STABILITY ANALYSIS OF THE AGRICULTURAL ARTICULATED VEHICLE BASED ON INTERVAL METHOD
/
基于区间数学法的农业铰接车的稳定性分析

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Abstract: *The agricultural mechanization operation is highly efficient, but many types of agricultural mechanical equipment are difficult to move or to operate on hilly terrain and mountains. Thus, these machines play a limited role. This study designs an agricultural articulated vehicle whose front and rear bodies exhibit relative yawing and a rolling degree of freedom; this vehicle can perform agricultural construction operations on different terrains. The agricultural articulated vehicle may roll over during operations such as digging; therefore, its stability should be determined. This research builds a dynamic model of the vehicle during digging through homogeneous coordinate transformation and with the use of the multi-body dynamics method. The interval method is applied to the stability analysis for such operations and describes the uncertain parameters that affect stability as a bounded interval. The stabilities of the front and the rear bodies are analyzed as follows: first, the influence of tire deformation on the rear body stability of the vehicle is determined. The critical rollover angle of the vehicle decreases with an increase in tire deformation. Second, the influence of different digging material weights on the front body stability of the vehicle is measured. Finally, the results obtained with the interval analysis method indicate that the vehicle remains stable while digging if the values of yaw angle φ and roll angle θ are located in the minimum envelope zone of [−42°, 42°] and [−28°, 28°]. The analysis results can guide structural improvement, provide an early warning for rollover during digging, and extend the application range of the proposed vehicle in agricultural construction.*

Keywords: *Agricultural articulated vehicle; Multi-body dynamics; Interval analysis; Stability*

摘要： *农业机械化生产虽然效率高，但是在地形复杂的丘陵地带和山区，很多农用机械设备移动和操作困难，难以发挥作用。设计了一种农业铰接车，它的前后车体相对有横摆和扭转自由度，可以在不同的地形下进行农业工程作业。农业铰接车在农业作业，例如挖掘过程中容易产生倾翻，需要研究挖掘作业过程中的稳定性，本文采用齐次坐标变换和多刚体动力学方法，建立了农业铰接车挖掘作业时的动力学方模型。在挖掘作业稳定性分析中，应用区间数学法，把影响稳定性的不确定参数作为一个数学区间，然后结合车辆的稳定性判据，分别对农业铰接车前后车体的稳定性进行了分析，首先，分析了轮胎变形对后车体倾翻稳定性的影响；其次，分析了挖掘物质量的不同对前车体倾翻稳定性的影响。最后，通过仿真分析，在挖掘作业过程中，保证车辆不产生倾翻，前后车体的横摆角φ 和侧偏角θ 落在[-42°，42°]和[-28°，28°]的包络区域内。分析结果可以为 农业铰接车结构设计的改进和挖掘稳定性预警提供设计方法和参考，拓展它在农业工程中的应用范围。*

关键词: *农业铰接车；多刚体动力学；区间分析法；稳定性*

INTRODUCTION

Wheeled construction machineries have been applied to various aspects of agricultural production, such as grapery ditching, soil loosening and preparation in upland fields, the exploitation of low-lying and easily waterlogged wasteland, water supply, and dredging work in paddy fields, given that these processes are difficult to perform manually. Moreover, efficiency is low and the operational hazard is high. In north China, the construction machineries should also be used for farmland water conservancy in the winter because of the frozen surface layer. The use of construction machineries has effectively economized on manpower, shortened product time, and saved cost. Nonetheless, many types of agricultural mechanical equipment are difficult to move and control on hilly terrains and mountains; thus, these machines play a limited role. In this study, we design a vehicle whose front and rear bodies are joined by a universal hinge; this vehicle displays relative yawing and a rolling degree of freedom. Therefore, the wheels can make

引言

轮式工程机械已经用到农业生产的各个方面，如葡萄园挖沟作业；旱田的松土、整地；地势低洼、易涝的荒地开发；水田给水清淤工程，人工作业相当困难，而且效率低，作业危险性大；在北方冬季里进行农田水利施工，地表冻层，只有采用工程机械可以施工。工程机械应用在农业生产上，节约了大量劳动力，又缩短了生产时间，节省成本。但是在地形复杂的丘陵地带和山区，由于很多农用机械设备移动和操作困难，难以发挥作用。因此我们设计了一种农业铰接车，它的前车体和后车体采用铰链关节连接，前后车体相对有横摆和扭转自由度，在复杂的农田地形下车轮充分和地面接触，增加了车辆的运动和作业的稳定性，能更好的适应不同农业生产的地形环境，它的结构

sufficient contact with the complex farmland terrain, and vehicle stability improves for operating and digging operations. The vehicle can adapt to different terrain environments for agricultural production, is low-cost, possesses a simple structure, is convenient to operate, and is widely applicable. Nonetheless, the stability of agricultural construction vehicle operation on hills, mountains, and paddy fields containing silt must be determined; at present, studies on agricultural mountain vehicles mainly concentrate on static and dynamic stabilities to overcome obstacles [1, 2, 3, 8] and to warn against rollover in advance [7, 9, 12]. According to statistics, more than 90% of rollover accidents related to agricultural mountain vehicles occurred in operation [5, 11] rather than in running. Few studies have been conducted on vehicle rollover in agricultural construction operations although the complex operation environment and uncertain factors, such as vehicle position and pose, tire deformation, and digging state, influence the stability of agricultural construction vehicles. Furthermore, traditional stability analysis methods are applied only in specific states [4, 10]. The current study applies the interval method to an analysis of stability during digging, describes all factors that affect stability as a bounded interval, and derives a dynamic equation. The effect of all uncertain factors on vehicle stability is determined with the stability criterion in combination with the interval method. Researching vehicle stability in agricultural construction operation generates a reference for structure design improvement as well as for follow-up rollover protection and early warning system design. Thus, the safety and convenience of vehicle implementation in the agricultural construction operation are improved.

MATERIALS AND METHODS

Dynamic Model for an Agricultural Articulated Vehicle

The backhoe device of the agricultural articulated vehicle is driven by a hydraulic system, and the revolving mechanism is driven by two lever-type cylinders, as shown in Figure 1. This vehicle can excavate soil and rock at different heights, angles and distances, as depicted in Figure 2. Furthermore, the operating range of the agricultural articulated vehicle is presented in Table 1.

简单，操作方便，成本也低廉，应用更广泛。农业工程车辆在丘陵、山区或者淤泥的水田工作，需要研究它工作过程中的稳定性。目前国内外对于山地农业工程车辆的稳定性研究，主要集中在车辆静态稳定性研究和越障行驶中的动态稳定性研究[1,2,3,8]以及车辆侧翻预警研究[7,9,12]等。根据文献资料统计山地农业工程车辆的倾翻 90%以上都是在作业过程中发生的[5,11]，而工程车行驶过程中的倾翻较少。对于工程车在农业工程作业过程的倾翻研究，国内外相关文献资料极少；山地农业工程车挖掘作业环境复杂，车体位姿，轮胎变形，挖掘工作状态等不确定性因素多，常用的求解稳定性的数学方法，只能求解特定状态下的车体稳定性[4,10]。本文利用区间数学的方法，分析农业铰接车在工作过程中的稳定，把影响农业铰接车稳定性的任一不确定因素当成一个数学区间，建立动力学方程，结合铰接式车辆倾翻的稳定性判据，求解农业铰接车的动力学方程，得到在不确定因素影响下，农业铰接车前后车体姿态的范围。通过研究农业铰接车在农业工程作业中的稳定性研究，可以为农业铰接车的结构设计改进和后期防倾翻预警系统设计提供参考依据，能够使车辆更安全方便的应用于农业工程生产建设。

材料与方法

农业铰接车动力学模型

农业铰接车的挖掘装置通过液压系统驱动，挖掘臂回转系统采用杠杆式双液压缸驱动方式，如图 1 所示，可以实现农业铰接车在不同角度、不同高度、不同距离的挖掘作业，如图 2 所示，农业铰接车的作业装置作业范围如表 1 所示。

Fig.1 -Wasteland development operation of the vehicle

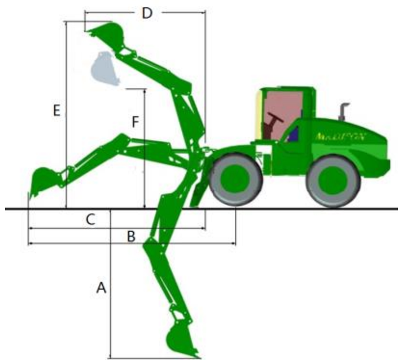

Fig.2 - Sketch of the vehicle operating range

Table 1

Parameters of the vehicle operating device

Definition of the parameter	Parameter values
A. Maximum digging depth	3.03 m
B. The horizontal distance between the bucket and the center of the front wheel	5.31 m
C. The horizontal distance between the bucket and the rotating center	4.11 m
D. The horizontal distance between the bucket on top and the rotating center	2.64 m
E. Maximum digging height	3.40 m
F. Maximum unloading height	2.85 m

Many uncertain factors are observed in this scenario, such as the velocity of the backhoe device, its angular velocity, and the attitude displayed during the operation process. The interval method is applied to solve the problem of stability given uncertain factors. The vehicle and the backhoe device are simplified as a rigid body, and the vehicle coordinate system is built through the multi-body dynamics method, as illustrated in Figure 3.

工作过程中，挖掘装置的速度、角速度及在工作区域位姿等影响农业铰接车稳定性的不确定参数众多，采用区间分析方法，能有效解决包含不定参数的系统分析问题。农业铰接车和其挖掘装置简化成刚体，通过多刚体动力学方法建模。建立农业铰接车坐标系，如图 3 所示。

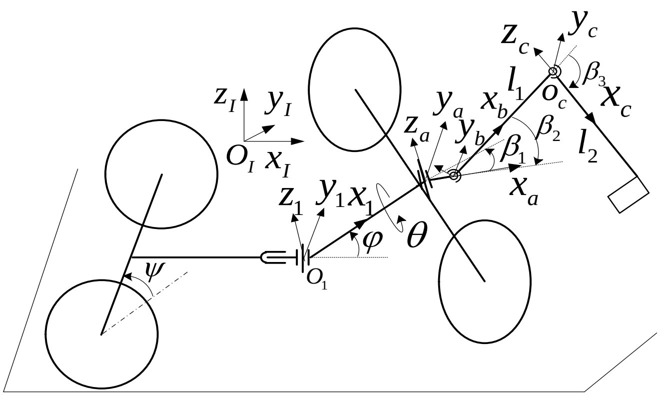

Fig.3 -Schematic for the vehicle coordinate system

where the origin of the inertial coordinate system $O_I x_I y_I z_I$ is the rear body centroid M_r and axle x_I is detected along the drive shaft of the rear body. Axle z_I is the vertical axis, and the front and rear bodies are joined by a universal hinge. The front body coordinate system $O_1 x_1 y_1 z_1$ originates at the yaw hinge, and the roll angle of the rear body is represented by ψ. The roll and yaw angles of the front body are denoted by θ and φ, respectively. The backhoe device and the front body are also connected through a universal hinge. The yaw angle between the backhoe device and the front body is represented by β_1, the pitching angle is denoted by β_2, and the angle between the boom and the bucket rod corresponds to β_3.

其中惯性系 $O_I x_I y_I z_I$ 原点在后车体质心 M_r， x_I 轴沿后车体传动轴方向。 z_I 轴铅垂向上。前后车体之间为万向铰连接，前车体本体坐标系为 $O_1 x_1 y_1 z_1$，原点在横摆铰接点处。后车体的侧偏角为 ψ，前车体的侧偏角为 θ，横摆角为 φ。挖掘装置和前车体通过万向铰连接，挖掘装置和前车体传动轴之间的横摆角为 β_1，俯仰角为 β_2，动臂和斗杆之间角度为 β_3。

The position vector of a generic point P on the vehicle in a generic coordinate system B is ${}^B\vec{p}$, which transforms a generic coordinate system A into ${}^A\vec{p}$. This vector is described in the form of homogeneous coordinates:

车辆上任一点 P，在某参考坐标系 B 中位置矢量为 ${}^B\vec{p}$，则变换到某参考坐标系 A 中位置矢量采用齐次坐标可表示为：

$$\begin{bmatrix} {}^A\vec{p} \\ 1 \end{bmatrix} = \begin{bmatrix} {}^A R_B & {}^A p_{Bo} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} {}^B\vec{p} \\ 1 \end{bmatrix} = {}^A T_B \begin{bmatrix} {}^B\vec{p} \\ 1 \end{bmatrix} \tag{1}$$

where ${}^{A}T_{B}$ is the matrix of position and orientation transformation from the B coordinate system to the A coordinate system, ${}^{A}R_{B}$ is the attitude matrix from the B coordinate system to the A coordinate system, and ${}^{A}\vec{p}_{Bo}$ is the position vector that indicates the origin of coordinate system B in coordinate system A.

When the vehicle excavates soil and rock, the body vehicles remain motionless, and the backhoe device alone moves. The boom and the bucket rod can be simplified as rods with lengths that are denoted by l_1 and l_2. The centroids of both the boom and the bucket rod are positioned at the middle points; moreover, the bucket and the digging mixture are simplified as the centralization mass of m_3. The driving moment of the backhoe device is represented by M_1, M_2, M_3, and the force of the backhoe device during digging is illustrated in Figure 4.

其中 ${}^{A}T_{B}$ 为坐标系 B 系到 A 系的位姿变换，${}^{A}R_{B}$ 为 B 系到 A 系的姿态变换矩阵，${}^{A}\vec{p}_{Bo}$ 为 B 系的原点在 A 系中的位置矢量。

农业铰接车在挖掘作业时，车体行进到工作位置固定，只有工作装置进行一系列的挖掘作业。工作装置的动臂和斗杆可简化成杆长度为 l_1，l_2 的杆，动臂和斗杆的质心在均在杆的中点，铲斗和挖掘物可看作 m_3 的集中质量点。挖掘装置的转动关节力矩为 M_1，M_2，M_3，则工作中，挖掘装置的受力如图 4 所示。

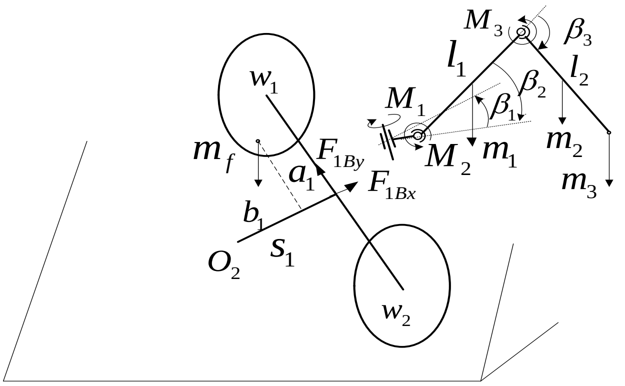

***Fig.4** -Forces exerted on the front body during digging*

In the inertial coordinate system, the position vector of centroid m_1 of the boom, centroid m_2 of the bucket rod, and centroid m_3 of the digging mixture are written as

在惯性坐标系下，动臂质心 m_1，斗杆质心 m_2，挖掘物质心 m_3 的位置矢量为：

$$\begin{bmatrix} {}^{O}\vec{p}_{m1} \\ 1 \end{bmatrix} = {}^{O}T_1 \, {}^{1}T_a \, {}^{a}T_b \begin{bmatrix} {}^{b}\vec{p}_{m1} \\ 1 \end{bmatrix} \quad (2)$$

$$\begin{bmatrix} {}^{O}\vec{p}_{m2} \\ 1 \end{bmatrix} = {}^{O}T_1 \, {}^{1}T_a \, {}^{a}T_b \, {}^{b}T_c \begin{bmatrix} {}^{c}\vec{p}_{m2} \\ 1 \end{bmatrix} \quad (3)$$

$$\begin{bmatrix} {}^{O}\vec{p}_{m3} \\ 1 \end{bmatrix} = {}^{O}T_1 \, {}^{1}T_a \, {}^{a}T_b \, {}^{b}T_c \begin{bmatrix} {}^{c}\vec{p}_{m3} \\ 1 \end{bmatrix} \quad (4)$$

The angle velocity of the boom l_1 and the bucket rod l_2 in the inertial coordinate system can be described as

动臂 l_1 和斗杆 l_2 在惯性系中的角速度可以表示为：

$${}^{O}\vec{\omega}_{m1} = {}^{O}R_1 \, {}^{1}R_a \dot{\vec{\beta}}_1 + {}^{O}R_1 \, {}^{1}R_a \, {}^{a}R_b \dot{\vec{\beta}}_2 \quad (5)$$

$${}^{O}\vec{\omega}_{m2} = {}^{O}R_1 \, {}^{1}R_a \dot{\vec{\beta}}_1 + {}^{O}R_1 \, {}^{1}R_a \, {}^{a}R_b \dot{\vec{\beta}}_2 + {}^{O}R_1 \, {}^{1}R_a \, {}^{a}R_b \, {}^{b}R_c \dot{\vec{\beta}}_3 \quad (6)$$

The velocity and the acceleration of m_1, m_2, m_3 can be written as

质心 m_1，m_2，m_3 的在惯性系下的速度和加速度可表述为：

$${}^{O}\vec{v}_{mi} = {}^{O}\vec{\omega}_{mi} \times {}^{O}\vec{p}_{mi} \quad (7)$$

$${}^{O}\vec{a}_{mi} = {}^{O}\vec{\varepsilon}_{mi} \times {}^{O}\vec{p}_{mi} + {}^{O}\vec{\omega}_{mi} \times {}^{O}\vec{v}_{mi} \quad (8)$$

where $i=1,2,3$, ${}^{O}\vec{\omega}_{m2}={}^{O}\vec{\omega}_{m3}$, ${}^{O}\vec{\varepsilon}_{m2}={}^{O}\vec{\varepsilon}_{m3}$, and $\vec{\varepsilon}=d\vec{\omega}/dt$ is angular acceleration.

We define β_1, β_2, β_3 as generalized coordinates. The driving moment M_1 , M_2 , M_3 is calculated by solving the Lagrange equation. When the vehicle performs a digging task under the assumption that the acceleration of the boom and of the bucket rod is represented by $\vec{\varepsilon}=0$, the kinetic energy of the boom and bucket rod system is written as

其中 $i=1,2,3$ ， ${}^{O}\vec{\omega}_{m2}={}^{O}\vec{\omega}_{m3}$ ， ${}^{O}\vec{\varepsilon}_{m2}={}^{O}\vec{\varepsilon}_{m3}$ ， $\vec{\varepsilon}=d\vec{\omega}/dt$ 为角加速度。

设 β_1、 β_2、 β_3 为广义坐标，采用拉格朗日方程求解驱动力矩 M_1 、 M_2 、 M_3，农业铰接车在挖掘工作时，假设动臂斗杆的加速度 $\vec{\varepsilon}=0$，则 农业铰接车动臂斗杆系统的动能可写为：

$$T=\frac{1}{2}\sum_{i=1}^{3}m_i\,{}^{O}\vec{v}_{mi}^{T}\,{}^{O}\vec{v}_{mi}+\frac{1}{2}\sum_{i=1}^{2}{}^{O}\vec{\omega}_{mi}^{T}\vec{J}_{mi}\,{}^{O}\vec{\omega}_{mi} \tag{9}$$

The potential energy in the system is written as

系统势能可写为：

$$V=\sum_{i=1}^{3}m_i\,{}^{O}\vec{p}_{mi}{}^{T}\vec{g} \tag{10}$$

The driving moment is calculated as

求得驱动力矩为：

$$M_i=\frac{d}{dt}\left(\frac{\partial L}{\partial\dot{\beta}_i}\right)-\frac{\partial L}{\partial\beta_i} \tag{11}$$

where $L=T-V$ is the Lagrange function.

When D'Alembert's principle is applied, the force and the moment of the front body as induced by the backhoe device can be computed as

其中 $L=T-V$ 为拉格朗日函数。

根据达郎贝尔原理可求得挖掘装置对前车体的作用力和作用力矩为：

$$\vec{F}_{1B}={}^{1}R_{O}\sum_{i=1}^{3}m_i\,{}^{O}\vec{a}_{mi} \tag{12}$$

$$\vec{M}_{1B}=-({}^{1}R_a\vec{M}_1+{}^{1}R_a\,{}^{a}R_b\vec{M}_2+{}^{1}R_a\,{}^{a}R_b\,{}^{b}R_c\vec{M}_3) \tag{13}$$

The vehicle consists of two parts. During digging, the front body may be influenced by the boom and bucket rod motion. When the force on wheels w_1 and w_3 is zero, the front and rear bodies tip over at approximately O_2W_2 and O_2W_4 , respectively, as shown in Figure 5.

由于农业铰接车分为前后两部分，挖掘作业时，由于动臂斗杆的影响，前车体最容易发生倾翻。此时有轮 w_1、w_3 受力为零，前车体绕 O_2W_2 轴倾翻，后车体绕 O_2W_4 轴倾翻，如图 5 所示。

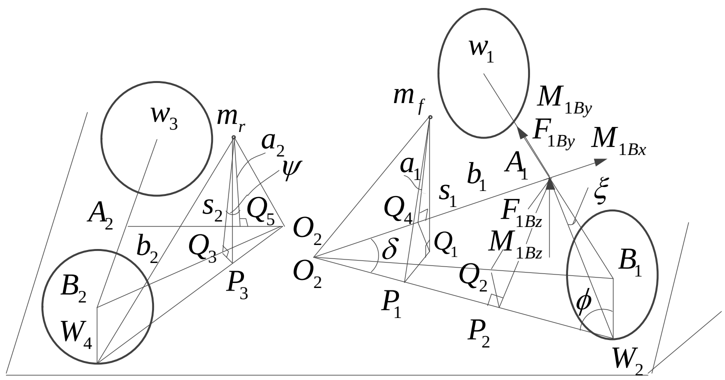

Fig.5 - Schematic of vehicle stability

The front and rear bodies of the vehicle are stationary during digging; this problem is one of static stability.

When the rear body remains stable, its centroid m_r should be located in plane $\Delta A_2B_2O_2$. Thus, the centroid can be calculated as

设农业铰接车的前后车体在挖掘作业过程中静止不动，前后车体属于静态稳定性问题。

当后车体保持稳定时，后车体质心 m_r 应落到平面 $\Delta A_2B_2O_2$ 内，可得：

$$Q_3P_3>0 \tag{14}$$

The roll angle of the rear body in stable state can be computed by using the geometric relation presented in Figure 5.

通过图 5 中几何关系，可得后车体保持稳定时的侧偏角为：

$$\psi < \arctan\left(\frac{s_2 d(s_2 - b_2)\sqrt{d^2 + s_2^2}}{d^2 s_2 a_2 - 2d^2 b_2 r + s_2^3 r + s_2^3 a_2 - s_2^2 b_2 r} - \frac{\delta}{2d}\frac{s_2}{\sqrt{d^2 + s_2^2}}\right) \tag{15}$$

where s_2 is the length of the rear vehicle, d is half of wheel track, a_2 is the length between centroid m_r and point Q_5 which is the centroid m_r projection onto the rear drive shaft, b_2 is the length between point A_2 and point Q_5, r is the wheel radius, δ is the static deformation of the tire when the vehicle is unstable. In the experiment, the measured value of the tire radius is [0.736 m, 0.775 m].

When the front body is in stable state, the moment of mass center m_f, which is relative to axle O_2W_3, is greater than the other moments that are also relative to this axle. Therefore, we can obtain

其中 s_2 是后车体长度，d 是车轮距的一半，a_2 是质心 m_r 与点 Q_5 的长度，其中 Q_5 是质心 m_r 在后驱动轴上的投影，b_2 是点 A_2 和点 Q_5 之间的长度，r 是车轮半径，δ 为失稳时轮胎的静变形量。通过测量，轮胎滚动半径变化量为[0.736m, 0.775m]。

前车体保持稳定时，前车体质心 m_f 绕轴 O_2W_3 的力矩大于其它外力绕轴此轴的力矩。因此，通过计算可得：

$$\sqrt{\frac{(s_1 - b_1)^2(\delta^2 + d^2)}{(s_1^2 + d^2)\cos^2\varphi} + \frac{s_1^2 a_1^2 \sin^2\theta}{s_1^2 + d^2 + r^2}} \times m_f \vec{g} > \sqrt{\frac{(s_1^2 + d^2)(r^2 + d^2)}{s_1^2 + d^2 + r^2}} \times \vec{F}_{1Bz} - \vec{r} \times \vec{F}_{1By} + \vec{M}_{1Bx} - \vec{M}_{1By} - \vec{M}_{1Bz} \tag{16}$$

where s_1 is the length of the front vehicle, a_1 is the length between centroid m_f and point Q_4 which is the centroid m_f projection onto the front drive shaft, b_1 is the length between point A_1 and point Q_4.

When the vehicle is performing a digging task, the uncertain interval parameters are the angles β_1, β_2, β_3 among the boom, the bucket rod, and the rotational velocity $\dot{\beta}_1$, $\dot{\beta}_2$, $\dot{\beta}_3$, the mass of the digging material m_t, and the deformation of the tire δ. To obtain acceptable stability in the digging operation, the allowable range of roll angle θ and yaw angle φ should be determined with the interval method.

其中 s_1 是前车体长度，a_1 是质心 m_f 与点 Q_4 的长度，其中 Q_4 是质心 m_f 在前驱动轴上的投影，b_1 是点 A_1 和点 Q_4 之间的长度。

对于农业铰接车来说，当在进行挖掘作业时，动臂斗杆间夹角 β_1、β_2、β_3，以及转动角速度 $\dot{\beta}_1$、$\dot{\beta}_2$、$\dot{\beta}_3$，m_t，轮胎变形量 δ 分别为不确定的区间参数，要保证挖掘作业过程中前车体的稳定性，需要求解在不确定区间参数中，农业铰接车的前车体侧偏角 θ 和横摆角 φ 的允许范围。

Analysis of Stability in Digging Operations based on the Interval Method

Interval analysis theory has been applied to many project fields [6, 13]. The closed interval of a real number can be described as

区间法分析农业铰接车挖掘作业稳定性

目前区间分析理论已经应用于很多工程领域[6,13]。实的闭区间可表示为：

$$X^I = [\underline{X}, \overline{X}] = \left\{X \in R \mid \underline{X} \le X \le \overline{X}\right\} \tag{17}$$

where $\underline{X}$ and $\overline{X}$ are the interval endpoints.

When the center interval method is applied, Eq. (17) is rewritten as

其中 $\underline{X}$，$\overline{X}$ 分别称为区间的下端点和上端点。

采用中心区间法，式（17）可以表示为：

$$X^I = X^C + \Delta X\, e_\Delta \tag{18}$$

where X^C is defined as the middle point of the interval, ΔX is defined as the interval radius, and the range of e_Δ is [-1, 1].

The vehicle parameters can be written in the interval form

其中 X^C 称为区间中点，ΔX 称为区间半径，e_Δ 的范围为[-1, 1]。

农业铰接车的设计参数，可以用区间法表示为：

$$\begin{cases} \beta_1 = \dfrac{\pi}{2} e_\Delta \\ \beta_2 = \dfrac{\pi}{6} + \dfrac{\pi}{6} e_\Delta \\ \beta_3 = 0.39\pi + 0.39\pi e_\Delta \\ \dot{\beta}_1 = \dfrac{\pi}{3} e_\Delta \\ \dot{\beta}_2 = 0.055\pi + 0.055\pi e_\Delta \\ \dot{\beta}_3 = 0.055\pi + 0.055\pi e_\Delta \\ m_t = 300 + 300 e_\Delta \\ \delta = 0.7555 + 0.0195 e_\Delta \end{cases} \tag{19}$$

Eq. (19) is substituted into Eq. (16) to yield

把式（19）代入式（16），可得到如下函数：

$$F(\theta, \varphi, e_\Delta) > 0 \tag{20}$$

When the agricultural articulated vehicle is performing a digging task, the values of the parameters that maintain front body stability must satisfy the boundary conditions of Eq. (20). The vehicle parameters are l_1 = 2.4 m, l_2 = 2 m, m_1 = 600 kg, m_2 = 450 kg, m_f = 7000 kg, and m_r = 5900 kg. The track value is $2d$ = 1.6 m, and the wheel radius value is r = 0.75 m, S_1 = 0.72 m, S_2 = 2.2 m, a_1 = 0.65 m, a_2 = 0.65 m, b_1 = 0.2 m, and b_2 = 1.4 m. The range of the roll angle θ is [−40°, 40°], and the range of the yaw angle φ is [−42°, 42°].

当农业铰接车挖掘时，保证前车体稳定性，车体相关参数的取值需要满足式（20）。农业铰接车设计参数为：l_1 =2.4m ， l_2 =2m ， m_1 =600kg ， m_2 =450kg ， m_f =7000kg， m_r =5900kg，轮距 $2d$ =1.6m，车轮半径 r =0.75m ， S_1 =0.72m ， S_2 =2.2m ， a_1 =0.65m ， a_2 =0.65m， b_1 =0.52m， b_2 =1.4m，侧偏角 θ 的设计范围为[-40°，40°]，横摆角 φ 的设计范围为[-42°，42°]。

RESULTS

Influence of Tire Deformation on Rear Body Stability

When the roll angle of the rear body vehicle increases, tire deformation changes. As a result, the actual rollover angle shifts as well. The relationship between the rollover angle of the rear body and tire deformation is calculated with Eq. (15); the result is shown in Figure 6.

The rollover angle of the rear body decreases when tire deformation increases in Figure 6. Thus, this deformation should be considered in the analysis of vehicle stability during agricultural construction to enhance the accuracy of the calculated rollover angle. If tire deformation is disregarded, then the calculated rollover angle is large; when this angle is used in the design of a vehicle rollover warning system, rollover accidents occur easily.

结果

轮胎变形量对后车体稳定性的影响

农业铰接车的后车体的侧偏角的增大，后车体左边和右边轮胎上的受力产生变化，轮胎的变形量就随着改变，造成后车体实际失稳的侧偏角也会不同。根据后车体侧偏的稳定性公式（15），可得后车体保持稳定的侧偏角随轮胎变形量的变化关系如图 6 所示。

由图 6 可知，轮胎变形量的增大，后车体失稳时的侧偏角会变小。所以在挖掘作业稳定性分析时，必须要考虑轮胎才能使计算的保持稳定性的侧偏角更准确，如果不考虑轮胎变形，会使计算的稳定侧偏角偏大，按照此计算结果进行倾翻预警设计，容易造成车辆翻车事故。

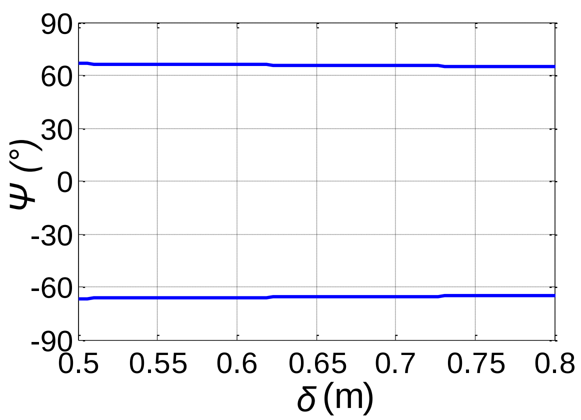

Fig.6 - Variation in the rollover angle of the rear body with tire deformation

Influence of Digging Material Weight on Front Body Stability

When the agricultural articulated vehicle is working, the digging material weight in the bucket strongly affects front body stability. If $\varphi = 0$, then the boom and bucket rod of the digging device are motionless, $\beta_2 = 0.12\pi$, $\beta_3 = 0.3\pi$, the rotation angular velocity around the Z_a axis is denoted by $\dot{\beta}_1$, and $\dot{\beta}_1 = 0.3\pi / s$. When the digging material weight is m_t =0 kg and m_t =300 kg, the stability curves of the front body are as depicted in Figure 7.

挖掘物质量对前车体稳定性的影响

当农业铰接车进行作业时，抓斗中挖掘物质量对前车体的倾翻稳定性有很大的影响，若给定 $\varphi = 0$，挖掘装置的动臂和斗杆相对不动，$\beta_2 = 0.12\pi$，$\beta_3 = 0.3\pi$ 只有绕 Z_a 轴的转动角速度 $\dot{\beta}_1$，设 $\dot{\beta}_1 = 0.3\pi / s$，当铲斗中的挖掘物质量 m_t =0 kg 和 m_t =300 kg 时，可以分别求的前车体保持倾翻稳定性的曲线，如图 7 所示。

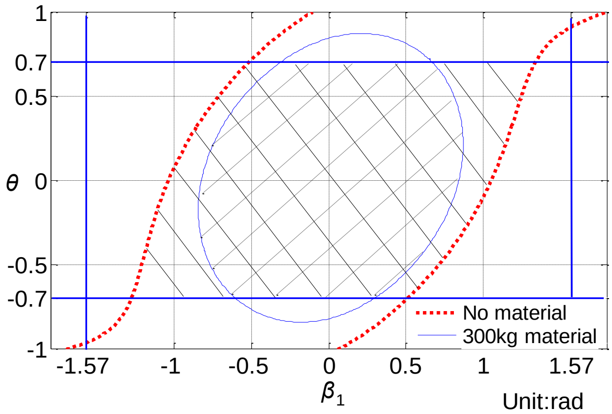

Fig.7 - Stability region of the front body under different digging material weights

According to the stability analysis results, the front body is stable when m_t =0 kg and if the roll angle of the front body θ and the rotation angle of the digging device β_1 are located in the elliptic region surrounded by a solid line. The same is true when m_t =300 kg, if θ and β_1 are located in the region surrounded by the dotted line. The range of θ is [−40°, 40°], and the range of β_1 is [−90°, 90°]; therefore, the stability region of the front body is the intersection of the same parameters under different conditions. This region is denoted by the areas marked with solid and dotted slashes in Figure 7.

根据稳定性分析结果，可知当 m_t =0 kg 时，前车体倾角 θ 和挖掘装置转角 β_1 在曲线围成的椭圆区域内，前车体是稳定的；当 m_t =300 kg 时，前车体倾角 θ 和挖掘装置转角 β_1 在曲线围成的区域内，前车体是稳定的；由于 θ 的范围为[-40°，40°]，β_1 的范围为[-90°，90°]，所以实际的前车体挖掘中的稳定性应该是同一参数不同取值范围的交集，为图 7 中的斜实线和斜虚线区域。

Influence of All the Parameters on Front Body Stability

As per an analysis of the influence of all the parameters on front body stability based on Eq. (19) and (20), the values of φ and θ should be located in the minimum envelope zone of $F(\theta, \varphi, e_\Delta) > 0$ when $e_\Delta \in [-1,1]$. The simulation result is presented in Figure 8.

The maximum value of roll angle θ, which maintains the stability of the front body, is ± 28°, as indicated in Figure 8, when the vehicle is performing a digging task and if the value of yaw angle φ is zero. The roll angle decreases with an increase in yaw angle. Thus, the value of the roll angle cannot reach the extreme point ±40°. When the value of the yaw angle reaches such points (lines 1 and 2 are displayed in Figure 8), roll angle value is ±18.7°. Therefore, the front body is stable when the values of roll angle θ and of yaw angle φ are located in the region shown in Figure8; otherwise, the front body faces the risk of tipping over.

前车体稳定性的所有参数综合影响分析

根据公式（19）和公式（20）分析所有参数对农业铰接车前车体稳定性影响，则 φ，θ 取值为：对于任意给定的 $e_\Delta \in [-1,1]$，φ，θ 应在函数 $F(\theta, \varphi, e_\Delta) > 0$ 的最小包络区域内，计算结果如图 8 所示。

由图 8 分析结果可知，在农业铰接车在挖掘作业时，当横摆角 φ 为 0 时，前车体保持稳定的最大侧偏角 θ 约为±28°，随着横摆角 φ 的增大，侧偏角 θ 减小，无法达到车体设计的侧偏角极限范围±40°。当横摆角 φ 增大到车体设计的极限位置时（图 8 中直线 1 和直线 2），侧偏角 θ 约为±18.7°。前车体侧偏角 θ、横摆角 φ 的值落在图中区域内时，前车体能保证挖掘时的稳定性，不会有倾翻风险，当落在区域外时，可能会有倾翻的风险。

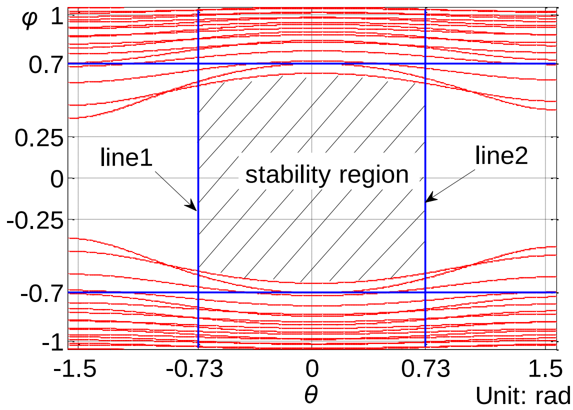

Fig.8 - Stable region of the front body during digging

CONCLUSION

Agricultural articulated vehicles can adapt well to different terrains; thus, such vehicles can be used on the rough terrains of upland fields, paddy fields, and mountainous and hilly areas during agricultural production. To protect the safety of the operator, the stability of this vehicle must be determined during agricultural production. We analyze the stabilities of the front and rear bodies when the vehicle performs a digging task. The factors that affect stability during digging operations are defined as the mathematical interval. Moreover, we build a stability analysis model in combination with multi-body dynamics, the interval method, and the stability criterion to analyze vehicle stability. The region of the roll and yaw angles is determined based on the simulation result and the yaw angle keeps the front body stable. Therefore, the simulation results can be fed back to a designer to optimize vehicle design and to provide data for early rollover warning design in agricultural production.

结论

农业铰接车因为地形适应能力强，在农业工程作业中可以应用在旱地、水田、山地等地形环境恶劣的地方，但是为了保证操作者的人身安全，研究农业铰接车在农业工程作业时的稳定性是十分重要的。本文对前车体和后车体在挖掘作业过程中的稳定性进行了单独分析。在建立静态稳定性分析模型时，把影响挖掘过程中稳定性的因素设为数学区间，结合多体动力学、区间数学方法和稳定性判据，来分析挖掘过程中的车体稳定性。通过仿真计算分析得到前车体在挖掘过程中不发生倾翻的侧偏角和横摆角区域范围。通过分析挖掘过程中的稳定性计算结果，可以反馈给设计人员改进和提高农业铰接车的结构设计，同时也可以为农业工程作业中车辆的倾翻预警装置研发提供参考。

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