Challenge Journal of Structural Mechanics

# Vertical and lateral horizontal forces of walking area running pedestrians

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**ABSTRACT**

The authors have noticed the newest observations and few analysis of excitation mechanism. Therefore they prepared the series of vertical and lateral forces measurements due to steps of walking, running persons on horizontal plane, on inclined plane and on stairway. They suppose that the import knowledge of the forces frequencies of step or strides for different walking velocities is the most important for the further analysis and analysis of the mechanisms. The time histories of lateral forces of a pedestrian were registered and statistical analyzed: the authors received the lateral force dependence on the walking velocity, on stride length an on the weight of pedestrian. The new research step is focused on lateral horizontal forces.

**ARTICLE INFO**

*Article history:*
Received 12 February 2016
Accepted 2 March 2016

*Keywords:*
Stride strength
Stride frequency
Step frequency
Dynamic coefficient
Effects of striding velocity

## 1. Introduction

ITAM investigated a large number of footbridges and asserted that these constructions were very sensitive to pedestrians' movements as their Eigen frequencies were close to the step frequencies of pedestrians due to the light weight of the footbridges investigated.

The older research focused on the magnitude of the vertical component of the stride strength and its dependence on various speeds of movement and step lengths.

## 2. Dynamic Load

The dynamic load has at least three components – one vertical, two horizontal and in the case of a curved pedestrian movement, one torsion component (Harper, 1962).

### 2.1. Vertical dynamic load

At first we deal with the most important one, i.e. the vertical. The dynamic load is expressed by the dynamic coefficient for a single person

$$\delta_p = \frac{maxF_{dyn}+F_{stat}}{F_{stat}}, \tag{1}$$

and for a group of people

$$\delta_{pc} = \frac{max\sum F_{dyn}+\sum F_{stat}}{\sum F_{stat}}, \tag{2}$$

where

F denotes strength.

The sum $\sum$ runs over all the strengths at a given time t which is chosen in such a way that $\delta pc$ is maximal.

We have also denoted by *stat*, respectively *dyn* the static, respectively dynamic parts of strengths.

#### *2.1.1. A single pedestrian*

*a) A pedestrian on a horizontal plane*

The vertical component of the strength reaches its maximum if the center of mass of the pedestrian is at its maximum over the horizontal plane. The so-called saddle point between two peaks (see Fig. 1) is the occasion when the center of mass is at its minimum, i.e. the pedestrian's two legs pass each other. If the pedestrian walks fast or runs, the saddle point does not occur and the two peaks merge (see Fig. 2).

In the figure, time is on the horizontal axis and the dynamic coefficient $\delta_p$ is on the vertical axis. We denote the duration of stride by $t_k$ and duration of step by $t_s$. The speed of walking is 1.1 $ms^{-1}$. The intersection of the

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ISSN: 2149-8024 / DOI: http://dx.doi.org/10.20528/cjsmec.2016.03.006

pressure functions (of time) of the left and right leg is the moment when both legs are touching the plane.

According to our measurements, the borderline between walking and running lies somewhere between 1.4 and 1.8 ms$^{-1}$; according to (Footbridge, 2002) and (Bachmann and Ammann, 1987) between 1.5 and 2.35 ms$^{-1}$. It appears in the series of subsequent strides in the way that the end of one stride and the beginning of the next one merge at one point (in the graph).

We have measured the walking of ten men and two women and can confirm that every individual has its own characteristic "handwriting" of walking.

In Fig. 3 we see the dependence of the dynamic coefficient $\delta_p$ on the step frequency $f_s = 1/t_s$ and stride frequency $f_k = 1/t_k$. The datasets are interpolated by polynoms through their means (dashed line) and through their maxima (full line).

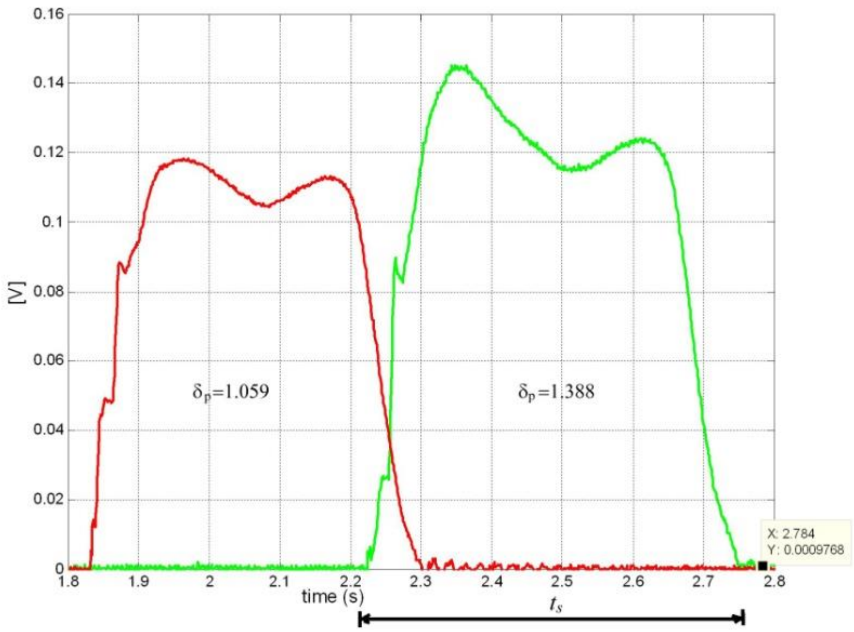


**Fig. 1.** Time histories of the left and right leg.

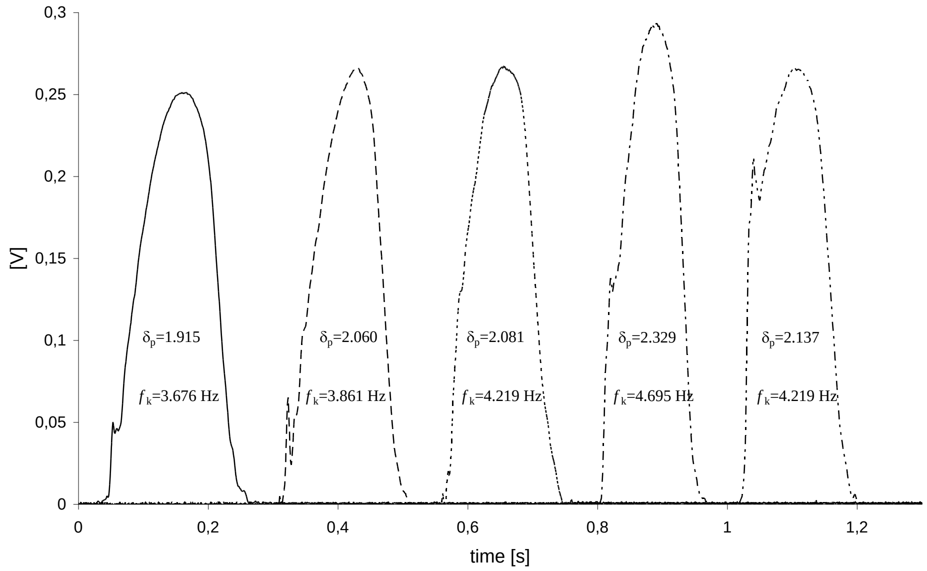


**Fig. 2.** An example of 5 strides when running (speed 3.4 ms$^{-1}$).

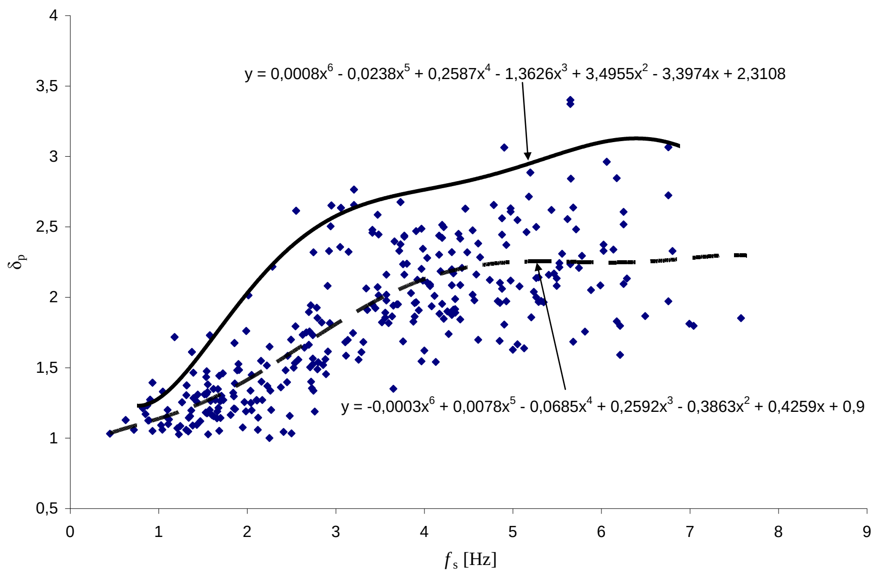


**Fig. 4.** The dynamic coefficient $\delta_p$ versus step frequency $f_s$.

In Fig. 4 we see the dependence of the step frequency $f_s$ and the stride length $l_k$ on the stride frequency $f_k$. From this dependence we can derive an approximate relation between $f_s$ and $f_k$

$$f_s \doteq 1{,}28 f_k \,. \tag{3}$$

Furthermore, the relation between the stride length $l_k$ and $f_k$ can be derived from that dependence.

In Fig. 5 we see the dependence of the dynamic coefficient $\delta_p$ and the striding velocity on stride frequency. It is apparent from this figure, which contains all the records of the individuals tested, that the aforementioned dependencies have a large variance; in spite of that it was possible to establish an approximate relation between the striding velocity $\dot{v}$ [m/s] and the step and stride frequencies

$$\dot{v} \doteq l_k * f_k \,, \tag{4a}$$

$$\dot{v} \doteq 0{,}8 f_k \doteq 0{,}6 f_s \,. \tag{4b}$$

The relations (3) and (4) do not capture any differences between men and women due to their approximate nature.

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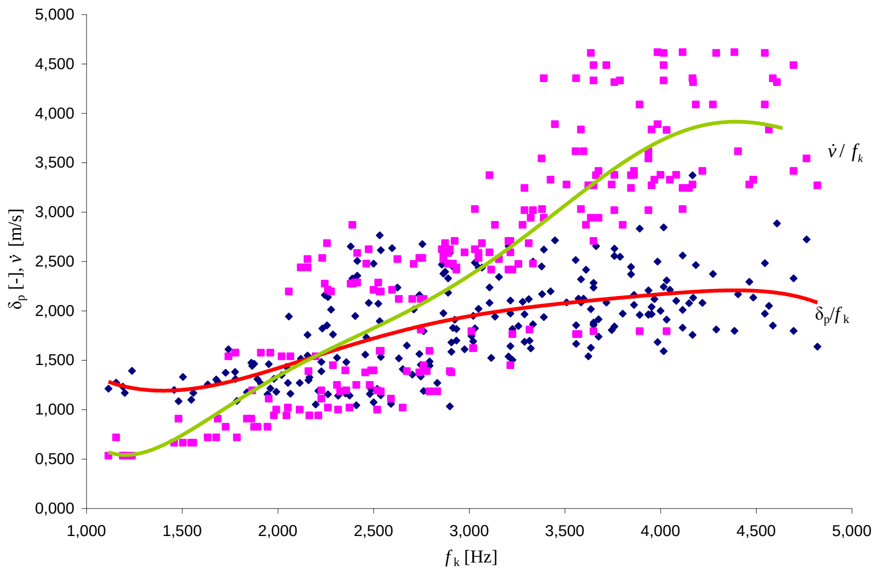


**Fig. 5.** The dependence of dynamic coefficient $\delta_p$ on stride frequency $f_k$ and striding velocity $\dot{v}$ on stride frequency $f_k$.

*b) A pedestrian on an inclined plane*

The next part of the experimental investigation was a measurement of the dynamic characteristics ($\delta_p$, $f_s$, $f_k$, $l_k$) for walking on an inclined plane. The selected slopes for experiments were (in percentage) 16%, 21.2%, and 33%.

In Fig. 6 we see the dependence of dynamic coefficient $\delta_p$ on the step frequency fs when the length of one step is 80 cm; it is obvious that if the slope is more than 16% the pedestrian is more careful, i.e., her dynamic load is smaller. Polynomial curves represent probable dependencies of the quantities involved.

Some of the results obtained:

- The vertical component of the strength as a function of time is roughly similar to the corresponding function in the case of the horizontal plane, i.e., they have two peaks for striding velocities $\dot{v} = 0.8 \div 1.5$ ms$^{-1}$ (the first one is usually higher than the second one when walking down and other way round when walking up), if the pedestrian walks quickly or runs they merge into one peak.
- "Decrease" of the dependence $\delta_p / f_s$ at $f_s \cong 3.5$ Hz occurs only if the slope is equal to 16%.
- The rate of frequencies $f_s$, and $f_k$, is more complicated – it differs from the formula (4b) which is valid for the horizontal plane only. The results of measurement are shown in table 1. It is apparent that the rate is higher when walking up. The slope and stride length have only minor effects.
- The rate of $\dot{v}$ and $\varnothing\, f_s$ is $\dot{v} / \varnothing\, f_s = 0.59 \div 0.86$ [m]; it was impossible to obtain more precise information from the measured values.

Note: The striding velocity of the pedestrian $\dot{v}$ is measured in direction of the inclined plane.

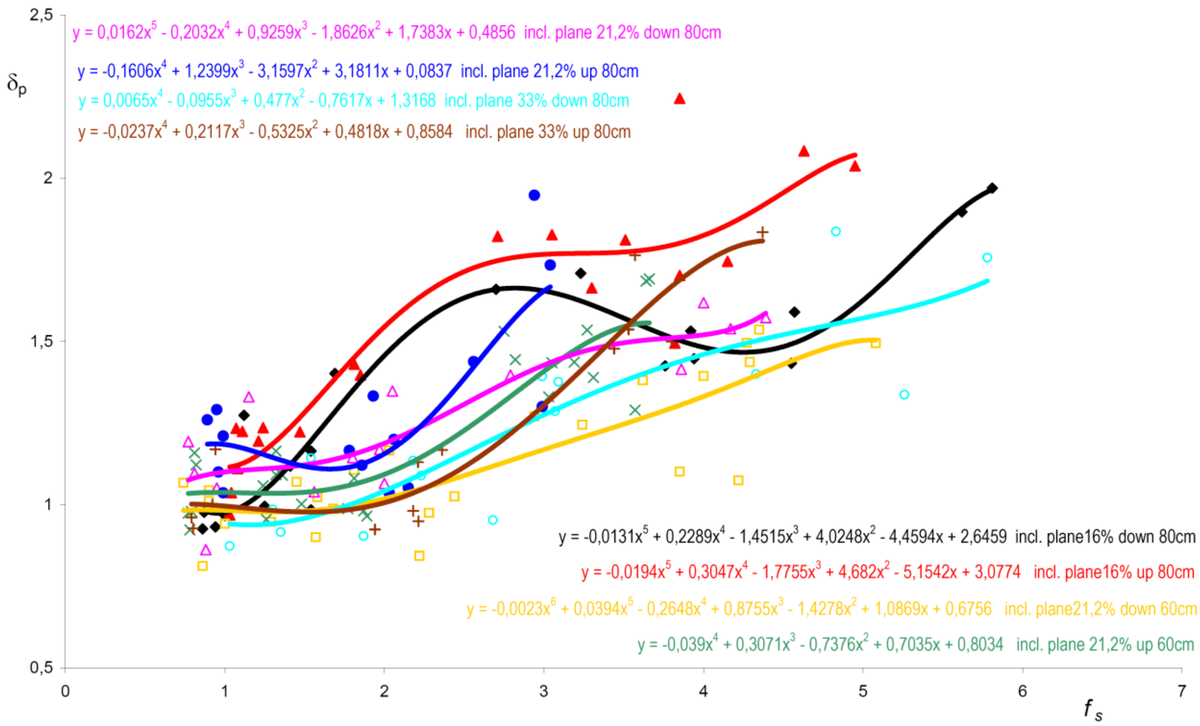


**Fig. 6.** The dynamic coefficient $\delta_p$ versus frequency $f_s$ – inclined plane.

**Table 1.** Rate $\varnothing\, f_s / \varnothing\, f_k$ as a function of stride length, slope of the ground, and walking direction.

| stride length 80 cm slope 16% | | stride length 60 cm slope 21.2% | | stride length 80 cm slope 21.2% | | stride length 80 cm slope 33% | |
|---|---|---|---|---|---|---|---|
| up | down | up | down | up | down | Up | down |
| 0.935 | 1.040 | 0.809 | 0.907 | 0.856 | 0.921 | 0.893 | 1.053 |

*c) A pedestrian on a staircase*

A staircase can be a part of a footbridge. This is why we have measured dynamic effects of a pedestrian on stairs when he or she moves up and down.

In Fig. 7 we depicted the dependence of the dynamic coefficient $\delta_p$ on the step frequency $f_s$.

From the measurement results it follows:

- men cause more dynamic effects when walking downstairs than upstairs
- women are more careful and when they walk downstairs they cause less dynamic effects than when they walk upstairs
- no effects of the height of heels (worn by women) were observed.

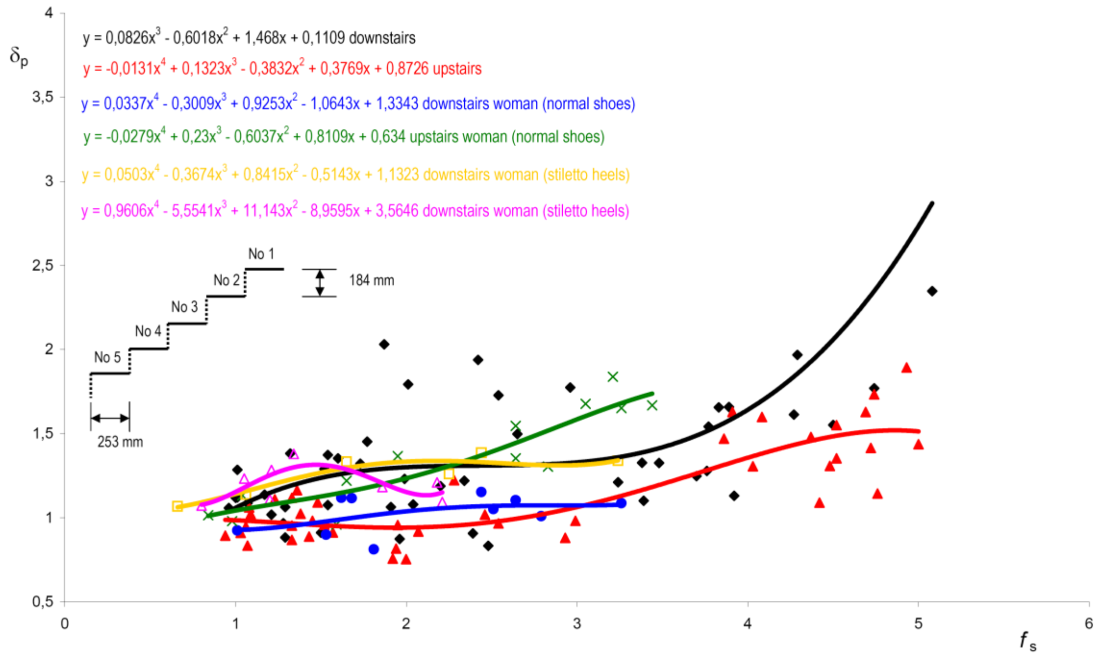


**Fig. 7.** The dynamic coefficient $\delta_p$ versus frequency $f_s$ .

The bar chart for slow walking has a similar shape as seen in Fig. 1 for footsteps on a horizontal plane. For fast walking, the two peaks in the bar chart merge.

Some of the results:

- the rate of frequencies $f_s$, and $f_k$ for striding velocities 0.25 ÷ 0.8 ms$^{-1}$ is

$$\varnothing\, f_s\, /\varnothing\, f_k = 0.77 \; to \; 1.19 \; . \tag{5}$$

- the rate of striding velocities (0.25 ÷ 0.8 ms$^{-1}$) to the average step frequency $f_s$ is $\dot{v}\, /\varnothing\, f_s = 0.24 \div 0.8$ m.

*2.1.2. A group of pedestrians on a horizontal plane*

We have investigated the vertical components of the strength exerted by a group of pedestrians because of our search for a theoretical expression of load and response of a footbridge in both synchronous and asynchronous cases. We have used five pedestrians walking side by side and in a variant setting three pedestrian side by side and two behind them (Fig. 11b showing the horizontal projection of sensors).

In both cases the distance between two outer sensors was 2 m, corresponding to the width of the footbridge between guardrails of 3 m. The “density” of pedestrians was 0.9 [person/m$^2$].

The Fig. 8a contains the record of vertical components of strengths exerted by five pedestrians when they walk with the velocity $\dot{v}$ =1.4 ms$^{-1}$ (the latter variant with 3 pedestrians in front and 2 behind them; Fig. 11b).

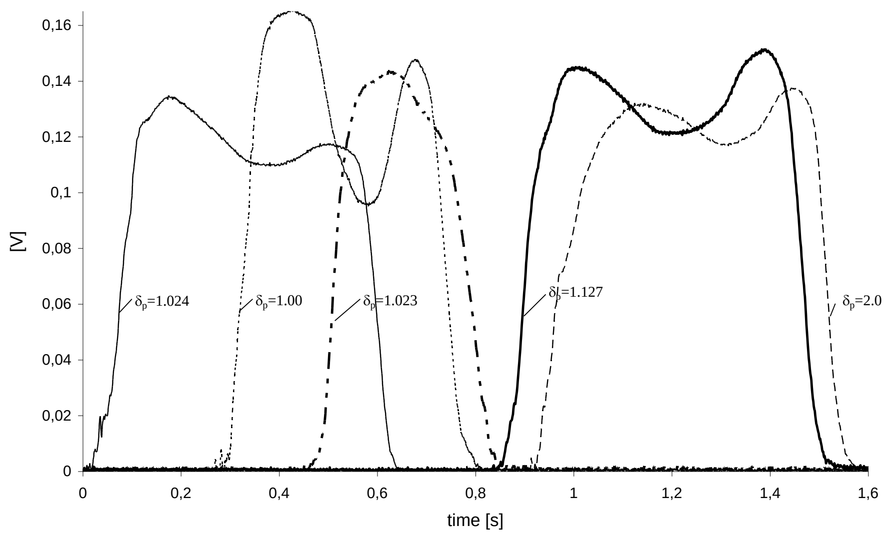


**Fig. 8a.** The vertical forces of five persons; $\dot{v} = 1.4\; m/s$ ; the sensors were located as in Fig. 11b.

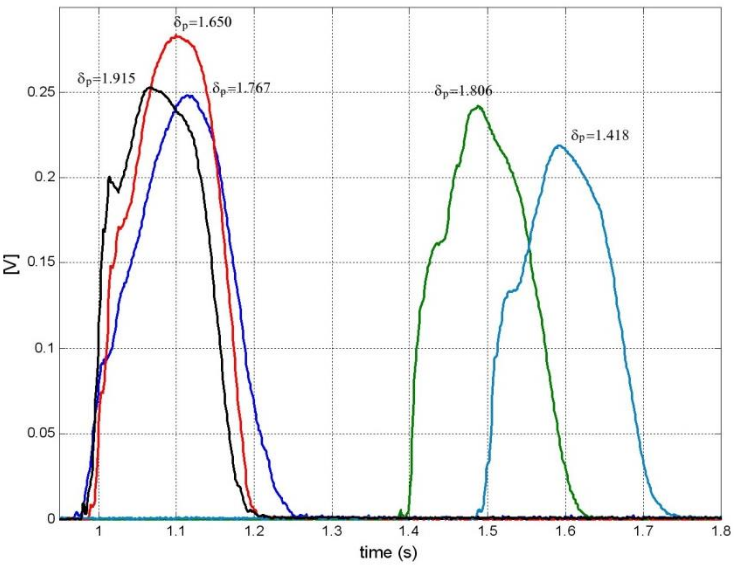


**Fig. 8b.** The vertical forces of five persons; $\dot{v} = 2.85\ m/s$ ; the sensors were located as in Fig. 11b.

The Fig. 8b shows the record of vertical components of strengths exerted by five pedestrians when they walk with the velocity $\dot{v} = 2.85$ ms-1 (the latter variant with 3 pedestrians in front and 2 behind them; Fig. 11b).

The Fig. 9a shows the record of vertical components of strengths exerted by five pedestrians when they walk with the velocity $\dot{v} = 1.5$ ms$^{-1}$, the Fig. 9b shows the situation with the velocity $\dot{v} = 3.2$ = 3.2 ms$^{-1}$. The setting of sensors was in agreement with the situation depicted in the Fig. 11c.

The dependencies of the dynamic coefficient $\delta pc$ on the step frequency fs for 2, 3, 4 and 5 people side by side, respectively is shown in the Fig. 10a.

A decrease of $\delta pc$ with the number of pedestrians reveals the Fig. 10b

Note: Due to the absence of a simultaneous stride in the situation with many pedestrians walking under normal conditions (in our case 5) the dynamic coefficient $\delta pc < \delta p$; the evidence of that is seen in Fig. 3 (mean values) and Fig. 10b.

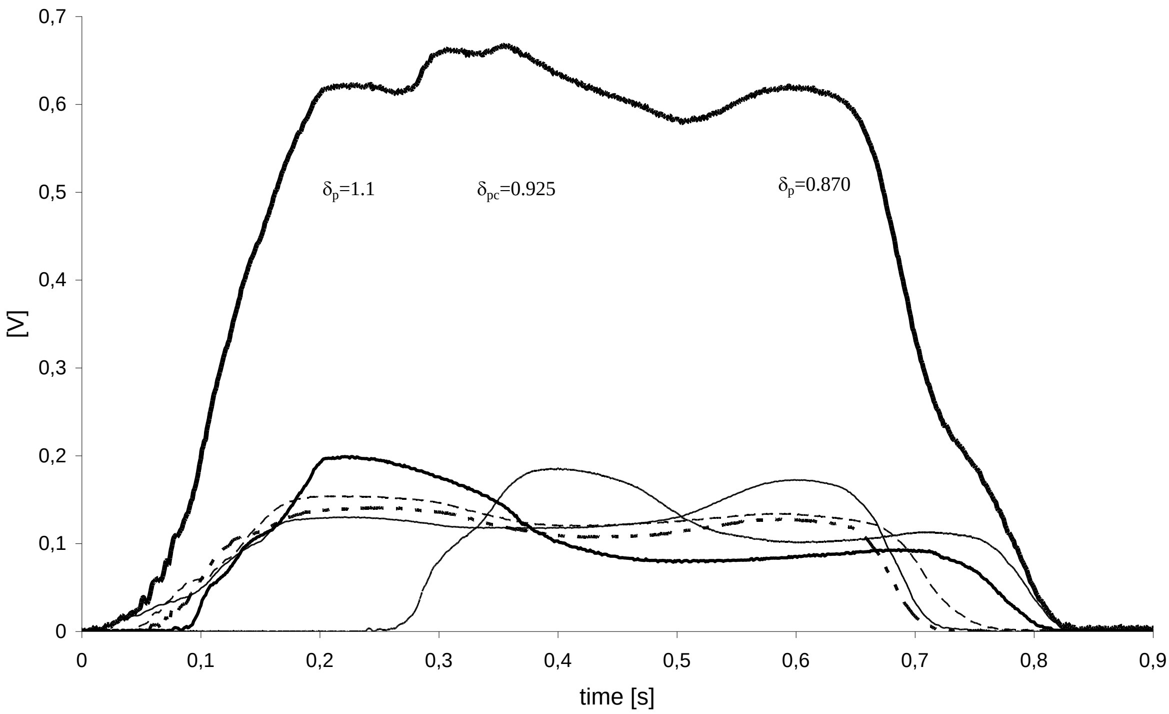


**Fig. 9a.** The vertical forces of five persons; $\dot{v} = 1.5\ m/s$ ; the sensors were located as in Fig. 11c. The symbol $\delta_{pc}$ = dynamic coefficient of all persons.

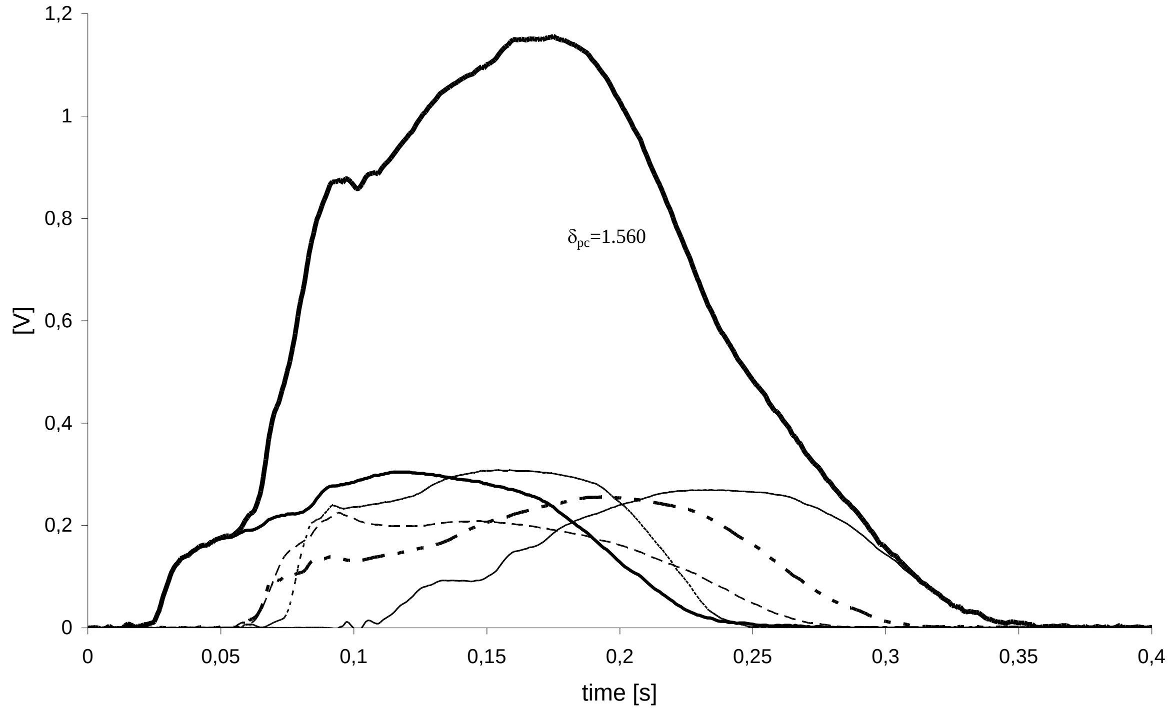


**Fig. 9b.** The same conditions as in Fig. 11c, but $\dot{v} = 3.2\ m/s$ .

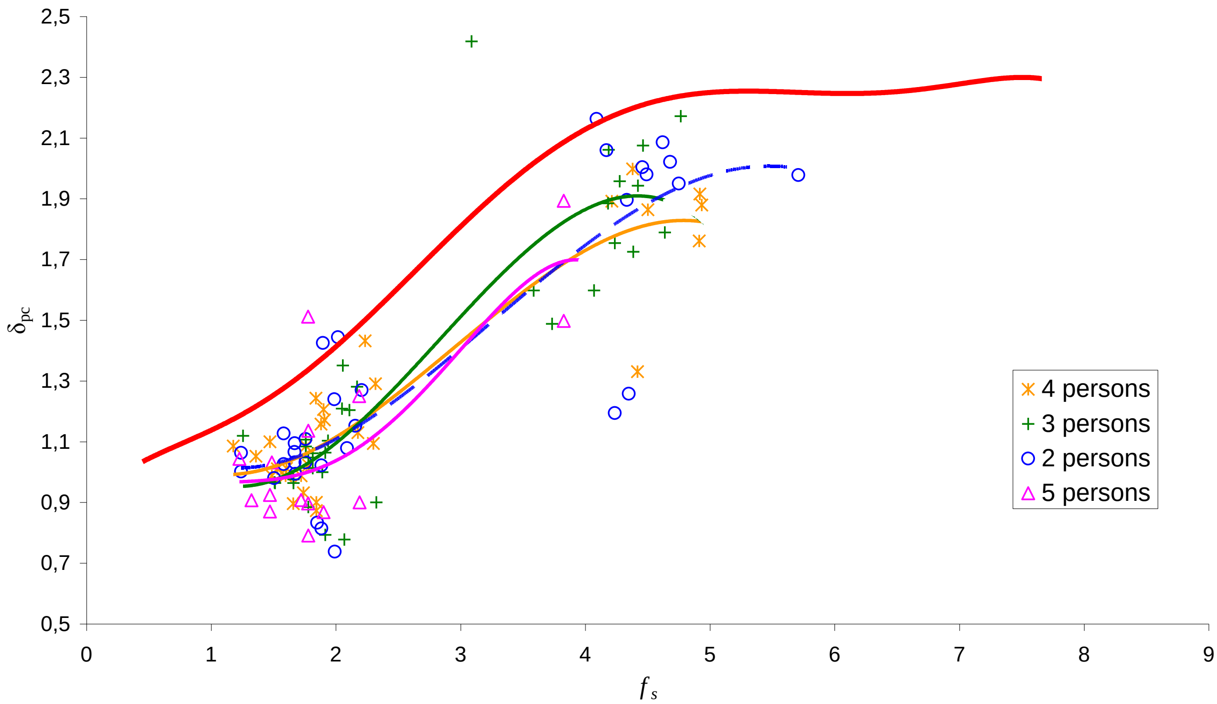


**Fig. 10a.** The dependencies of the dynamic coefficient $\delta_{pc}$ on the step frequency $f_s$ for 2, 3, 4 and 5 people side by side, respectively.

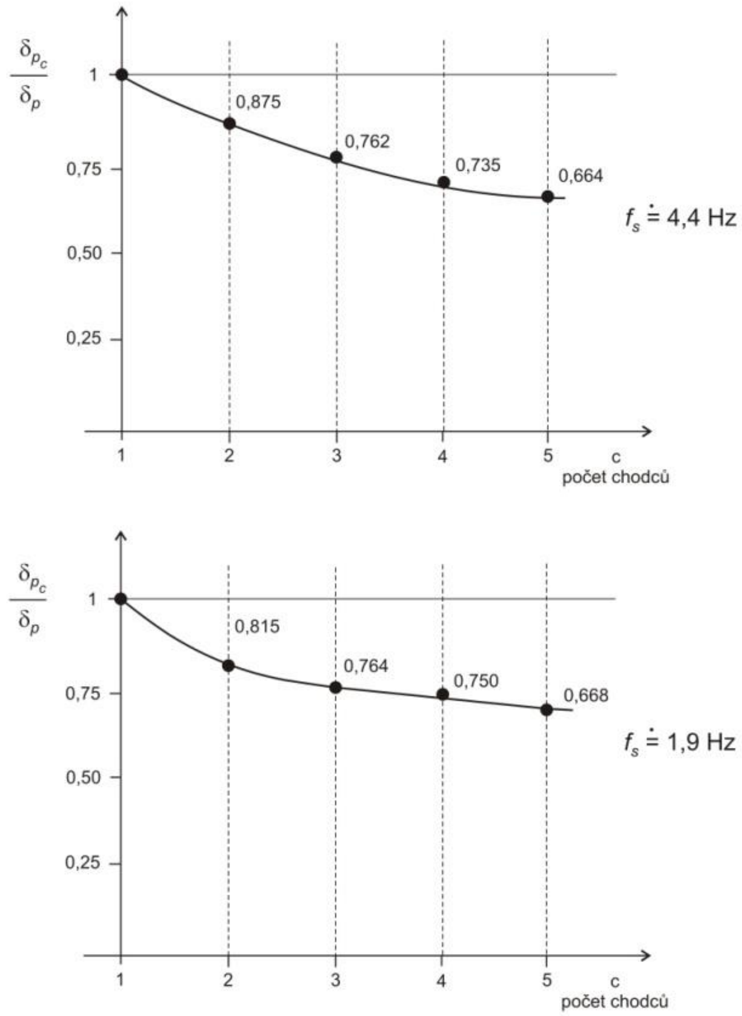


**Fig. 10b.** The dependence of the dynamic coefficient $\delta_{pc}$ on the number of pedestrians.

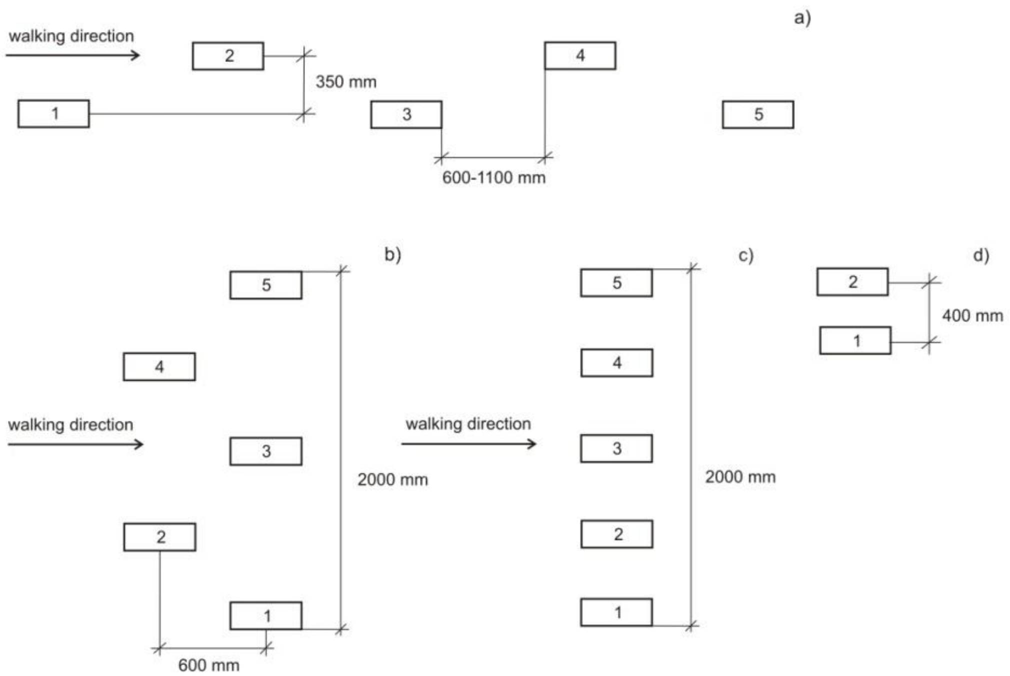


**Fig. 11.** The positions of sensors; (a) in the case of a single-person-walk; (b) and (c) in the case of five people walking; (d) for a vandal.

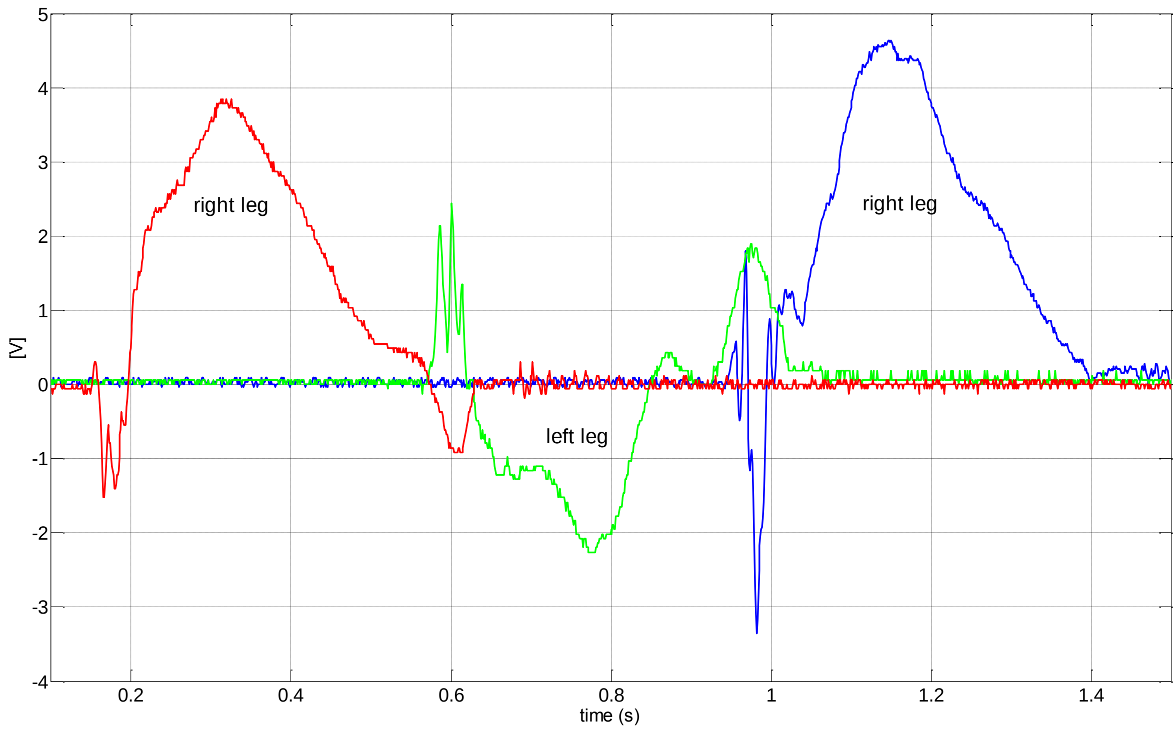


**Fig. 12.** Typical time histories of horizontal lateral loads.

*2.1.3. Sensors of the vertical stride strength*

The sensor is a steel plate, 5 mm thick, 395 mm long and 150 mm wide, supported on short sides as a simple beam. The deflection stress was measured by a strain gauges. The eigenfrequency of an unloaded sensor is 143 Hz and the logarithmic decrement of amplitude is $\vartheta \doteq 0.3$ ; also its eigenfrequency is sufficiently different from a step frequency. The deflection stress did not exceed 120 MPa. The positions of sensors are depicted in the Fig. 11.

### *2.2. Lateral horizontal load*

The lateral horizontal force depends on the weight of pedestrian on the speed of the walking and on the length of the stride.

On Fig. 12 are typical time histories of horizontal lateral loads (right, left, right leg); Harper described the shape with two peaks in (Footbridge, 2002).

### *2.3. Sensors of the lateral horizontal load*

The sensor is the series of three steel strips, 370 mm long, 30 mm wide, supported by boundary box. The defection stress of steel strips was measured by strain gauges. On the top of steel strips is the plate from soft material, which guarantees the participation of all strips. On the Fig. 13 is the relation between Volts and the lateral horizontal load. The sensor is shown on Fig. 14. On Fig. 15 is the position of sensors.

In Fig. 16 are results of our experiments. The weights of pedestrians was from 700 N up to 1125 N and the walking speed from 0,45 m/s up to 1,44 m/s (from 1,6 km/h up to 5,1 km/h)

## 3. Theory and Empirical Formula

### *3.1. The dynamic load in the vertical direction*

*3.1.1. Load exerted by a single pedestrian deterministically expressed*

There is a reliable formula for the dynamic increment (Footbridge, 2002) for $N = 20 \div 25$

$$F_{dyn} = c_z * N * \alpha * m_p * g\,, \tag{6}$$

where $c_z$ is the correlation coefficient (≈ 0.2) expressing the synchronization of steps with footbridge movements,

$N$ – the number of pedestrians

$\alpha$ – the dynamic coefficient of steps ($\alpha$ = 0.2÷0.5 for walking, α = 0.6 ÷ 1.4 for running)

$m_p$*$g$ = gravity force of a pedestrian.

*3.1.2. Load exerted by a continuous stream of pedestrians deterministically expressed*

Measurements of the vertical response have confirmed that the vertical components of the strengths have themselves two components in time: nonstationary and a stationary one. In case of the damping value $\xi =$ 0.015 (a common value) and the stride frequency $f_k = 2$ Hz, the maximal amplitude of the response occurs only after 60 steps (i.e., about 30 seconds after entering the footbridge), while 60% of the response occurs after 10 steps and 85% after 20 steps. Consequently, the nonstationary component is not important for long footbridges (Stoyanoff, 2002).

Let us assume that the continuous stream of pedestrians is formed by rows of 5 pedestrians (across the footbridge deck 3500 mm wide) which are $d = 0.6 \div 1$ m apart (it may be even more according to the step velocities). Let us moreover assume in agreement with Fig. 9a and 9b that pedestrians' strides in a row are simultaneous. Time shifts of six rows are expressed by a phase shift $\varphi$, which may be chosen for example as six multiples of 30 degrees between 0 and 180 degrees randomly attributed while the phase shifts of the first six-pack are denoted by $\varphi_1$ to $\varphi_6$, of the next one $\varphi_7$ to $\varphi_{12}$ etc.; the next rows of pedestrians follow till they fill up the whole footbridge deck. A scheme of the loading by six rows of pedestrians in time is drawn in Fig. 17; for better comprehension we have used an axonometric projection and time functions are plotted in coordinate systems with time axes perpendicular to the axis of the footbridge – we have limited ourselves to three such functions, only.

The vertical component of the strengths can be computed as

$$F_i(t) = F_i * \delta_{pc}|sin(\omega_i t + \varphi_i)| \ , \qquad (7)$$

where $\omega_i$ is an angular step frequency of the $i^{th}$ row of pedestrians.

Eq. (7) means that we deal with a standing system of varying loads instead of a continuous moving stream of pedestrians.

Then solving the response of a footbridge is a matter of routine.

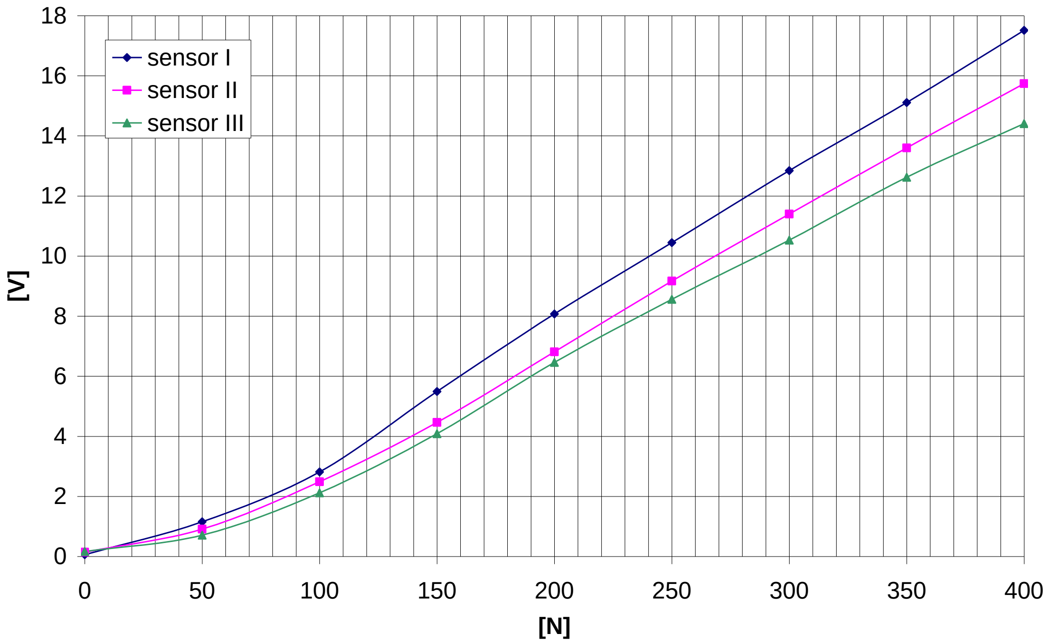


**Fig. 13.** Relation between Volts and the lateral horizontal load.


**Fig. 14.** The sensor.

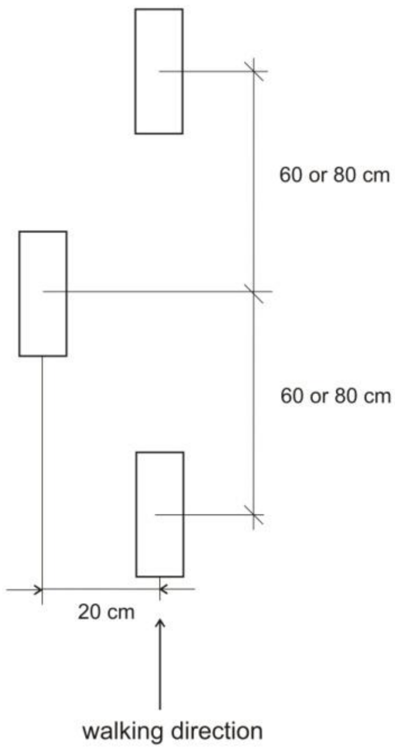


**Fig. 15.** Sensors positions for lateral horizontal loads.

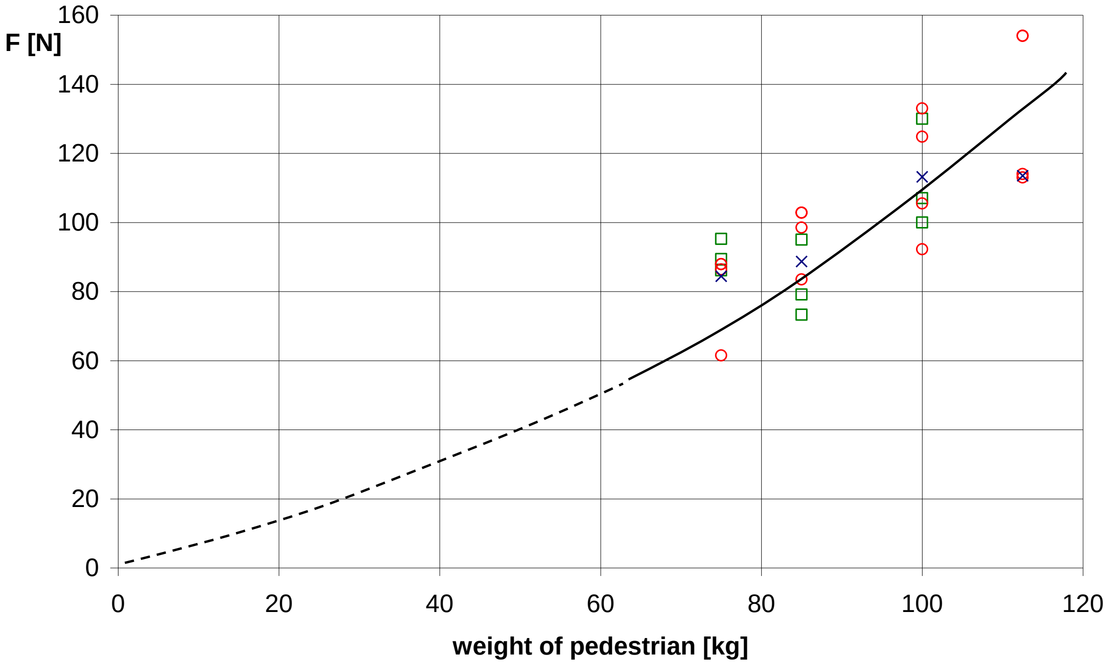


**Fig. 16.** Relation between weight of pedestrian and lateral horizontal load.
○ – *stride length 60cm,* □ – *stride length 80cm,* × – *the mean value*

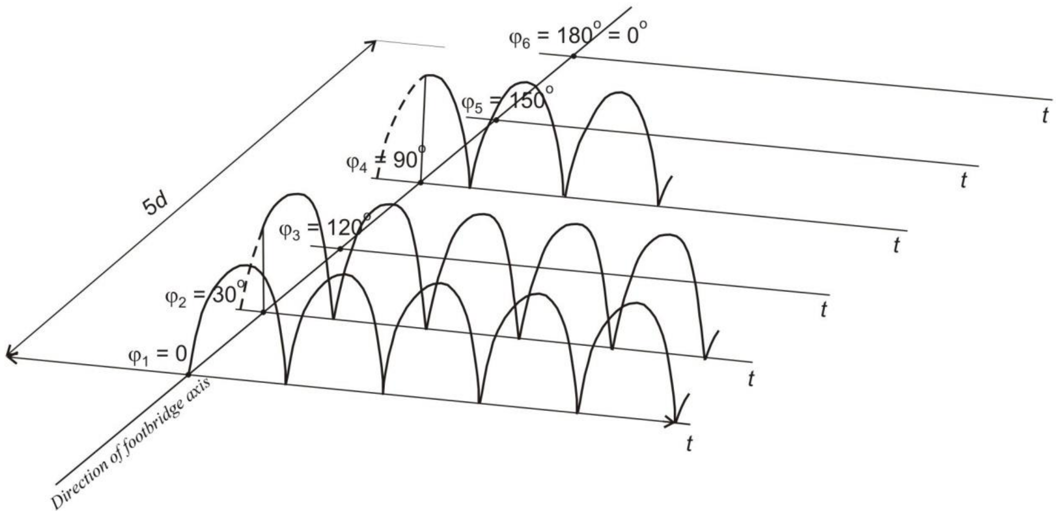


**Fig. 17.** Six rows of pedestrians.

*3.1.3. Vandals*

Footbridges, due to their small bending resistance in the vertical direction, tempt vandals to cause them to vibrate abnormally. The region of the lowest bending eigenfrequencies contributes to it, as they can be easily achieved by knee bends. Nevertheless, there is no danger of vibrations caused by a larger number of people, since it has been tested that they could not keep their knee bends in phase; but if there are just three to five of them they can succeed in coordinating their movements so that the load causing its response can exceed the acceptable vibration rates of other pedestrians; in exceptional cases the construction can be damaged.

Tests with sensors, described in section 2.3, have been conducted in the laboratory of the ITAM; loads have been represented by one test person (a subtle vandal, respectively a heavy one) who repeated knee bends in the frequency range 0.6 ÷ 4.5 Hz. In the case of frequencies under 0.6 Hz the dynamic coefficient of a vandal $\delta_v$ is small and the frequencies over 4.5 Hz cannot be achieved by human knee bends.

The dependence of the dynamic coefficient of a vandal $\delta_v$ on the knee bending frequency $f_v$ with an idealized time function $F_{vandal}(t)$ is plotted in the Fig. 18.

Assuming that the movement connected with a knee bend is very close to a harmonic movement, the load in a suitably chosen spot on the footbridge can be computed as

$$F_{vandal}(t) = F_{stat} * \delta_v * sin(2\pi f_v) * t \ , \qquad (8)$$

if $f_v$ is substituted with a bending frequency of the footbridge, e.g. $f_{(1)}$.

### *3.2. The dynamic load in the horizontal lateral direction*

Stoyanoff (2002) gave the formula for the load

$$F(t) = c_R * N * \alpha * w_P * cos\,\Omega t\ *, \qquad (9)$$

where
- $c_R$ (correlation coefficient ≈ 1)
- $N$ number of pedestrians ( 20 ⍰ 25 persons)
- $\propto$ dynamic coefficient (0,125)
- $w_P$ weight of the pedestrian
- $\Omega$ dominant walking circular frequency (commonly f = 1 Hz)

According to Matsumoto (Footbridge, 2002) the force per unit length $f_P(x,t)$ can be expressed as

$$f_P(x,t) = \frac{\sqrt{N}*\alpha*w_p}{L} * cos\,\Omega t\ , \qquad (10)$$

where
- $\propto$ = 0.04 (the footbridge without motion)
- L = the footbridge length
- $\Omega$ dominant walking circular frequency

## 4. Conclusions

The important results of measurements done in the ITAM laboratory and on the footbridges of various supportive systems follow:
- Mutual relations among the stride frequency, step frequency, step length, dynamic coefficient and the striding velocity depend on individual body characteristics of a pedestrian.
- The dynamic coefficient for a given pedestrian can be larger than for a group of pedestrians, if they do not move in a synchronous way.
- The obtained dynamic coefficients are of use for computations of load exerted by a single pedestrian, a group of pedestrians, a connected stream of pedestrians, and vandals, and for the computation of responses of footbridges with different supportive systems.
- Lateral horizontal forces of a pedestrian.

**Acknowledgements**

The support of the Grant Agency of the Czech Republic - grant No. 15-01035S and RVO 6837829 research plan are gratefully acknowledged.

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