading channels,” IEEE J. Select Areas Comm., vol. 18, no. 1, pp. 132–140, Jan. 2000.
[12] S. Bhashyam and B. Aazhang, “Multiuser channel estimation and tracking for long-code CDMA systems,” IEEE Trans. Comm., vol. 50, Issue 7, pp. 1081–1090, Jul. 2002.
[13] K. J. Kim and R. A. Iltis, “Joint detection and channel estimation algorithms for QS-CDMA signals over time-varying channels,” IEEE Trans.Comm., vol. 50, no. 5, pp. 845–855, May 2002.
[14] Abdelmonaem Lakhzouri , Elena Simona Lohan , Ridha Hamila , Markku Renfors, “Extended Kalman Filter channel estimation for line-of-sight detection in WCDMA mobile positioning,” EURASIP Journal on Applied Signal Processing vol. 2003, Issue 13, Pages 1268-1278
[15] Ulrich Klee, Tobias Gehrig, John McDonough,“ Kalman Filters for Time Delay of Arrival-Based Source Localization,” EURASIP Journal on Applied Signal Processing, vol. 2006, issue 1,pp.167-167, Jan.2006.
[16] Liu Shunlan, Ma yong, Zhou Haiyun “Passive location by single observer with the Unscented Kalman Filter,” IEEE Int. Symp. on Microwave, Antenna, Propagation and EMC Tech. for Wireless Communications, vol. 2, pp. 1186- 1189, Aug. 2005.
[17] Wan, E. A., and van der Merwe, R., “Kalman Filtering and Neural Networks. Adaptive and Learning Systems for Signal Processing,” Communications, and Control. Wiley, 2001, ch. 7 - The Unscented Kalman Filter, pp. 221-280.
[18] Alfriend, K.T. and Lee, D-J., “Nonlinear Bayesian Filtering For Orbit Determination and Prediction,” 6th US Russian Space Surveillance Workshop, St. Petersburg, Russia, 22-26 August 2005.
[19] M. Nørgaard, N.K. Poulsen, O. Ravn: *Advances in Derivative-Free State Estimation for Nonlinear Systems*, Technical report IMM-REP-1998-15, Department of Mathematical Modelling, DTU, 1998