STRUCTURAL INTENSITY METHOD APPLIED TO STUDY OF VIBRATIONS DAMPING
/
METODA INTENSIMETRIEI STUCTURALE APLICATĂ LA STUDIUL AMORTIZĂRII VIBRAȚIILOR

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ABSTRACT

Article describes a practical method for determining the vibration damping of the material based on the use of structural intensity measurements of longitudinal vibrations. Structural intensity is grounded on the analogy with the concept of acoustic intensimetry that uses the correlation between the signals obtained from two accelerometers. The structural intensity method can be applied, by computation or by measurement, for identification of the vibration propagation trace and for vibration control. In this paper, after developing the theory on calculation of longitudinal vibration power flow, one proposes an experimental method for determining the attenuation of longitudinal vibrations of materials according to their nature, size and method of fixing the structure that takes the vibrations transmitted by means of a vibration absorber.

REZUMAT

Articolul descrie o metodă practică de detarminare amortizării vibrațiilor unor materiale pe baza măsurătorilor de intensimetrie structurală a vibrațiilor longitudinale. Intensimetria structurală se bazează pe analogia cu conceptul de intensimetrie acustică, care utilizează corelația dintre semnalele care provin de la două accelerometre. Metoda intensimetriei structurale se poate aplica, prin calcul sau prin măsurători, pentru identificarea căilor de propagare a vibrațiilor şi pentru controlul vibrațiilor. În această lucrare, după dezvoltarea teoriei privind calculul fluxului de putere al vibrațiilor longitudinale, se propune o metodă experimentală pentru determinarea atenuării vibrațiilor longitudinale a materialelor în funcție de natura lor, dimensiuni şi modul de fixare al structurii care preia vibrațiile transmise, prin intermediul unui absorbitor de vibrații.

INTRODUCTION

Acoustic intensimetry bases have grown substantially as theoretical fundamentals as well as applications *(Fahy, 1989)*. In recent years, a more attention was directed to develop the method of structural intensimetry with practical applications *(Noiseux, 1970; Orăşanu et al, 2013)*. Measurements of vibration energy flow can give information about identification of the positions of vibration sources, energy transmitted from source in different ways and different types of vibrations waves *(Verheij, 1990)*.

Unlike numerical methods, which are based on theoretical models that allow it to approximate the energy of the transmitted vibration, such as the method of discretization developed in references *(Craifaleanu and Dragomirescu, 2015; Orăşanu and Dragomirescu, 2015)*, the proposed method consists in the processing of the signals obtained from direct measuring.

MATERIAL AND METHOD

1. Structural intensity

For the study of power transmission characteristics of structural vibrations it is not enough to make simple measurements of vibration levels, but it is necessary to determine the distribution of energy flow through structural intensity measurements. This flow represents the instantaneous rate of energy transfer per unit area in a given direction and is called the structural intensity. For this purpose it shall determine inter spectral acceleration density in two closely related sections, similar to the technique of the two microphones in acoustic intensimetry technique.

In recent years, one has made great progress in measuring sound power. In the field of vibrations, practical measurements rise many and difficult problems.

Structural intensity of vibrations, $\bar{I}$, in a given direction, is obtained by the time-average of the product of force *F* and the vibration speed vector, $\bar{v}(r,t)$:

$$\langle \bar{I} \rangle_t = \frac{1}{T}\int_0^T F(r,t)\cdot \bar{v}(r,t)dt \text{ or in complex form: } \langle \bar{I} \rangle_t = \frac{1}{2}\mathrm{Re}\left[\hat{F}\cdot \hat{\bar{v}}^*\right] . \tag{1}$$

The complex shape from (1) is used when the force and the particle velocities are treated as complex harmonic variables. The direction of intensity is the same with the direction of the particle resultant velocity. The expression of intensity depends on the type of vibration of a structure, namely: longitudinal, bending and torsional. In the following, we will establish a theoretical expression for the longitudinal vibrations of the beams.

2. Longitudinal vibrations of the beams

One studies the vibrations of the homogeneous beam, with a constant cross section that is acted by a longitudinal force. For an element of length *dx* (fig.1), we can write Newton’law

$$m\frac{\partial^2 u}{\partial t^2} = F + \frac{\partial F}{\partial x}dx - F , \tag{2}$$

where *u(x,t)* is the displacement of particles in section located at distance *x* and *F* is the normal force on the same section which it is assumed that varies linearly with the distance.

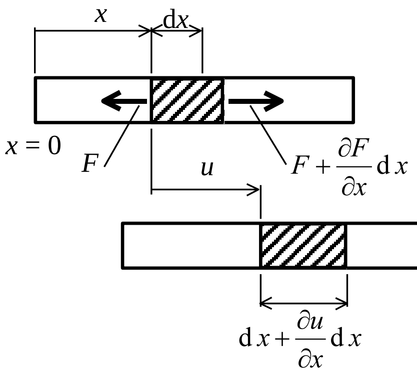


Fig. 1 - Toolbar element during longitudinal vibration

Relative elongation, by definition, the ratio between the elongation Δl and the original length *l*, it is written:

$$\varepsilon = \frac{\Delta l}{l_0} = \frac{dx + \frac{\partial u}{\partial x}dx - dx}{dx} = \frac{\partial u}{\partial x} , \tag{3}$$

so that Hooke's law becomes:

$$\frac{F}{A} = E\frac{\partial u}{\partial x} , \tag{4}$$

where *A* is the area of the bar’s section and *E* is the Young’s modulus.

Using the last three relationships one can obtain:

$$\rho A dx \frac{\partial^2 u}{\partial t^2} = EA\frac{\partial^2 u}{\partial x^2}dx \; , \tag{5}$$

where *ρ* is the density of the bar material.

The final relationship

$$\frac{\partial^2 u}{\partial t^2} = \frac{E}{\rho}\frac{\partial^2 u}{\partial x^2} \tag{6}$$

represents the equation of longitudinal vibrations of the beam. The report $\sqrt{E/\rho}$ is noted with c_L and has the dimensions of velocity. It is the speed of propagation of longitudinal vibrations and according to this, the transcribed form of equation (6) is:

$$\frac{\partial^2 u}{\partial t^2} = c_L^2 \frac{\partial^2 u}{\partial x^2} \; . \tag{7}$$

Structural intensity, for longitudinal vibrations in a beam, can be written successively

$$I_L = \langle F.v \rangle = \frac{EA}{c_L^2}\left\langle \frac{\partial^2 u}{\partial t^2} dx \int \frac{\partial u}{\partial t} dt \right\rangle = \frac{EA}{c_L^2}\Delta\left\langle (\frac{a_1+a_2}{2}) \int (a_2-a_1)dt \right\rangle \qquad (8)$$

where a_1 and a_2 are the accelerations at two nearby points 1 and 2, located at a distance Δ between accelerometers.

The relations (8) were obtained using the finite approximations *(Pavic, 1992)*:

$$a \cong \frac{1}{2}(a_1+a_2) \text{ and } \int a dt \cong \int (a_2-a_1)dt$$

$$F = \frac{EA}{\Delta r}(u_1-u_2) \qquad (9)$$

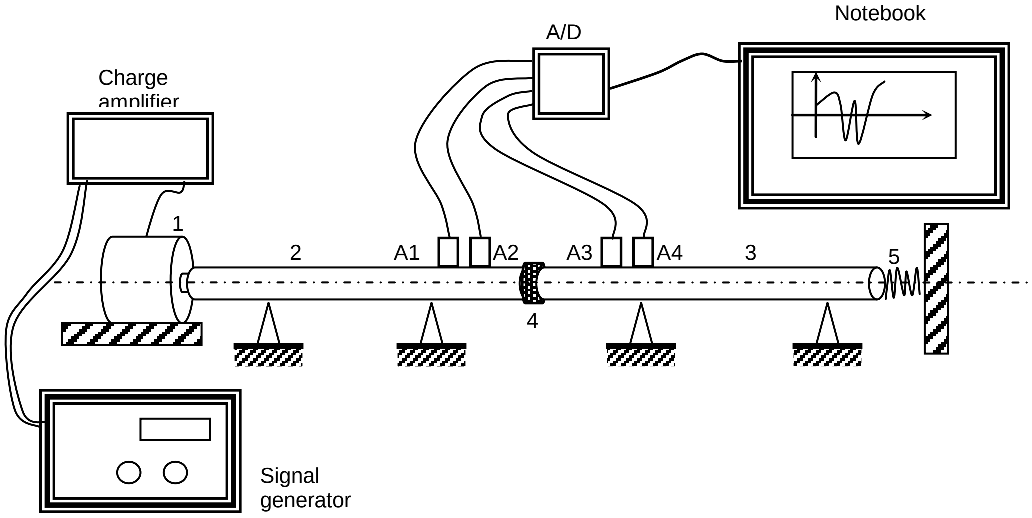
$$v = \frac{du}{dt} \cong \frac{v_1+v_2}{2} = \frac{1}{2}\int (a_1+a_2)dt$$

For harmonic wave, with angular velocity ω, the normal force is

$$F = \frac{EA}{i\omega\Delta}(a_1-a_2) \qquad (10)$$

and the vibration velocity of a particle will be written

$$v = \frac{a_1+a_2}{2i\omega}. \qquad (11)$$



a)

Fig.2 - The block diagrams of the damped measurement
1.Shaker; 2,3 Beam ; 4 Sample; 5 Spring; A1-A4 Accelerometers; A/D

Structural intensity can also be measured using cross-correlation method between the two acceleration signals. Here the vibration intensity vector component, in the direction of the two accelerometer will be

$$I_L(\omega) = -\frac{EA}{\omega^3\Delta}\operatorname{Im}[G(a_1,a_2)], \qquad (12)$$

where $\operatorname{Im}[G(a_1,a_2)]$ is the imaginary part of cross-spectrum between the two accelerometers signals.

For bending waves in a rod, the energy flow may be written as

$$I_B(\omega) = \frac{2\sqrt{Bm}}{\omega^2\Delta}\operatorname{Im}[G(a_1,a_2)]2\pi/\lambda, \qquad (13)$$

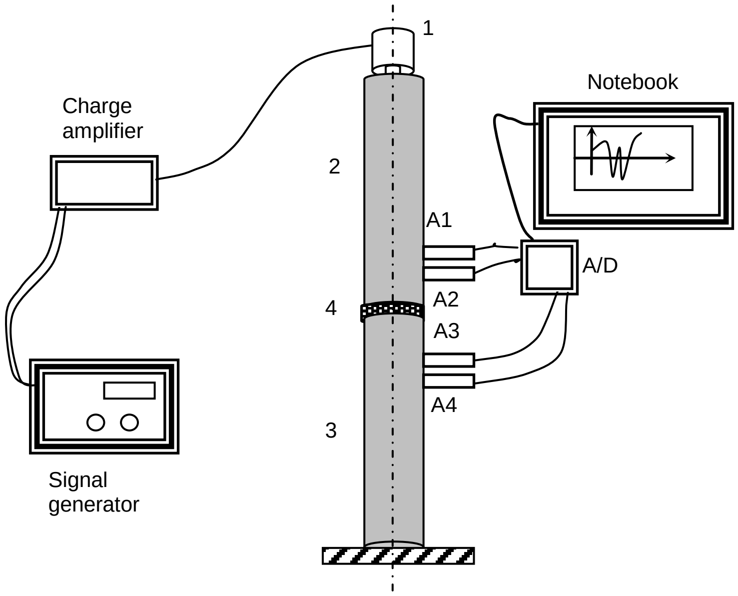
where:

B is bending stiffness, *λ* is the wave length and *m* is mass per unit length *(Craifaleanu et al., 2014; Orăşanu and Craifaleanu, 2011; Zhao, 1988)*.

RESULTS

In the first experiment there were used two horizontal bars with an outer diameter of 20 mm. Between the two bars a sample of cork with the role of damping longitudinal vibration transmitted from the first to the second bar has been fixed. The connection between the second bar and the outside has been carried out by means of a spring (fig.2,a).

The second experiment was carried out by means of two vertical cylindrical bar with an outer diameter of 100 mm. Between the two bars was fixed too a sample of cork with the purpose of vibration damping, the second rod being rigidly fixed to the foundation (fig.2,b).



b)

Fig. 2 - The block diagrams of the damped measurement
1-Shaker; 2,3-Beam ; 4-Sample; 5-Spring; A1-A4 - Accelerometers; A/D

In both cases, the used accelerometers A1-A2 and A3-A4, of delta-share type (B&K 4507), having the phase difference of not more than 4°, were mounted so that their axes coincide with the axes of the bars and the mini shaker. The experiments were carried out also, for other types of material (rubber, felt) of different thickness (0.5 cm, 1 cm and 2 cm).

The signal generated by the mini shaker is a white noise signal. Vibration energy flow (vibration intensity) expressed by relation (12) as the cross-spectra density of two accelerometer signals is shown in fig.3 and fig.4, in the frequency range of 1-1000Hz, for a 1cm sample of cork.

Significant differences occur in the resonance frequencies of the bar that takes over the damped vibrations.

CONCLUSIONS

The measurements made on the flow of vibration energy can provide qualitative and quantitative information on vibration damping of longitudinal direction by a vibration damper. Differences between the two experimental devices (fig.3 and fig.4) are not significant where there are no resonance frequencies of the bar behind the damper. It might occur increases of vibration power at the resonance frequencies of the structure behind the damper.

Fig.3 - Power flow vibration spectrum
corresponding to the experiment from fig.2a, for sample of cork of 1 cm

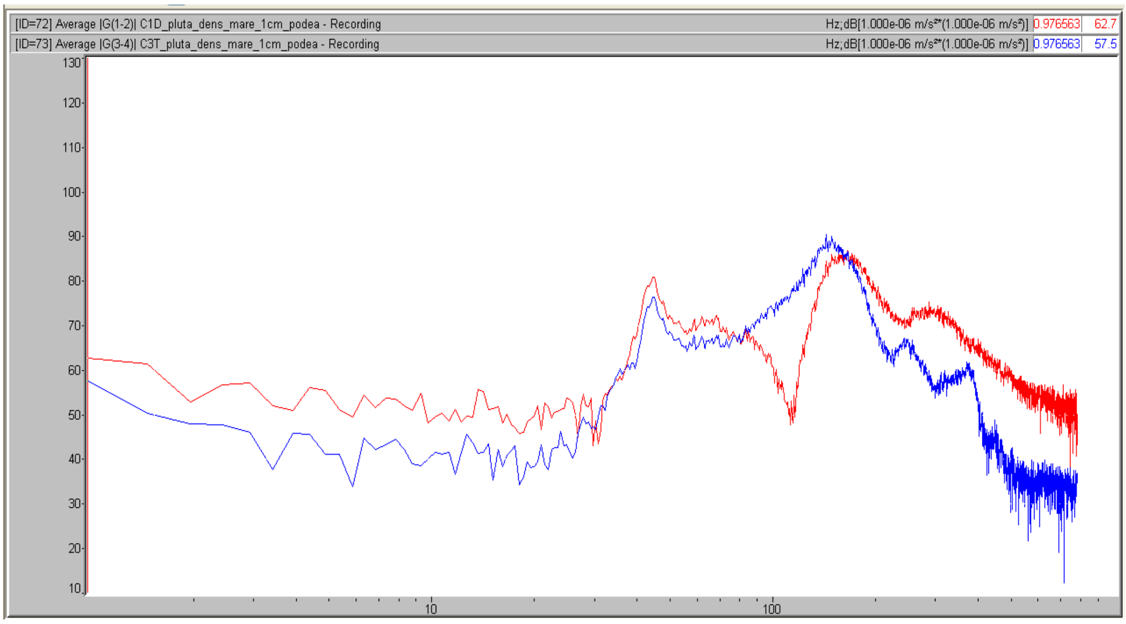


Fig.4 - Power flow vibration spectrum
corresponding to the experiment from fig.2b, for sample of cork of 1 cm

Measurements of structural intensity depend on:

- errors of tools by the phase difference between the measured accelerations and corresponding electrical signals;
- the transducer mass must be much smaller than the structural apparent mass;
- the spacing between the accelerometers produces errors proportional with ($\frac{2\pi}{\lambda} \cdot \Delta$), where λ is the wave length.

For the study of bending vibration damping, the excitation and measurement direction of accelerations are chosen perpendicularly to bars.

For torsional vibrations, things are more complicated, both from the point of view of the excitation and of the transducer, which is specific for measurement of torsional vibrations.

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