DISCRETE MODELLING OF SURFACES OF EQUAL SLOPES BY MEANS OF NUMERICAL SEQUENCES

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ДИСКРЕТНЕ МОДЕЛЮВАННЯ ПОВЕРХОНЬ ВІДВАЛІВ ВИЗНАЧЕНОГООБ'ЄМУ ПОДВІЙНИМИ ЧИСЛОВИМИ ПОСЛІДОВНОСТЯМИ

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ABSTRACT

This paper presents an approach, the mathematical algorithms and software implementation of processes of discrete geometric modelling of open storage of bulk materials, such as surfaces of the equal slope, taking into account the geometrical and technological requirements for newly created objects and the physical and mechanical properties of bulk materials forming them. This approach and developed algorithms will not only make possible to technologically project the construction of open storage of bulk materials in time and provide a variety of geometrical requirements to objects, but also to make recommendations for the selection and organization of transporting machines work which are necessary for.

РЕЗЮМЕ

У роботі запропоновано підхід, математичні алгоритми та програмну реалізацію процесів дискретного геометричного моделювання за допомогою числових послідовностей відкритих складів сипких матеріалів як поверхонь однакового відкосу, з урахуванням геометричних, технологічних вимог до створюваних об'єктів та фізико-механічних властивостей сипких матеріалів, що їх утворюють. Такий підхід та розроблені алгоритми дозволять не тільки технологічно запроектувати зведення відкритих складів сипких матеріалів у часі та забезпечити цілу низку геометричних вимог, що висуваються до об'єктів, а й виробити рекомендації щодо підбору та організації роботи необхідних для цього транспортуючих машин.

INTRODUCTION

When designing the open storage of bulk materials in a defined area that is limited by curved contour of complex geometry, with the defined ceiling height of storage, the building of such structure models is actual as for the organization of lifting machines work so as for the optimization of the storage of given objects. Such models will make possible correctly and technologically design the construction of open storage in time and provide a series of geometric requirements imposed on objects *(Kupriiashkin A., 2015)*.

The main physical and mechanical property of bulk material, e.g. the ability of its elements to roll down the sloping surface is the angle of natural inclination, the angle that is formed between the plane of the base and the cone generatrix by free fall of bulk materials onto the horizontal plane *(Boitsov Y.U. and Kartalis N., 2013; Klein G., 1977; Vaisberh L., Demidov I. and Ivanov K., 2015)*.

This characteristic of the bulk material is basic when creating geometric models of surfaces of the equal slopes as the ratio of the base size, its geometry and height of storage, its volume and mass are directly dependent on this angle (see Fig. 1).

Fig. 1 - Photos of existing dumps of bulk materials

Geometrically, the problem of modelling of open storage of bulk materials as surfaces of equal slopes, based on the curvilinear contour of a defined area, with a certain height of storage leads to the construction: of the lower curvilinear contour, the upper contour that is equidistant *(Hozbenko V. and Lytkina E., 2010)* to the lower one and is the trajectory of motion of the load device.

So the development of new efficient algorithms for constructing models of surfaces of equal slopes considering besides technological requirements (selection, calculation and organization of transport vehicles), geometrical requirements (height, volume, mass etc.) and even the physical and mechanical properties of bulk materials which form them is quite challenging problem.

MATERIAL AND METHOD

Development of methods of discrete modelling by the scientists working, in the field of applied geometry made it possible to simplify the solution of a number of practical problems concerning the designing of complex spatial structures and objects.

One of the methods of discrete geometric modelling of equable structures, that has several advantages over all existing methods, is the formation of images using mathematical tools of numerical sequences *(Pustiulha V., 2006).* In works *(Pustiulha V. et al., 2011; Pustiulha V. et al., 20112014)* efficient algorithms of discrete modelling of storage surfaces of the equal slope using one-dimensional numerical sequences, namely discrete models of closed curves with the defined geometric properties, models which are equidistant to these curves, ensuring the absence of irregular points are proposed. However, the development of algorithms for discrete modelling of curved surfaces of the equal slopes as integral geometric objects that are models of open storage of bulk materials with certain technological initial conditions remains rather challenging problem.

The process of designing an open storage of bulk materials is proposed to carry out by the following steps. First - the formation of a discrete model of the closed curvilinear contour with certain geometric properties, including: smoothness of molded object, passing through a series of basic, points of reference, providing a defined area of the modelled contour. Second - analysis of discrete analogues of curvature at points of discrete model, natural slopes angles of bulk material and consequently, determination of the parameters of equidistant that will serve as the trajectory for lift transporting machines to form uniform surface slopes at a defined closed contour. Third - the formation of a coherent discrete model of the equal surface slopes counting the volume, weight and height of the simulated open storage of bulk material.

Theoretical studies are based on the guidelines of discrete geometry, mathematical tools of numerical sequences, differential geometry and theory of bulk materials. Modelling of surfaces of equal slopes with predetermined requirements was carried out by the software MathCAD.

RESULTS

Into the basis of algorithm for forming a discrete model of closed circuit of surface fundamentals slopes were set by the same system of one-dimensional numerical sequences of such type:

$$\begin{cases} x_n = \left(1-\dfrac{n}{N}\right)x_1 + \dfrac{n}{N}x_N + \dfrac{n}{N}\sum\limits_{v=1}^{N-1}\sum\limits_{s=1}^{v} kP_s^x - \sum\limits_{v=1}^{n-1}\sum\limits_{s=1}^{v} kP_s^x; \\ y_n = \left(1-\dfrac{n}{N}\right)y_1 + \dfrac{n}{N}y_N + \dfrac{n}{N}\sum\limits_{v=1}^{N-1}\sum\limits_{s=1}^{v} kP_s^y - \sum\limits_{v=1}^{n-1}\sum\limits_{s=1}^{v} kP_s^y, \end{cases} \quad (1)$$

where:

x_1, x_N, y_1, y_N are the border restrictions;

N - serial number of the circuit unit;

kP_S^x, kP_S^y - components of functionally distributed load in the units, parameters of which provide the number of restrictions to the geometry of the model, including the defined area of a closed contour, like this:

$$S_k = \frac{1}{2}\cdot\left|\sum_{n=1}^{N-1}\left(x_n \cdot y_{n+1} - x_{n+1}\cdot y_n\right)\right| \quad (2)$$

where:

x_n, y_n are the values of the coordinate points of the components of a discrete model of a closed curve;

n - series number of the unit;

N -number of the end unit, $x_n = x_1, y_n = y_1$;

S_k- surface area.

According to the presentation of discrete model of the lower base of surface of the equal slope, the upper base is formed as the trajectory of movement for the lifting device of bulk materials. Omitting the details which are shown in *(Pustiulha V. et al., 2014)*, the system of formulas of equidistant curve (see Fig. 2), that is a discrete model of the upper base of a searched surface is represented as:

$$\begin{cases} xe_n = x_n \pm \dfrac{le \cdot dy_n}{\sqrt{(dx_n)^2 + (dy_n)^2}}; \\ ye_n = y_n \pm \dfrac{le \cdot dx_n}{\sqrt{(dx_n)^2 + (dy_n)^2}}, \end{cases} \tag{3}$$

where *le* is parameter of distance between the points of the base (1) and equidistant (3) curves that are determined from geometrical and technological requirements.

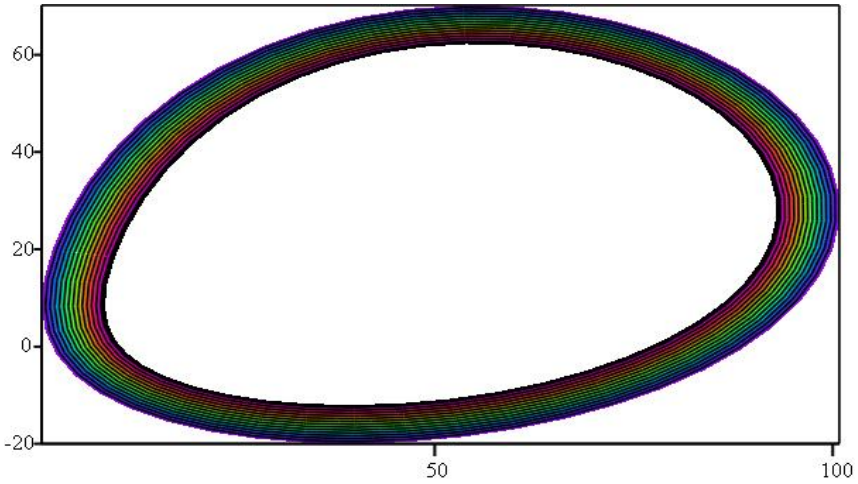

Fig. 2 - Discrete model of set of equidistant curves

At the third stage, namely formation of an integral discrete model of the surface of equal slopes, considering the research results *(Pustiulha V. et al., 2007; Pustiulha V. et al., 2009; Pustiulha V. et al., 2013)*, in parametric form can be represented as follows.

Using supportive contours x_n, xe_n with double numerical sequence $X_{n,k}$ we build the cylindroid like this:

$$X_{n,k} = \left(\left(1-\frac{n}{N}\right)x_1 + \frac{n}{N}x_N + \frac{n}{N}\sum_{v=1}^{N-1}\sum_{s=1}^{v} kP_s^x - \sum_{v=1}^{n-1}\sum_{s=1}^{v} kP_s^x\right)\frac{k}{N} + \left(x_n \pm \frac{le \cdot dy_n}{\sqrt{(dx_n)^2 + (dy_n)^2}}\right)\left(1-\frac{k}{N}\right) \tag{4}$$

Discrete model of this cylindroid is shown in Fig. 3.

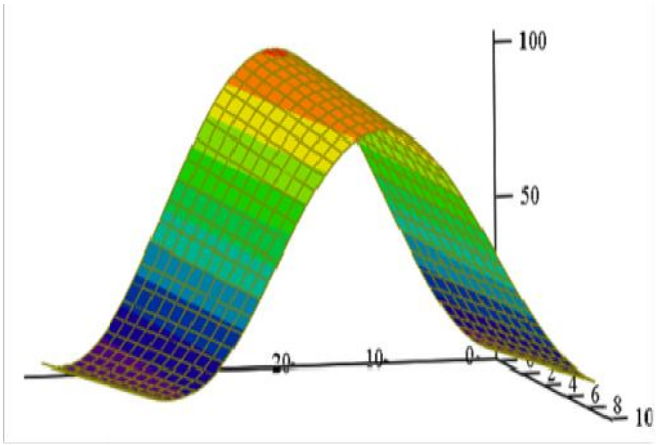

Fig. 3 -Discrete model of surface of equal slope by the component of X

Using supportive contours y_n ye_n for the component of Y we build the second cylindroid $Y_{n,k}$ like this:

$$Y_{n,k} = \left(\left(1-\frac{n}{N}\right)y_1 + \frac{n}{N}y_N + \frac{n}{N}\sum_{v=1}^{N-1}\sum_{s=1}^{v} kP_s^y - \sum_{v=1}^{n-1}\sum_{s=1}^{v} kP_s^y\right)\frac{k}{N} + \left(y_n \pm \frac{le\cdot dx_n}{\sqrt{(dx_n)^2+(dy_n)^2}}\right)\left(1-\frac{k}{N}\right) \quad (5)$$

Discrete model of this cylindroid is shown in Fig. 4.

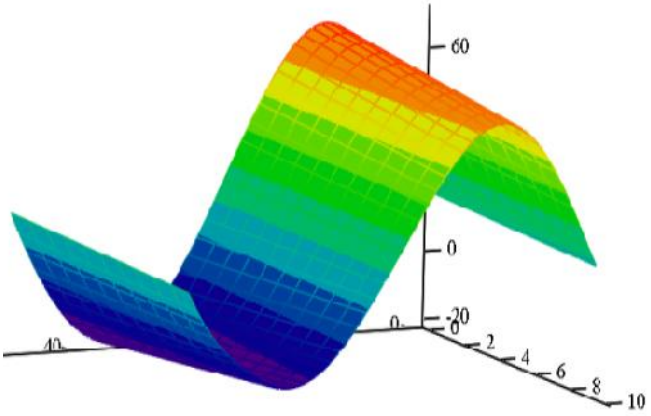

Fig. 4 -Discrete model ofsurfaceofequalslopebythecomponentof Y

The third component Z is given by a double numerical sequence $Z_{n,k}$ in the form:

$$Z_{n,k} = G\cdot\frac{k}{N} + H\cdot\left(1-\frac{k}{N}\right) \quad (6)$$

where G is parameter of the base of a discrete model of the surface and H is model height.

Then the discrete model of the surface of equal slope can be represented by a system of three double numerical sequences (4), (5), (6), a visual representation of which is shown in Fig. 5.

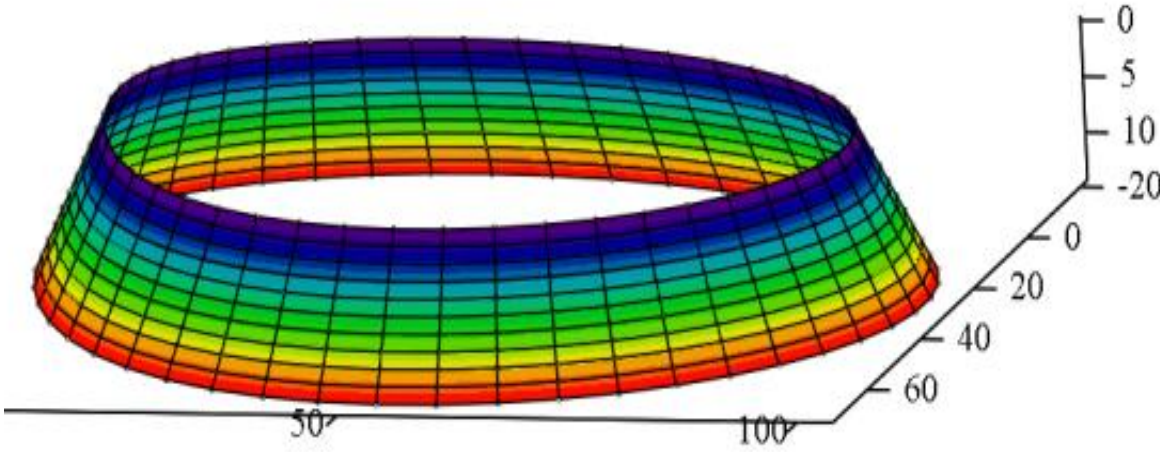

Fig. 5 - Generalized discrete surface model

Each of the generators of the represented discrete model of surface can be interpreted as the contact line of formative cone of natural slope of bulk material at a certain time interval. So, the generalized discrete model of an open storage of bulk material with the defined geometric properties, which are taken into account and technological requirements of its construction is shown in Fig.6.

An important characteristic of a discrete model of open-storage for bulk materials is its volume. It is proposed to make the calculation of storage volume as follows. Suppose V_{zc} is a compartment volume of the outer direct cylinder with the base that is described by the system of numerical sequences x_n, y_n (1).

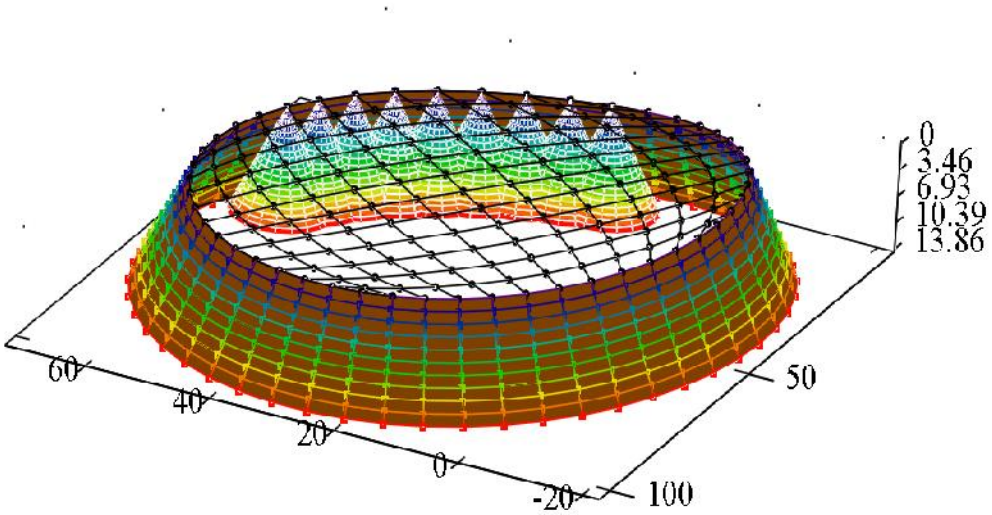

Fig. 6 - Modelling of stepwise process formation of open storage for bulk materials

$$V_{zc} = S_k \cdot H = \frac{1}{2}\left|\sum_{n=1}^{N-1}\left(x_n \cdot y_{n+1} - x_{n+1} \cdot y_n\right)\right| \cdot H \tag{7}$$

where S_k is the area of discrete model of closed curve of the supportive contour and H is height of the cylinder.

Volume of the inner direct cylinder with the base that is described by the system of numerical sequences of xe_n, y_{en} (3) is defined similarly.

$$V_{vc} = S_{ke} \cdot H = \frac{1}{2}\left|\sum_{n=1}^{N-1}\left(xe_n \cdot ye_{n+1} - xe_{n+1} \cdot ye_n\right)\right| \cdot H \tag{8}$$

where S_{ke} is the area of discrete model of equidistant.

So, the full volume of discrete model of open storage for bulk materials can be obtained from:

$$V_s = \frac{V_{zc} + V_{vc}}{2} = \frac{1}{4} \cdot H \cdot \left|\sum_{n=1}^{N-1}\left(xe_n \cdot ye_{n+1} - xe_{n+1} \cdot ye_n\right)\right| + \left|\sum_{n=1}^{N-1}\left(x_n \cdot y_{n+1} - x_{n+1} \cdot y_n\right)\right|. \tag{9}$$

Knowing the bulk density of a certain bulk material γ_0 the total mass of stored bulk material can be determined.

$$m_s = V_s \cdot \gamma_0. \tag{10}$$

Technologically, the reproduction of the model might look like: working part of transporting machine *(Tsyz I. et al., 2015)* should discretely move on the calculated trajectory, the boundary of which is the mode of equidistant (upper base of surface of equal slopes) at a rate that is calculated depending on the given performance (m^3/h). Machines and volumes of basic cones (see Fig. 6) are part of the developed discrete models. In addition, taking into account eq.(9), we can determine the total time required to create an open storage for bulk material.

CONCLUSIONS

This paper reports an approach, mathematical algorithms and software implementation of processes of discrete geometric modelling using double numerical sequences of open storage of bulk materials as surfaces of equal slopes, considering not only technological and geometric requirements for construction of these objects but also physical and mechanical properties of bulk materials which form them.

The process of designing an open storage for bulk materials is proposed to be carried out in three stages. First - the formation of a discrete model of the closed contour with certain geometric properties, second - definition of parameters of equidistant that will serve as the trajectory of loading device for forming uniform surface slopes at a defined closed circuit and create an algorithm for its design, the third - the creation of a uniform discrete surface model with the same slopes calculating volume, mass, height of the modelled open storage of bulk materials and time to create it.

This approach and developed algorithms will not only technologically design the construction of open storage of bulk materials in time and provide a series of geometric requirements that apply to objects, but also make recommendations which concern the selection and organization of work of transporting machines which are necessary for.

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