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MATHEMATICAL MODEL FOR THE STUDY AND DESIGN OF A ROTARY-VANE GAS REFRIGERATION MACHINE

This paper presents the mathematical model for calculating rotary-vane gas refrigerating machine operating cycle main parameters that have an impact on the unit operation, machine control and working processes occurring in it at the specified criteria. The procedure and graphical method for the rotary-vane gas refrigerating machine (RVGRM) are proposed. Parametric study of the main geometric variables and temperature variables on the thermal behavior of the system is analyzed. The model considers polytrope index for the compression and expansion in the chamber. Graphs of the pressure and temperature in the chamber of the angle of rotation of the output shaft are received. The possibility of inclusion in the cycle regenerative heat exchanger is appreciated. The change of the coefficient of performance machine after turning the cycle regenerative heat exchanger is analyzed. It is shown that the installation of a regenerator into the RVGRM cycle results in more than 30% COP increasing. The simulation results show that the proposed model can be used in the process of design and optimization of Stirling gas refrigerating machine.

Keywords: *Stirling gas refrigerating machine; Rotary-vane gas refrigerating machine; Mathematical model; Energy efficiency; Air.*

NOMENCLATURE

A	mechanical work (J)
a	parameters of the cam (rad)
b	parameters of the cam (rad)
c	coefficient of dead volume
COP	coefficient of performance
c_v	gas specific heat for constant volume (kJ/kgK)
T	temperature (K)
Q	heat (J)
ψ_v	angle size of the vane (rad)
V	volume (m^3)
p	pressure (bar)
α	shaft angle (rad,°)
R	specific gas constant (kJ/kgK)
M	mass (kg)
η	efficiency
n	polytropic index

I. INTRODUCTION

All over the world in the area of energy saving and problem solving aimed at reducing anthropogenic effects on the environment, refrigeration machines that are used natural working medium as refrigerants become increasingly important [1]. For this reason Stirling gas refrigerating machine get topical priority.

The design of the compressor unit Stirling gas refrigeration machine are based on the reciprocating motion of the pistons, sold through a crank mechanism (CSV) or by means of a linear actuator. For achieving the tightness chambers it requires the installation of the contact seal in the case of CSV use, and "contactless" pistons, which are part of the linear actuator, is achieved by increasing the complexity of mechanical design of these machine, that in own turn. This complexity of the system of cranks and levers, piston Stirling gas refrigeration machine, along with other reasons, restricts their increased use [2].

To improve the design of the piston Stirling gas refrigeration machine structural optimization rotary vane gas refrigeration machine (RVGRM) is carried out. With less system elements than the CSV by 60 %, better balance and more simple manufacturing techniques using cause of a cylinder form of rotary vane machines are capable to be more preferable direction in Stirling machine improvement [3].

II. DESIGN ASPECTS OF THE RVGRM

The mechanical design of rotary vane gas refrigeration machine is shown in Figure 1. The vane group of RVGRM consists from a cylindrical corpus 1 in which two coaxially mounted rotor (outer and inner), two seals corpus 2 and the two end caps 4. The rotor inner and the rotor outer are internal assembly units, each of which consists of a shaft 6, 7 respectively, four pistons 5, four clamping plates 8 and four seals 3. Rotors form inside the corpus 1, four working chambers of variable volume, which simultaneously carried out four working process: the communication of the working fluid from the cooler through a window in the end cap 4, the expansion of the working fluid, the working fluid communication with the freezer through a window in the end cap 4, compressing the working fluid message with a cooler.

Thus each machine working chamber formed by the following parts: two shafts 6, 7, two end caps 4, the two pistons 5 and the corpus 1.

Compactness coefficient of RVGRM main volume (the ratio of working volume equivalent to the volume of the machine) reaches 15-20%, while the maximum value of this parameter for the piston gas refrigeration machine (V – shaped with a crank) is 1-2 %. Such a large (several times) benefit of the specific mass indicators opens up prospects for the use of this type machines. It gives us the opportunity to use rotary vane machines for domestic refrigeration appliances.

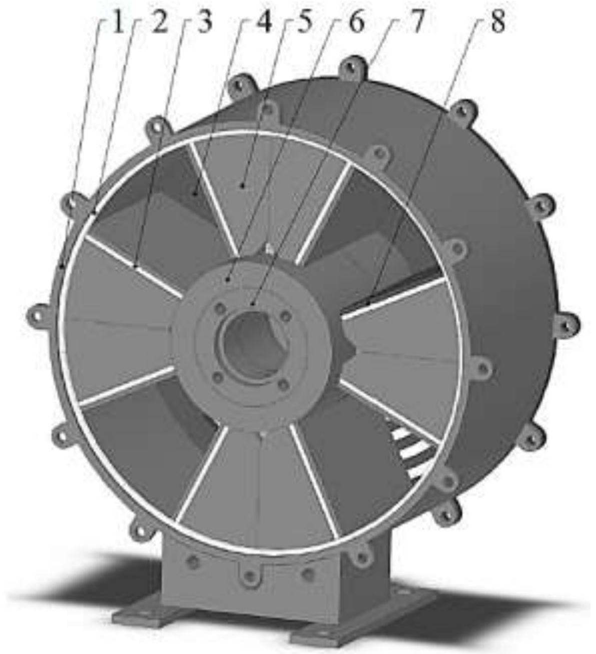


Figure 1 *–The scheme of RVGRM:*
1 – corpus; 2 – corpus seal; 3 – sealing blade;
4 – end cap; 5 – vane; 6, 7 – shafts; 8 – pressure plate.

III. MATHEMATICAL MODEL

The study of existing methods of calculation shows that the theoretical studies of Stirling gas refrigeration machine limited to consideration of the ideal cycle. Mathematical models of the cycle do not take into account the character of the heat flows (non-stationary) devices in the machine. In article [4] provided a detailed review of systems, based on the Stirling cycle machines.

In this paper we propose a mathematical model for calculating the basic parameters RVGRM cycle affecting the operation cycle, the machine control and operating processes occurring in it for the given criteria.

The analysis RVGRM of thermodynamic cycle consisting of two modules connected to the output shaft shifted by 45°, allowing the following main processes see Figure 2:

1) a—b –working fluid compression;

2) b—c (c—d) – transfer of the working fluid through the cooler in the other module with heat removal;

3) d—e – expanding cooled working fluid;

4) e—a – transfer of the working fluid through the freezer to another heat supply module.

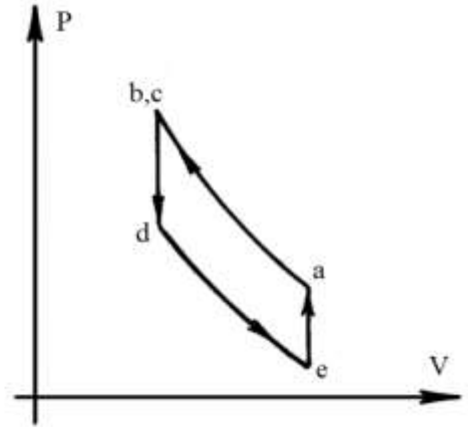


Figure 2 *– The thermodynamic cycle RVGRM*

From the structural dimensions of the rotary-vane group machines and motion conversion parameters can be expressed by the variation of volume of the chamber between the vanes:

$$V(\alpha)=(2a+2b\cdot cos2\alpha-\psi_{л})c \quad (1)$$

Spend calculation of the volume of the key points of the cycle in which all processes are in time corresponding to rotation of the output shaft (45°). In this case, the working chamber volume will be equal to:

$$V_a=V_e=(2a+2b+\psi_{л})c \quad (2)$$

$$V_b=V_d=(2a-\psi_{л})c \quad (3)$$

Because the ideal models for calculating the adiabatic index is used which depends on the pressure and temperature of the working fluid, and do processes of compression and expansion occur in reality, the exchange of heat with the environment. Consequently for more accurate calculations, the equations need to replace the adiabatic index for two different polytropic index for the compression process n_1 and the expansion process n_2 .

Beginning the process of expansion corresponds to the minimum angle between the axes of the vanes under consideration chamber, and hence the minimum of its volume. By the end of the expansion process chamber volume increases about two times or more.

The pressure change during a polytropic expansion can be described as:

$$p_{de}(\alpha)=p_d\left(\frac{V_d}{V(\alpha)}\right)^{n_2} \quad (4)$$

The temperature during expansion changes under the law:

$$T_{de}\left(\alpha\right)=T_{d}\left(\frac{V_{d}}{V(\alpha)}\right)^{n_2-1} \qquad (5)$$

At the moment of opening windows connecting the module to the freezer, the working fluid begins the process of transferring from one module to another through the load heat exchanger with heat supply. In the process of heat supply pressure and an increase in temperature.

Analogously the enlargement process will produce calculations for polytropic compression process:

$$p_{ab}\left(\alpha\right)=p_{a}\left(\frac{V_{a}}{V(\alpha)}\right)^{n_1} \qquad (6)$$

$$T_{ab}\left(\alpha\right)=T_{a}\left(\frac{V_{a}}{V(\alpha)}\right)^{n_1-1} \qquad (7)$$

At the moment of the opening of windows, connecting the machine module with cooler and receiver begins the process of suction and discharge the working fluid. Based on the design of the machine, the working fluid through injecting cooler into a sufficiently large volume of the receiver compared to the volume of the chamber at the opening of the window. Simplistically it can be assumed that the pressure in this process decreases to the working fluid pressure in the receiver. The same can also be considered with respect to the suction: suction is performed from the receiver volume is much larger volume of the chamber and the pressure can be considered constant and equal to the pressure in the receiver.

Intended that the cooler design provides the required heat transfer down to ambient temperature.

The work required to compress the working fluid can be calculated as:

$$A_{com}=\int_{0}^{\frac{\pi}{4}}p_{ab}(\alpha)\frac{dV(\alpha)}{d\alpha}d\alpha \qquad (8)$$

The work required to expand the working fluid can be calculated as:

$$A_{exp}=\int_{0}^{\frac{\pi}{4}}p_{de}(\alpha)\frac{dV(\alpha)}{d\alpha}d\alpha \qquad (9)$$

An indicator the work of thermodynamic cycle:

$$A_{i}=A_{com}+A_{exp} \qquad (10)$$

By using ideal gas law conditions can be express the value of the working mass:

$$M_{a}=\frac{p_{a}\cdot V_{a}}{R\cdot T_{a}} \qquad (11)$$

Since the supply of heat is carried out during a-e, and the heat removal process b-d, the cycle COP can be calculated as follows:

$$COP1=\frac{Q_0}{A_i}=\frac{M_a\cdot c_v(T_a-T_e)}{A_i} \qquad (12)$$

Taking into account the return to the cycle of the heat exchanger when used with efficiency $\eta_r = 0.9$, COP machine will increase to a value:

$$COP2=\frac{M_a\cdot c_v(T_d-T_e-\eta_r\left(T_d-T_a\right))}{A_i} \qquad (13)$$

The coefficient of performance machine ideal cycle is determined by the relation:

$$COP_s=\frac{T_a}{T_d-T_a} \qquad (14)$$

Equation (14) coincides with the expression for calculating the coefficient of performance an ideal reverse Stirling cycle.

The degree of thermodynamic perfection can be calculated as follows:

$$\eta_1=\frac{COP1}{COP_s} \qquad (15)$$

$$\eta_2=\frac{COP2}{COP_s} \qquad (16)$$

The input parameters for the calculation is: indicators polytropic compression and expansion of air, on the basis of the range of pressures and temperatures can be defined $n_1 = 1{,}4$, and $n_2 = 1{,}38$, $R = 287\,J/(kg\cdot\text{К})$ $c_v = 723\,J/(kg\cdot\text{К})$.

The numerical solution of differential equations was performed in Mathcad system with a built-in function Odesolve, designed to solve linear differential equations with respect to the highest derivative Runge-Kutta method.

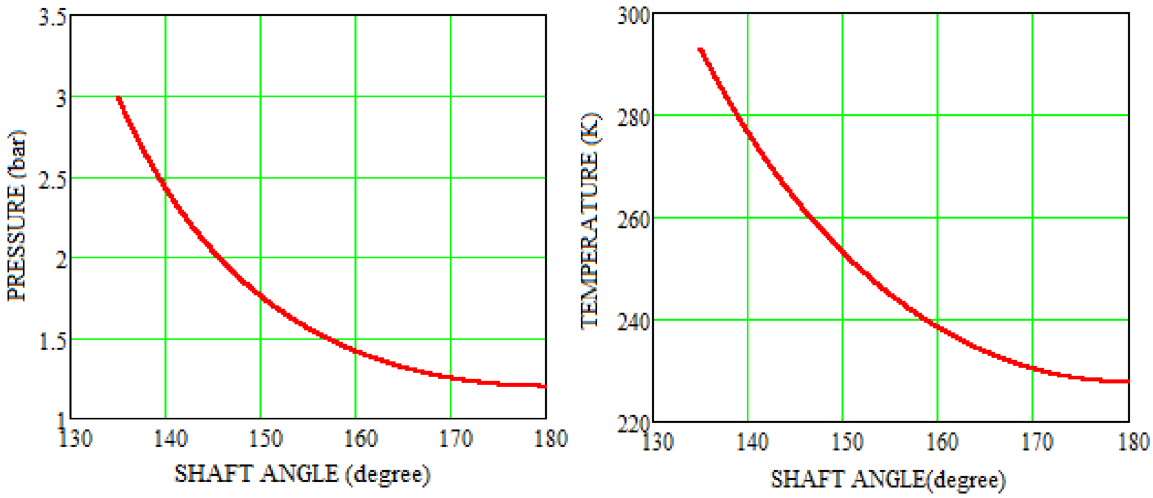
IV. RESULTS AND DISCUSSIONS

We perform calculations for rotary-vane gas refrigerating machine cycle, using the above formulas. RVGRM expansion process flowing when the angle of rotation of the output shaft $135^{\circ} \le \alpha \le 180^{\circ}$. Pressure and temperature at the beginning of the expansion process are $p_d = 3\,bar$ and $T_d = 293{,}15\,K$. Graphs of pressure and temperature in the chamber during the expansion process are shown in Figure 3.

Given graphs show that the pressure in the chamber during expansion changes from $p_d(135^{\circ}) = 3\,bar$ to $p_e(180^{\circ}) = 1{,}2\,bar$, and temperature from, $T_d(135^{\circ}) = 293{,}15\,K$ to $T_e(180^{\circ}) = 227{,}68\,K$.

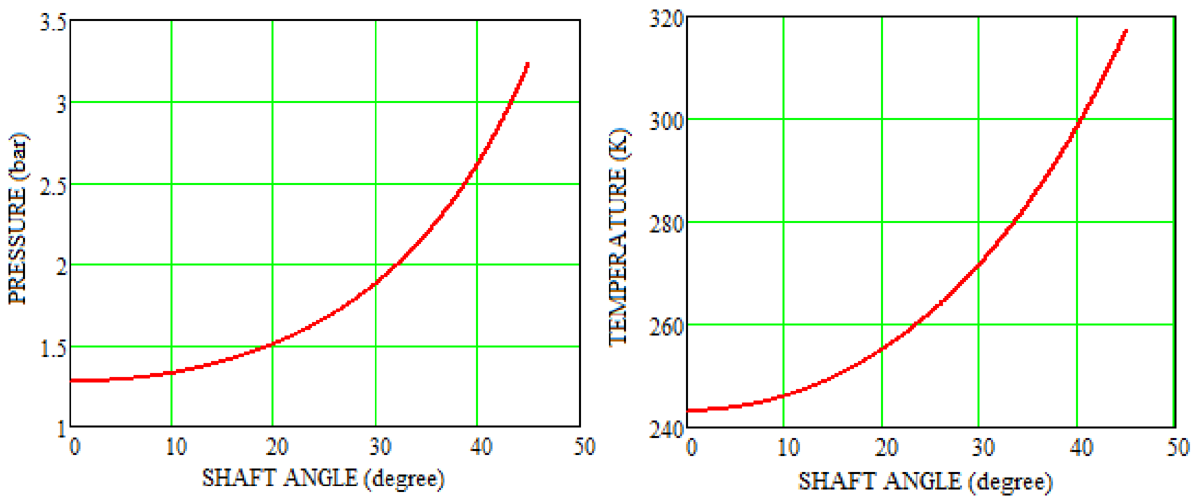
Compression process in RVGRM proceeds at an angle of rotation of the output shaft $0^{\circ} \le \alpha \le 45^{\circ}$. Pressure and temperature at the beginning of the compression process are equal $p_a = 1{,}3\,bar$ and $T_a = 243{,}15\,K$.

Graphs of pressure and temperature in the chamber during the compression process are shown in Figure 4. Given graphs show that the pressure in the chamber during compression changes from, $p_a(0^{\circ}) = 1{,}3\,bar$ to $p_b(45^{\circ}) == 3{,}2\,bar$, as well temperature from, $T_a(0^{\circ}) = 243{,}15\,K$ to $T_b(45^{\circ}) = 317{,}26\,K$. Here are the results of calculations for RVGRM cycle. The results are shown in Table 1.



a) b)

Figure 3 *– Graphs of pressure and temperature dependence:*
a) pressure in dependence on the machine angle of rotation of the output shaft;
b) temperature in dependence on the machine angle of rotation of the output shaft.



a) b)

Figure 4 *– graphs of pressure and temperature dependence:*
a) pressure in dependence on the machine angle of rotation of the output shaft;
b) temperature in dependence on the machine angle of rotation of the output shaft.

Table 1 – Results from one cycle of a rotary-vane gas refrigerating machine

Technology performance attribute	**Value**
Minimum temperature in the machine, T_e	227,68 K
Maximum temperature in the machine, T_b	317,26 K
Heat Transfer to chamber, Q_0	20,77 J
Heat Transfer to chamber, Q_{02}	27,49 J
An indicator work, A_i	6,57 J
COP1	3,16
COP2	4,18
Degree of thermodynamic perfection, η_1	0,65
Degree of thermodynamic perfection, η_2	0,86

Note that the in temperatures below -30°C efficiency of RVGRM are considerably higher than the vapor-compression systems. This complication is associated with the need to move to a two-stage compression in the vapor-compressor plants, which leads to an increase in the cost of these facilities.

Thus, the use of rotary vane gas chiller allows us to operate in a wider range of cooling temperatures (0 to -40°C) in one machine and obtain high energy efficiency.

V. CONCLUSIONS

A mathematical model for calculating the basic physical processes that occur in a rotor-blade gas refrigeration machine is built. The graphs of pressure and temperature dependencies in the working chamber of the angle of rotation of the output shaft are built. It is shown that the installation in RVGRM cycle heat exchanger a coefficient of performance 3.16 brings to an increase in COP by more than 30%.

The research results show that the proposed model can be used for the design and optimization of rotor-vane gas refrigeration machine.

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МАТЕМАТИЧНА МОДЕЛЬ ДЛЯ ДОСЛІДЖЕННЯ І ПРОЕКТУВАННЯ РОТОРНО-ЛОПАТЕВОЇ ГАЗОВОЇ ХОЛОДИЛЬНОЇ МАШИНИ

У статті пропонується математична модель розрахунку основних параметрів робочого циклу роторно-лопатевої газової холодильної машини, які впливають на роботу установки, управління машиною і робочими процесами, що протікають в ній при заданих критеріях. Запропоновано методику аналізу графічним методом роторно-лопатевої газової холодильної машини (РЛГХМ). Вивчено вплив геометричних і температурних змінних на термічну поведінку системи. Модель враховує показник політропи для процесів стиснення і розширення в робочій камері. Отримано графіки залежностей тиску и температур в робочій камері від кута повороту вихідного валу. Оцінено можливість включення в цикл регенеративного теплообмінника. Проаналізовано зміну коефіцієнта перетворення машини після включення в цикл регенеративного теплообмінника. Показано, що установка регенератора в цикл РЛГХМ призводить до збільшення COP більш ніж на 30%. Результати моделювання свідчать, що запропонована модель може бути використана для проектування і оптимізації газової холодильної машини Стірлінга.

Ключові слова: *Газова холодильна машина Стірлінга; Роторно-лопатева холодильна машина; Математична модель; Енергоефективність; Повітря.*