DYNAMICS OF FLEXIBLE ELEMENTS OF DRIVE SYSTEMS WITH VARIABLE CONTACT POINT TO THE PULLEYS
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ДИНАМІКА ГУЧКИХ ЕЛЕМЕНТІВ ПРИВІДНИХ СИСТЕМ ІЗ ЗМІННОЮ ТОЧКОЮ ДОТИКУ ДО ШКІВІВ

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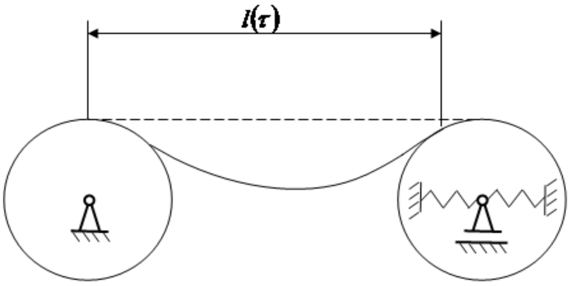
ABSTRACT
The developed technique makes possible to investigate the impact on oscillations of flexible elements of drive systems and transportation of nonlinear forces, the speed of longitudinal movement and perturbations of boundary conditions. Based on the obtained results it is proved that even for the linear analogue system the slowly time-dependent variable of the distance between the flexible elements (SE) contact points and the pulleys causes the change of the basic parameters of the waves. The limits of applying the wave theory of motion in the case of nonlinear oscillations of flexible elements of drive systems under slowly varying boundary conditions are expanded. The basic computations to analyze the main parameters of the dynamic process depending on motion speed of flexible element, tension force, and the ratio describing the motion principle of the contact point of flexible element and the pulley, are made.

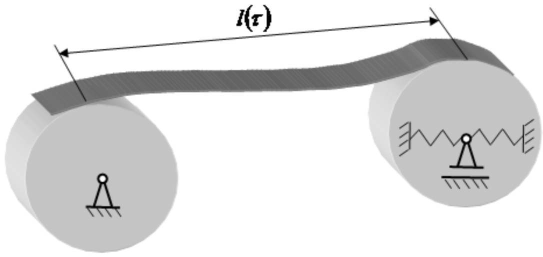
РЕЗЮМЕ
Розроблена методика дає можливість дослідити вплив на коливання гнучких елементів систем приводу та транспортування нелінійних сил, швидкості поздовжнього руху та збурень крайових умов. Отримані результати показують, що навіть для лінійного аналогу системи повільнозмінна в часі величина віддалі між точками контакту ГЕ та шківів спричиняє зміну основних параметрів хвиль. Розширено межі застосування хвильової теорії руху на випадок нелінійних коливань гнучких елементів привідних систем із повільно змінними крайовими умовами. Отримано базові співвідношення для описання визначальних параметрів динамічного процесу у залежності від швидкості руху гнучкого елементу, сили натягу, співвідношення, яке описує закон руху токи контакту гнучкого елементу та шківа.

INTRODUCTION
The wave theory of motion in recent decades has become a new development to describe the various processes and phenomena *(Goroshko O. O., 2012; Chen L. Q. et al, 2004; Chen L. Q., 2005; Dodd Ret al, 1988; Kharchenko Y.V., Sokil M.B., 2006; Mytropolskyi Y.A., 1995; Mytropolskyi Y. O., Sokil B.I., 1998; Mytropolskyi Y.A., 1998; Mytropolskyi Y.A., Lymarchenko O.S., 1998; S. Ponomareva W.T., van Horssen, 2004; Sokil M.B., 2012).* As for its application in the theory of oscillations, the research concerning the dynamics of nonlinear continuum should be mentioned primarily (*Chen L. Q. et al, 2004; Chen L. Q., 2005*; *Kharchenko Y.V., Sokil M.B., 2006; S. Ponomareva. W.T., van Horssen, 2004; Sokil M.B., 2012)*. The nonlinear continuum, which is widely used in engineering, includes the flexible elements (SE) of drive systems and transportation. The peculiarity of their operation is that they are characterized by longitudinal component of motion speed. Based on the wave theory of motion adapted to the dynamics of such systems, it is possible to explain many interesting phenomena that are not inherent in their simplified equivalents, i.e. SEs that do not account the longitudinal component of the motion speed and actually existing nonlinear power factors *(Mytropolskyi Y.A., 1998; Mytropolskyi Y.A., Ymarchenko O.S., 1998; Mytropolskyi Y.A., Moseenkov B. I., 1976).* In particular, even the SE motion speed constantly causes the change of the main parameters of this element's oscillations. At the same time, in many of the cited studies there was an assumption that the SE length *(Goroshko O. O., 2012)* or the distance between the contact points (for one-dimensional models) or correspondently the SE line of contact to the head and driven pulleys or drums is invariable (*Chen, L. Q., 2004; Chen L. Q., 2005*; *Kharchenko Y.V., Sokil M.B., 2006; S. Ponomareva. W.T., van Horssen, 2004; Sokil M.B., 2012*). This assumption, with reasonable accuracy, is true when the axes of

the head and driven pulleys or drums are stationary. This allows using classical boundary conditions in the appropriate mathematical models of the dynamics of the process. At the same time, while operating various kinds of mechanisms and systems in which SE carries out the transfer motion, the assumption requires clarification. This primarily concerns the mechanisms whose axes of head and driven drums (or one of them) are movable, i.e. spring loaded (see. Fig.1 and Fig.1.b).



a)



b)

Fig.1 - Driven gears with movable SE contact points to the pulley (a) or to the drum (b)

In this case, coordinates of the SE contact points and pulleys or drums and the distance between them are variable. The construction of mathematical models of the dynamics of SE and their solutions require refined formulation of the problem, a correct representation of boundary conditions and taking into account the variable tension force caused by the SE. The studies of this problem are carried out in such a context.

MATERIAL AND METHOD

It is known *(Chen L. Q., 2005, Sokil M.B., 2012)*, that the differential equation of SE oscillation of low bending stiffness, which is characterized by a constant component of speed V , can be represented as

$$u_{tt} + 2Vu_{xt} - \left((\alpha(\tau))^2 - V^2\right)u_{xx} = \varepsilon f\left(u, u_x, u_t, u_{xx}\right) \tag{1}$$

where:

ε - the small parameter;

$\alpha(\tau)$ - the slowly variable function, which is determined through a variable tension force $T(\tau)$ and the SE linear weight ρ : $(\alpha(\tau))^2 = T(\tau)/\rho$,);

$\varepsilon f(\tau, u, u_x, u_t, u_{xx})$ - the known analytic function that describes the nonlinear forces, and the small parameter specifies the small value of the SEs in comparison with the linear constituent of restoring force.

In (1), the function $u(t,\tau,x)$ determines the deviation from the SE equilibrium position with Euler coordinate x *(S. Ponomareva. W.T., van Horssen, 2004)* at an arbitrary point of time t .

Here, for simplicity, we will consider the case for which the SE contact point and right pulley is a slowly varying function of time $l = l(\tau)$, $\tau = \varepsilon t$ - "slow" time. In this case, the boundary conditions for equation (1) take the form

$$u(t,\tau,x)\big|_{x=0} = u(t,\tau,x)\big|_{x=l(\tau)} = 0 \tag{2}$$

The task is to determine the influence of parameters $V, \alpha(\tau), l(\tau)$ and functions $\varepsilon f(\tau, u, u_x, u_t, u_{xx})$ on the dynamics of SE.

The solution of the formulated problem is associated with solving the boundary problem (1), (2). The maximum value of non-linear forces is small as compared to the maximum value of term $(\alpha(\tau))^2 u_{xx}$ (see Restrictions on nonlinear forces). Thus, for its development, the general ideas of perturbation methods can be used *(J. Cole., 1972)*. The SE can be most effectively used to describe the analytically undisturbed movement, i.e. to find solution of equation under the boundary conditions, which are analogous to (2).

$$u_{tt}^0 + 2Vu_{xt}^0 - \left((\alpha(\tau))^2 - V^2\right)u_{xx}^0 = 0 \tag{3}$$

Even a relatively simplified mathematical model of the dynamic process of the researched object to build the solution does not allow the direct applying of the main backgrounds of classical Fourier and d'Alembert methods for partial differential equations *(Mytropolskyi Y.A., Moseenkov B. I.,1976).* Despite the "non classicality" of boundary conditions (2), single-dynamic process of unperturbed problem with sufficient degree of accuracy can be interpreted as an imposition of different length of waves but the same frequencies. Thus, the solution of the boundary problem (3), (2) is assumed to have the form

$$u^0(t,\tau,x)=a\Big(\cos\big(\kappa(\tau)x+\omega(\tau)t+\varphi_0\big)-\cos\big(\chi(\tau)x-\omega(\tau)t-\varphi_0\big)\Big) \tag{4}$$

In the dependence (4) $\omega(\tau)$ - frequency, a - amplitude of direct and reflected waves, $\kappa(\tau),\chi(\tau)$ - their wave numbers, and φ_0 - initial phase of the waves. The formal difference of the given description of the dynamic process in SE with "no classical" boundary conditions in comparison with "classical" ones is the dependence of wave numbers $\kappa(\tau),\chi(\tau)$ and frequencies $\omega(\tau)$ on slow time τ. Through formal computations, which are similar to the case of classical boundary conditions *(Kharchenko Y.V., Sokil M.B., 2006; Sokil M.B., 2012)*, we get the value of specified parameters

$$\kappa(\tau)=\frac{k\pi}{\alpha(\tau)l(\tau)}\big(\alpha(\tau)+V\big),\ \chi(\tau)=\frac{k\pi}{\alpha(\tau)l(\tau)}\big(\alpha(\tau)-V\big),\ \omega(\tau)=\frac{k\pi}{\alpha(\tau)l(\tau)}\big((\alpha(\tau))^2-V^2\big), \tag{5}$$

where the constant $k=1,2,...$ points the wave mode.

Note 1. Based on the linear boundary problem describing the unperturbed motion, its multi-frequent solution can be recorded without much difficulty.

The nonlinear forces and boundary conditions simultaneous effect give the solution of specified approximated task of function $u(t,\tau,x)$, which can be represented in the form

$$u(t,x)=a\Big[\cos\big(\kappa(\tau)x+\psi\big)-\cos\big(\chi(\tau)x-\psi\big)\Big]+\varepsilon U_1(\tau,a,\psi,x) \tag{6}$$

where:

$\psi=\omega(\tau)t+\varphi$, $U_1(\tau,\psi,x)$ - unknown analytic periodic ψ function satisfying boundary conditions arising from (2), i.e.,

$$U_1(\tau,a,\psi,x)_{|x=0}=U_1(\tau,a,\psi,x)_{|x=l(\tau)}=0 \tag{7}$$

In addition, the nonlinear forces cause the change of the dynamic process's amplitude and frequency. Laws of changing the given parameters, as in *(Mytropolskyi Y.A., Moseenkov B. I.,1976)*, will be set by differential equations

$$a_t=\varepsilon A_1(\tau,a)...\ \varphi_t=\varepsilon B_1(\tau,a). \tag{8}$$

Right parts of last ratio, that is, functions $A_1(\tau,a)$, $B_1(\tau,a)$ and $U_1(\tau,a,\psi,x)$ are arranged in such a way that the solution in the form of presentation (6) with the proposed degree of accuracy will satisfy the original boundary problem (1), (2). The above mentioned provides a dependence binding the desired function, that is, $A_1(\tau,a)$, $B_1(\tau,a)$ and $U_1(\tau,a,\psi,x)$

$$\begin{aligned}&\omega^2(\tau)\frac{\partial^2U_1}{\partial\psi^2}+2V\omega(\tau)\frac{\partial^2U_1}{\partial\psi\partial x}-((\alpha(\tau))^2-V^2)\frac{\partial^2U_1}{\partial x^2}=F_1(\tau,a,x,\psi)-a\frac{d\omega(\tau)}{d\tau}\Big(\sin\big(\kappa(\tau)x+\psi\big)+\sin\big(\chi(\tau)x-\psi\big)\Big)+\\&+2\Big\{A_1(\tau,a)\big(\omega(\tau)+\kappa(\tau)V\big)(\sin\big(\kappa(\tau)x+\psi\big)+\big(\omega(\tau)-\chi(\tau)V\big)\sin\big(\chi(\tau)x-\psi\big))+\\&+aB_1(\tau,a)(\big(\omega(\tau)+\kappa(\tau)V\big)\cos\big(\kappa x+\psi\big)-\big(\omega-\chi V\big)\cos\big(\chi(\tau)x-\psi\big))\Big\}\end{aligned} \tag{9}$$

where:

$F_1(\tau,a,x,\psi)$ corresponds to the function $f(\tau,u,u_t,u_x,u_{xx})$ provided that function $u(t,\tau,x)$ and its derivatives accept only the main meanings in the equations arising from (6).

After uncomplicated transformations right parts of the differential ratio (6) take the form

$$\begin{aligned}&A_1(\tau,a)\big(\omega(\tau)+\kappa(\tau)V\big)(\sin\big(\kappa(\tau)x+\psi\big)+\big(\omega(\tau)-\chi(\tau)V\big)\sin\big(\chi(\tau)x-\psi\big))+\\&+aB_1(\tau,a)(\big(\omega(\tau)+\kappa(\tau)V\big)\cos\big(\kappa x+\psi\big)-\big(\omega-\chi V\big)\cos\big(\chi(\tau)x-\psi\big))=\\&=\big(A_1(\tau,a)\cos\psi-aB_1(\tau,a)\sin\psi\big)\Big[\big(\omega(\tau)+\kappa(\tau)V\big)\sin\kappa(\tau)x+\big(\omega(\tau)-\chi(\tau)V\big)\sin\chi(\tau)x\Big]+\end{aligned} \tag{10}$$

$$+\left(A_1(\tau,a)\sin\psi + aB_1(\tau,a)\cos\psi\right)\left[\left(\omega(\tau)+\kappa(\tau)V\right)\cos(\kappa(\tau)x)-\left(\omega(\tau)-\chi(\tau)V\right)\cos(\chi(\tau)x)\right]$$

$$a\frac{d\omega(\tau)}{d\tau}\left(\sin\left(\kappa(\tau)x+\psi\right)+\sin\left(\chi(\tau)x-\psi\right)\right)=a\frac{d\omega(\tau)}{d\tau}\left(\left(\sin\kappa(\tau)x+\sin\chi(\tau)x\right)\cos\psi+\right.$$

$$\left.+\left(\cos\kappa(\tau)x-\cos\chi(\tau)x\right)\sin\psi\right).$$

For unambiguous definition the unknown functions $A_1(\tau,a)$ and $B_1(\tau,a)$ of differential equation (9) should be imposed on the function $U_1(\tau,a,\psi,x)$. The additional condition is stipulated: it cannot be one of the additions of proportional $\sin\psi$ and $\cos\psi$. The physical meaning of specified statements is as follows: the amplitude of the wave process coincides with the amplitude of its first mode. The above statements will come true in the following case

$$\int_0^{2\pi} U_1(\tau,a,\psi,x)\begin{Bmatrix}\sin\psi\\ \cos\psi\end{Bmatrix}d\psi = 0 . \tag{11}$$

The partial derivatives of specified function have similar properties. This allows deriving a system of linear algebraic equations and functions $A_1(\tau,a)$ and $B_1(\tau,a)$ from the differential equation (9)

$$\rho(\tau,x)A_1(\tau,a)+aq(\tau,x)B_1(a)=\frac{a}{2}\frac{d\omega}{d\tau}r(\tau,x)-\frac{\varepsilon}{2\pi}\int_0^{2\pi}F_1(\tau,a,\psi,x)\cos\psi d\psi ,$$

$$q(\tau,x)A_1(a)-a\rho(\tau,x)B_1(a)=\frac{a}{2}\frac{d\omega}{d\tau}p(\tau,x)-\frac{\varepsilon}{2\pi}\int_0^{2\pi}F_1(\tau,a,\psi,x)\sin\psi d\psi , \tag{12}$$

where

$$\rho(\tau,x)=\left(\omega(\tau)+\kappa(\tau)V\right)\sin\kappa(\tau)x+\left(\omega(\tau)-\chi(\tau)V\right)\sin\chi(\tau)x)),\ p(\tau,x)=\left(\cos\kappa(\tau)x-\cos\chi(\tau)x\right),$$

$$q(\tau,x)=\left(\omega(\tau)+\kappa(\tau)V\right)\cos\kappa(\tau)x-\left(\omega-\chi(\tau)V\right)\cos\chi(\tau)x,\ r(\tau,x)=\left(\sin\kappa(\tau)x+\sin\chi(\tau)x\right).$$

Note 2: We consider the case for which nonlinear system forces cause the change only in time of the basic parameters of the wave process ("short systems"). A more general case for which the determinative parameters of waves also depend on the linear variable (the case of "long" systems) can be another subject of research.

RESULTS

The conducted research allows to use the averaging out device *(Mytropolskyi Y.A., 1972)* using the variable x in relation to the system of differential equations (12). Due to obtained results, the functions describing the basic parameters of the wave process can be defined by the following formula:

$$A_1(\tau,a)=\frac{1}{2\pi l(\tau)\left[\left(\omega(\tau)+\kappa(\tau)V\right)^2+\left(\omega(\tau)-\chi(\tau)V\right)^2\right]}\int_0^{l(\tau)}\left\{\rho(\tau,x)\left[a\frac{\pi d\omega(\tau)}{d\tau}r(\tau,x)-\varepsilon\int_0^{2\pi}F_1(\tau,a,\psi,x)\cos\psi d\psi\right]\cos\psi d\psi-\right.$$

$$\left.-q(\tau,x)\left[a\frac{\pi d\omega(\tau)}{d\tau}p(\tau,x)-\varepsilon\int_0^{2\pi}F_1(\tau,a,\psi,x)\sin\psi\, d\psi\right\}\right]dx$$

$$B_1(\tau,a)=\frac{1}{2\pi a l(\tau)\left[\left(\omega(\tau)+\kappa(\tau)V\right)^2+\left(\omega(\tau)-\chi(\tau)V\right)^2\right]}\int_0^{l(\tau)}\left\{\rho(\tau,x)\left[a\frac{\pi d\omega(\tau)}{d\tau}p(\tau,x)-\varepsilon\int_0^{2\pi}F_1(\tau,a,\psi,x)\sin\psi d\psi\right]\cos\psi d\psi-\right.$$

$$\left.-q(\tau,x)\left[a\frac{\pi d\omega(\tau)}{d\tau}r(\tau,x)-\varepsilon\int_0^{2\pi}F_1(\tau,a,\psi,x)\sin\psi\, d\psi\right\}\right]dx. \tag{13}$$

A special case of specified dependencies at $V=0, l(\tau)\equiv l_0, \alpha(\tau)\equiv\alpha_0$ (α_0, l_0 - steel) are the known in literature results *(Mytropolskyi Y.A., Moseenkov B. I., 1976)* concerning nonlinear oscillations of flexible one-dimensional media with fixed ends. Thus, in the first approximation the dynamic process of SE with its slowly varying contact point of right end and the pulley is described by dependence (6) in which the parameters a and ψ are determined in accordance with ratio (8) and (13).

The first improved approximation describes the impact of nonlinear forces on the form of waves; so the function $U_1(\tau,a,\psi,x)$ should be defined. Taking into account the imposed conditions, it can be represented as

$$U_1(\tau,a,\psi,x)=\sum_m\sum_{n,n\neq 1}U_{1mn}(\tau,a)X_m(\tau,x)exp(in\psi) \quad (14)$$

where the system of functions $\{X_m(\tau,x)\}$ must be complete and materialize the boundary conditions (2). The system of functions $\{X_m(\tau,x)\}=\left\{\sin\frac{m\pi}{l(\tau)}x\right\}$ satisfies such conditions. In this case, the unknown coefficients $U_{1mn}(\tau,a)$ are linked by the system of linear algebraic equations

$$\left\{n^2\omega^2-((\alpha(\tau))^2-V^2)\left(\frac{s\pi}{l(\tau)}\right)^2\right\}U_{1sn}(\tau,a)-2V\omega(\tau)\sum_{m=1}^{j}\frac{2s}{s^2-m^2}ni\frac{m\pi}{l}U_{1mn}(\tau,a)=-F_{1sn}(\tau,a),\ m+s-\text{the odd} \quad (15)$$

where $F_{1sn}(\tau,a)=\dfrac{1}{2\pi l(\tau)}\int_0^{2\pi}\int_0^{l(\tau)}F_1(\tau,a,x,\psi)\exp(-in\psi)X_s(x)dxd\psi.,1\leq s\leq j.$

As a rule, the first modes of oscillation greatly influence the dynamic process. Therefore, a system of algebraic equations (15) is sufficiently limited by the first few terms of the expansion. In this case, to find its solution is not difficult. In particular, if we use only the first two terms of the expansion of a function $U_1(\tau,a,\psi,x)$ in a series of system functions $\{X_m(\tau,x)\}$, we obtain

$$U_{11n}(\tau,a)=-\frac{1}{\Delta}\left\{\left[n^2\omega^2-(\alpha^2-V^2)\left(\frac{2\pi}{l}\right)^2\right]F_{11n}(\tau,a)-\frac{8\pi ni}{3l}V\omega F_{12n}(\tau,a)\right\}$$

$$U_{12n}(\tau,a)=-\frac{1}{\Delta}\left\{\left[n^2\omega^2-(\alpha^2-V^2)\left(\frac{\pi}{l}\right)^2\right]F_{12n}(\tau,a)+\frac{8\pi ni}{3l}V\omega F_{11n}(\tau,a)\right\} \quad (16)$$

where $\Delta=\left[n^2\omega^2-(\alpha^2-V^2)\left(\frac{\pi}{l}\right)^2\right]\left[n^2\omega^2-(\alpha^2-V^2)\left(\frac{2\pi}{l}\right)^2\right]-\frac{64}{9}n^2\omega^2V^2$

CONCLUSIONS

The developed technique makes it possible to investigate the impact on oscillations of flexible elements of drive systems and transportation of nonlinear forces, the speed of longitudinal movement and perturbations of boundary conditions. Based on the obtained results it is proved that even for the linear analogue system the slowly time-dependent variable of the distance between the SE contact points and the pulleys causes the change of the basic parameters of the waves. In addition, under certain conditions, it can contain sustained dynamic process when SE is converted to unstable process. Thus, with decreasing SE tension force (at a constant speed and constant SE contact points and pulleys) the amplitude of oscillations increases. The process becomes unstable in a speed value. Simultaneously, the results can be the basis for developing methods of influence of periodic disturbance on SE oscillations with slowly varying distance between its points of contact to the pulleys.

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