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PROSPECTIVE PHYSICS TEACHERS' USE OF MULTIPLE REPRESENTATIONS FOR SOLVING THE IMAGE FORMATION PROBLEMS

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Abstract. *The aim of this research is to determine the role of multiple representations such as problem picture, ray diagram, and equation used by the students in solution of the image projection problems. The study is performed by the survey method. The participants of this study were prospective physics teachers of a faculty of education from a state university in Turkey. Fifty-three teacher candidates solved open-ended questions and 20 of them were also interviewed.*

The results have shown that the students used two different types of solution method; in solution 1, they used the solution method where "problem pictures, mirror/lens equations and ray diagrams" are used, and in solution 2, they used the solution method where "problem pictures and mirror/lens equations" are used. The results show that students mostly prefer the second solution method, however, the students who prefer the first solution method are more successful than them. Also, it is found that the students have various alternative ideas about the usage of these multiple representations. The reasons of these alternative ideas are discussed at the end of the study.

Key words: *alternative ideas, image formation, multiple representations, ray diagram.*

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Introduction

The Importance of Representations

As Lasry and Aulls (2007) stated, demonstrating an object or a phenomenon by various illustrations may facilitate its understanding. When considered from this point of view, it is quite obvious that the symbolic representations where conceptual knowledge or problem is given in science or mathematics are intended for increasing the comprehension. In other words, correct understanding of what the symbolic representations mean is an important part of education. Alibali, Phillips and Fischer (2009) expressed that while solving problems, children may use incorrect or inefficient strategies, because they are unable to achieve to represent key features of the problems accurately.

The studies related to the effect of the use of multiple representations in problem solving on the development of problem solving performances (Dufresne et al. 1997; Heuvelen and Zou 2001; Meltzer 2005) or the effect of knowledge on the problem representations (McNeil and Alibali 2004; Rittle-Johnson and Alibali 1999; VanHeuvelen 1991) support the importance of the representations. These studies reveal that there is an important correlation between the conceptual understanding and problem solving process and the multiple representations. Rittle-Johnson et al. (2001) stated that developed problem representations would bridge over the rote knowledge to conceptual knowledge.

Revealing conceptual understandings or problem solving strategies of the students is possible by finding out how they used these representations. The fundamental problem of this research is to determine the students' alternative ideas and problem solving strategies by benefiting from these symbolic representations used by the student. Alibali et al. (2009) displayed that both features of a presented problem encoded by the problem solver, and the

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activated knowledge from long-term memory are included within the problem representations. In this present study, multiple representations have been defined as external representations used in practice such as mathematical, diagrammatic or graphical representations as indicated by Meltzer (2005).

Multiple Representations in Physics Education and the Underlying Reasons to Choose Studying about How Students Use these Representations

Usage of multiple representations (MRs) is a very important part of physics education. In other words, representing a physics problem by words, equations, graphs, or pictures can significantly affect the performance of students on that problem (Kohl and Finkelstein 2006). Rosengrant et al. (2009) stated that we must help our students learn to construct different representations and to use them for problem solving in order to make them expert problem solvers.

Various studies emphasized the importance of MRs in learning physics and solving problems. Multiple representation studies done by physics education research community between 2003-2005 were categorized into three categories by Rosengrant et al. (2007). And the next studies can be classified into these groups. According to this, MRs studies can be grouped as the studies (Heuvelen and Zou 2001; Kohl et al. 2007; Kohl and Finkelstein 2006; Van Heuvelen 1991) where MRs were used to help students learn concepts, solve problems and use representations, the studies (Kohl and Finkelstein 2008; Rosengrant et al. 2009; Rosengrant et al. 2006) about how MRs were used by students in problem solving, and the studies (Kohl and Finkelstein 2006; Meltzer 2005;) where MRs were used in constructing the problem. Also, some earlier studies have focused on how experts and novices differ in their use of MRs (Kohl and Finkelstein 2008; Rosengrant et al. 2009).

Whereas there are many studies reporting that the usage of multiple representations in physics education literature increase students' learning. As stated in the studies of Kohl and Finkelstein (2008), Rosengrant et al. (2009), and Rosengrant et al. (2006), there are only a few studies reporting how students use these representations, the thinking processes used by them meanwhile, and the quality of the representations developed by the students. However, as McDermott stated, special difficulties with various representations used by the students should be defined (Meltzer 2005). In his study, Meltzer (2005) revealed these difficulties on the problems presented by various representations. In the study by Rosengrant, Heuvelen and Etkina (2009), the usage of one of the representations, the qualities of free body diagrams used by the students especially, and the reasons of the usage were analysed in detail.

In this study, the students' usage of multiple representations in optics problem solutions will be analyzed.

Why Geometrical Optics Problems Were Chosen in this Study?

All of the aforementioned researches related to MRs in physics education have been in electricity, magnetism, work-energy or force and motion. However, this study focussed on the representations used in geometrical optics problems. In the study, the representations will be used as external representations. External representations in physics include words, pictures, diagrams, graphs, computer simulations, mathematical equations, etc. (Rosengrant et al. 2006).

In geometrical optics, these representations will be considered as ray diagrams, mirror/lens equations, and problem picture. Light-Ray Tracing (ray diagram) is one of the geometrical optics techniques which analyzes the interaction of light with optical instruments based on divergence and convergence of light (Isık 2008). A ray diagram is a tool used for determining the location, size, orientation, and type of image formed by an optical device.

The mirror/lens equations (Ohanian 1989) are the equations used for calculating the image distance by means of focal length and object distance which occurred by the usage of the ray diagram based graphical method in order to calculate the place of the image. The most important characteristics of these equations are the sign conventions which are a bit different for mirrors and lenses. In addition to this, the problem picture has a quite important place in the geometrical optics problems as well as in many physics problems. General view of the optical system, that is the shape and place of the optical devices and drawing of principal axis and distance drawings on it are quite important in problem solutions.

In geometrical optics, there are many important studies related to especially diagrammatic representation of light and image projection which are done on various age groups (Colin et al. 2002; Colin and Viennot, 2001; Galili et al. 1993; Galili and Hazan 2000; Goldberg and McDermott 1987; Goldberg and McDermott 1986). As a result of these studies, significant misconceptions of students were determined relating to the nature of light rays, their knowledge about the image, image formation in the mirror or lens systems; the relation between the observer and the image; the difference of images in plane and concave mirrors and the roles of lenses and mirrors. These studies also revealed, that students at various levels from primary school to the college level had difficulty in drawing the correct ray diagram, understanding the sight at a plane mirror, understanding the image formation process in a plane mirror. However, usage of problem picture, ray diagram, and mirror / lens equation representations in problem solutions, furthermore, usage of them in different type of optical problems where optical systems consisting of two optical devices exist, were analysed in none of these studies. Moreover, the quality of the representations used by the students in geometrical optics problem solutions, and the correlation of it with the students' problem solving success were not investigated. However, the analysis of image formations in the optical systems where more than one optical instrument is used, is asked in many physics textbooks and the examinations at the secondary school and college levels. In these systems the procedure of the multiple image formation of a single object is analysed and at this step students have many difficulties. The tools which are put in for the solution of these problems are mostly problem pictures, light ray diagrams and equations. This also indicates, that students' difficulties should be investigated in these representations.

Purpose of the Study

The purpose of this study is to analyse how the students use multiple representations (problem picture, ray diagram, mirror/lens equation) in the solutions of the image projection problems constructed by more complex optical systems where two optical devices exist and then to reveal the alternative ideas developed by the students related to the usage of them. The research questions which direct this research are:

1. How often do the students use multiple representations for solving these problems?
2. What are methods of solving these problems regarding their use as multiple representations?
3. How does the correctness of their solutions vary regarding their methods of solving?
4. What is the quality of the multiple representations they used during the solution of these problems?
5. What are the students' alternative ideas related to the use of multiple representations in their solutions?

Methodology of Research

The current investigation was in survey model to describe the understanding of multiple representations of the prospective physics teachers while solving image formation problem of geometrical optics. The nature of student failures in solving geometrical optics problems prompted the investigation of how students construct and apply representations to the geometrical optics problems. The data of the research were gathered in autumn 2012.

Participants

The participants of this study were prospective physics teachers of a faculty of education from a state university in Turkey. Fifty-three teacher candidates participated in the study, and 20 of them were also interviewed. The average age of the participants was between 20 and 22. The students who par-

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ticipated in the study had completed optics course at least one semester before. Optics course is given as five theoretical and two laboratory hours at physics education department.

The course includes the subjects of electromagnetic waves and optics, reflection, refraction, dispersion, phase, geometrical optics, optical instruments, polarization, optical activity, birefringence, interference, Fraunhofer and Fresnel diffraction. This course is very similar to courses called mostly optics or optics and waves given at some western European or American Universities regarding either its credits or content. Nevertheless, the course includes brief introduction to some modern optics application (such as lasers, masers and holography) but the quantum optics. In addition, the content of these subjects is deeper than the optics sections in general physics courses, but more superficial than the optics courses which are specially given. The same situation is valid for the optics laboratory. In the laboratory applications, recipe type experiments are conducted towards ray optics, wave optics, interference and polarization. The students enrolled in this course did not receive the optics subjects in general physics course in their previous years at university and their prior knowledge about optics is based on the physics course they received at secondary school. Therefore, this course should be considered as basic optics course at first step. Another reason for not delivering this course at advanced level is that the students are educated to be physics teachers in secondary schools. The highest disadvantages of the course are the misconceptions due to the practical solutions to the problems which are developed as a result of anxiety to the limited time during the university settlement examination, the recipe type, low capacity experiments which are conducted as a result of inadequate circumstances.

In geometrical optics part of this course, refractive and reflective optical devices and image projections in the optical systems constituted by these optical devices are discussed in detail, and in laboratory part, experiments are carried out about refraction index, the focal length of reflective and refractive systems, and the place of the image in optical systems. The lectures were given by the lecturers using discussion method. Ray diagrams are frequently used in image projection and distance equations have been used from four numerical solutions.

Instrument

Data of the research were obtained by using a questionnaire form and a semi-structured interview form which includes three types of image projection problems in geometrical optics (see Appendix A). These are the problems where the place of the object is known, but the place of the image is asked (N=2), the place of the image is known, but the place of the object is asked (N=1), and the places of the optical devices within the optical system are asked (N=1). Problems were constructed on the optical systems, including at least two optical devices, where mirrors and lenses are used. The problems include many events which can be acquired during the analysis of image formation procedure where more than one image is formed from a single object. These are re-reflection and refraction, drawing of backswept rays, the real images of virtual images, the virtual images of real images, drawing of the optical system, use of sign and etc. Problems especially are defined verbally so that allowing students organise their own problem pictures which are assumed to be one of the multiple representations.

By the help of two experts, four problems in medium difficulty and appropriate to students' levels, at least one from each problem types above, were selected among 12 problems constructed. In order to check the comprehensibility of the problems, a pilot experiment was carried out with five students, and the incomprehensible points were corrected. Selected four questions were given to 53 students on a question form, and 60 minutes were given for the solution of the problems. The students were asked to write down every representation that they think of, or use in the solutions of the problems. This application is done both for displaying the performance of the students in the representations they used according to the problem type, and for determining the students to be selected for the interview. After the analysis of the students' answers, 20 students were selected from the groups constituted according to the representations they used in the problem solutions. A semi-structured interview was carried out with these students three months after the application intended to make them resolve the same questions.

Data Analysis

Teacher candidates answered the questions by writing their solutions on the paper where four questions exist. In the analysis of these answers, first of all, the usage frequency of the problem picture, ray diagram and mirror/lens equations which are the representations used by them in their solutions were determined according to the problem type. As a result of this, two solutions used by the students for each type of problem were determined. After this stage, in order to determine the quality of the representations, each representation was coded using the rubric in Table II, a similar ratio rubric was also used for the same aim by Rosengrant et al. (2009). Data were separated for two different solution methods. The answers were recoded in a one-month interval with this rubric. This encoding performed at two different times had an inter-rater reliability of 0.80.

At the end of these analyses, in order to evaluate the students' mistakes in detail, interview data performed with 20 students were selected, as at least 10 students from both solution methods corresponding to each problem type were analyzed by qualitative data analysis method for each problem type. Interview data were collected by means of the notes taken by the researcher, sound recordings, and the solution paper of the students. During the interview, the students were asked to think aloud while solving the problems, and when their ideas were not clear or they had difficulty in thinking aloud they were asked some probing questions (such as "why did you think like that?", "what if…?, "how would you do….?"). Drawings and explanations containing mistakes within data obtained were encoded and called as "alternative ideas", and these ideas were classified into categories in itself, and each category was named by the researcher. In order to validate the interview data the name and the context of the categories were discussed with an expert of physics education and the common results were concluded. The answers were recoded in a one-month interval by the same researcher. The level of agreement between this coding, considered as the reliability of the procedure, was found as 0.86.

Table 1. Rubric for coding representations.

Representations	1 Inadequate	2 Needs improvement	3 Adequate
Problem pictures	Representations are constructed, but contain major errors or misconceptions such as focal point displayed behind the convex mirror, a lens with a single focal point, the convex mirror drawn as a concave mirror, etc.	The problem picture drawn is not wrong, but contains missing points such as undrawn focal point, or non-symmetrical principal axis drawn, etc.	Shapes and places of optical devices are drawn correctly, and focal point and center on the principal axis are correctly determined.
Lens/ mirror equation	Representations are constructed, but contain major errors or misconceptions such as missing or extra equation, wrong signs taken etc.	The equations written are not wrong, except that there are some mistakes, such as, the mistakes due to operational mistakes, wrong placed distances although displayed correctly in drawing, etc.	Equations are written correctly, distances and signs are taken correctly, and the operations are done correctly.
Ray diagram	Representations are constructed, but contain major errors or misconceptions such as single ray usage, confusing reflection and refraction with each other, etc.	The rays drawn are not wrong, drawn ray diagram is correct, but deficient, not all of the images are drawn by ray diagram.	Rays are drawn adequate and correctly as to form all images.

Results of the Research

Document Analysis

The features of the problems given to the students to solve are displayed in Table 2. Usage frequencies according to the problem type of problem picture, ray diagrams and mirror/lens equation which students use while solving optical problems are presented in Table 3.

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Table 2. Problem types related to image projection.

Problem types (Total number of the problems)	The features of the problem	Total number of solutions
First type problem (2)	The place of the object is known, but the place of the image is asked	53 x2= 106
Second type problem (1)	The place of the image is known, but the place of the object is asked	53
Third type problem (1)	The place of the optical devices is asked	53
Total		212

Table 3. Usage frequencies of the representations used in the solution of each problem type.

Problem types (Total number of solutions for this problem)	Representations			
	Pictures (f)	Ray diagrams (f)	Focus Equation (f)	Unanswered (f)
First type Problem (N=106)	87% (92)	29 % (31)	87% (92)	13% (14)
Second type problem (N=53)	83 % (44)	30 % (16)	83 %(44)	17 %(9)
Third type problem (N=53)	83 % (44)	36 % (19)	83 %(44)	17 % (9)
Total (212)	85% (180)	31%(66)	85%(180)	15% (32)

According to Table 3, all of the students who solved the problem used the problem picture and mirror/lens equation for all three problem types. Table 3 displays that the students use two types of solution methods for these three problem types. In the first type of solution method, the students used problem picture, ray diagram, and focal point equation; and in the second type of solution method, the students used problem picture, and focal point equation without using ray diagram. In this study, these solution methods will be called as "First Solution Method (FSM)" and "Second Solution Method (SSM)".

Table 4 shows the distribution of the students' answers according to these solution methods for each problem type.

Table 4. Distribution of the solution methods according to the problem type.

Problem types (Total number of solutions for this problem)	FSM % (N)	SSM % (N)	Unanswered % (N)
First type Problem (N=106)	29 (31)	58 (61)	13 (14)
Second type problem (N=53)	30 (16)	53 (28)	17 (9)
Third type problem (N=53)	36 (19)	47 (25)	17 (9)
Total (212)	31 (66)	54 (114)	15(32)

It is understood from Table 4, that the FSM is frequently used in the third type of problem (36%), and the SSM is frequently used in the first type of problem (58%). Moreover, it is seen that the second solution method (SSM) (54%) is preferred more than the first solution method (FSM) (31%).

From here on, the answers whose solutions are not understood or not solved, i.e., the "unanswered" part in Table 4 will not be taken into consideration since they can not be classified into any solution method.

Distribution of the correctness or wrongness of the results reached by the solution methods used according to the problem types are given in Table 5 and Table 6. Here, not only the "Adequate" answers, but also the solutions corresponding to "needs improvement" part in Table 1, which does not contain any mistake, are taken as a correct answer.

Table 5. Total values of correct and incorrect responses for the first solution method.

Problem types (Total number of solutions for this problem.)	Correct % (N)	Incorrect % (N)
First type Problem (N=31)	61 (19)	39 (12)
Second type problem (N=16)	69 (11)	31 (5)
Third type problem (N=19)	53 (10)	47 (9)
Total (66)	61 (40)	39 (26)

According to Table 5, the students who used the first solution method are most successful, the second type of problem (69 %), and the least successful are those, who used the third type of problem (53%).

Table 6. Total values of correct and incorrect responses for the second solution method.

Problem types (Total number of solutions for this problem.)	Correct % (N)	Incorrect % (N)
First type problem (N=61)	33 (20)	67 (41)
Second type problem (N=28)	29 (8)	71 (20)
Third type problem (N=25)	40 (10)	60 (15)
Total (N=114)	33 (38)	67 (76)

According to Table 6, the students who used the second type of solution method are most successful, in the third type of problem (40%), and the least successful are in the second type of problem (29%).

Table 5 and Table 6 display, that the students who used the first solution method (61%) are more successful than the students who used the second solution method (33%).

The results of the analysis for determining the comprehensibility level of the representations used by the students are given in Table 7.

Table 7. Students' success rates of the representations used in the problem solutions.

Representations (Total number of solutions which this representation was used.)	Success rate		
	1 Inadequate %	2 Needs improvement %	3 Adequate %
Pictures (N=180)	15 (27/180)	70 (126/180)	15 (27/180)
Ray diagrams (N=66)	65 (43/66)	21 (14/66)	14 (9/66)
Mirror/lens Equations (N= 180)	57 (102/180)	10 (19/180)	33 (59/180)

Table 7 shows, that the students have mistakes related to the usage of problem picture, ray diagram, and mirror/lens equation representations. They have quite a lot inadequacies especially in the subject of the ray diagram (65%) and focal point equation (57%) usage. Details of these will be given in interview analyses. In addition to this, it is seen for the problem picture, that in the question containing lenses, 65% of the students had shown the lenses by arrow sign, and the rest of them had drawn glass lenses having two spherical surfaces. These two drawings are evaluated similarly at this stage, but different results caused by these two drawings are encountered, as mentioned in discussion part.

The results of the analysis done to observe the change in the comprehensibility of the representations according to problem solution methods are given in Table 8 and Table 9.

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Table 8. Students' success rates of the representations who used the first solution method.

Representations (Total number of solutions which this representation was used.)	Success Rate		
	1 Inadequate %	2 Needs improvement %	3 Adequate %
Pictures (N=66)	27 (18/66)	32 (21/66)	41 (27/66)
Ray diagrams (N=66)	65 (43/66)	21 (14/66)	14 (9/66)
Mirror/lens Equations (N= 66)	38 (25/66)	9 (6/66)	53(35/66)

Table 9. Students' success rates of the representations who used the second solution method.

Representations (Total number of solutions which this representation was used.)	Success Rate		
	1 Inadequate %	2 Needs improvement %	3 Adequate %
Picture (N= 114)	8 (9/114)	92 (105/114)	-
Mirror/lens equation (N=114)	68 (77/114)	11 (13/114)	21(24/114)

According to Table 8 and Table 9, there are some inadequacies related to representations while using both solution methods. However, adequacies of the students who used the FSM in usage of the mirror / lens equation (53%) are quite higher than the students who used the SSM (21%). Moreover, problem pictures of the students who used the FSM are more adequate than the students who used the SSM.

The quantitative analyses performed above displayed that there are some inadequacies of representation usage in both solution methods. In the next part, the interview analyses will be given, and what kind of difficulties the students have while using representations in the first solution method and the second solution method will be discussed.

Interview Analysis

This part is composed of the detailed analysis of semi-structured interview, which tried to figure out the alternative ideas which students had behind their multiple representations. As a result of the investigation of the students' answers, interviews were performed with 20 students selected, provided that 10 students correspond to one type of solution methods for each problem. As a result of the analysis of data obtained from the interviews, students' alternative ideas are classified into certain categories. Frequencies of these alternative ideas within the total number of solutions are given in the tables. According to this, Table 10 shows the alternative ideas of 10 students who solved all problems by the FSM related to ray diagram representation usage in their solutions. Table 11 shows the alternative ideas related to the usage of problem picture and lens/mirror equation representations in the solutions of 10 students who used the SSM and 10 students who used the FSM; and Table 12 shows the common alternative ideas of both student groups related to the subject of "reality" and "virtuality". The alternative ideas given in tables were obtained from their oral explanations and drawings during the interview. In order to support these ideas which are thought to be understood better visually, some samples of students' solutions during the interview are given (Figure 1-7).

Table 10. Alternative ideas of the students who used the first solution method related to ray diagram representation.

Categories	Students' alternative ideas	FSM (f)
1. Image projection from image by means of the rays	1.1. Image behaves like a light source, and emits light in all directions. (Fig. 1b)	17
2. Image projection by means of the rays	2.1. Image can be formed by means of a single light ray. (Fig. 1;2;4)	29
	2.2. A light ray coming from a point of the object can form a different point of the image.	8
	2.3. The image is formed on the point where a light ray coming from the object intersects the principal axis. (Fig. 1;2)	29
	2.4. The image is formed on the point where any two light rays intersected each other.	3
3. Behaviour of spherical mirrors on light	3.1. The light ray coming parallel to the convex mirror reflects and passes from the focal point in front of the mirror.	3
	3.2. The light ray coming parallel to the convex mirror refracts and passes from the focal point behind the mirror.	2
	3.3. Refraction of rays coming to the mirrors	7
4. Behaviour of lenses on light	4.1. Reflection of rays coming to the lenses	6
	4.2. The light ray coming parallel to the concave lens refracts and passes from the focal point behind the lens.	4
	4.3. The light ray coming from behind of the lens is not affected by the lens. (Fig. 2)	4
5. Virtual image projection in lenses	5.1. The convex lens which forms a virtual image behaves like a concave lens. The convex lens forms virtual image by diffusing light. (Fig. 1a)	4
	5.2. The lens forms image in front of it by reflecting the rays. (Fig. 1c)	5

It is seen from Table 10, that most of the students have misconceptions on image projection by a single ray, and image projection where the rays intersected the principal axis. In addition to this, the students have more misconceptions about the behaviour of lenses on light as compared to the mirrors.

One of the common explanations of these students having the alternative idea of 1.1 related to this situation is like that "… now as the new object is this (i_1) we have to send new rays from this."

Alternative idea 2.1 is found in many solutions. As can be seen from the Figures1, 2, and 4, the students use only one ray in order to obtain the image. Moreover, the students formed the images where the rays crossed the principal axes in a way to support the alternative idea of 2.3 as seen from the Figures 1 and 2. This situation was usually met with the students who tried to obtain image by using only a single ray.

Table 11. Alternative ideas of the students who used the first and the second solution method related to problem picture and mirror / lens equation representations.

Categories	Students' alternative ideas	FSM (f)	SSM (f)
6. Property of the optical system	6.1. The focal point of the convex mirror is in front of it.	3	2
	6.2. There is a focal point both in front of and behind the convex mirror.	2	1
	6.3. Convex lens has focal point only in front of it.	2	7
	6.4. Optical system has only optical devices.	-	6

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Categories	Students' alternative ideas	FSM (f)	SSM (f)
7. Using mirror / lens equation	7.1. Distances are taken/displayed according to the points where the rays or their projections intersected the principal axis. (Fig.1; 2)	29	-
	7.2. All image distances on the opposite side of the object are taken as negative.	5	9
	7.3. "Minus" (-) sign means that the image is on the opposite side of the object. (Fig. 1c;7)	7	15

It is seen from Table 11, that the students have difficulties in deciding the place of the focal point especially in a convex mirror. As in the mirror and lens equations, it is seen that the students who use the FSM make mistakes in determining the distances; whereas the students who use the SSM have more misconceptions about the signs.

As it can be seen from the alternative idea 7.3, some students often used the expression of "the place for this image is on the other side of the object" for the "negative" distances in their solutions. In the examples of Figures 1c and 7, some students placed the image in the wrong side by having this alternative idea based on the "negative distance" they found in their solutions.

Table 12. Alternative ideas of the students who used the first and the second solution method related to virtuality and reality subjects.

Categories	Students' alternative ideas	FSM (f)	SSM (f)
8. The place of the virtual image in lenses and mirrors.	8.1. The virtual images occurring on convex lens and concave mirror are in the same side of the object.	3	5
	8.2. The virtual image occurring on concave lens and convex mirror is in the opposite side of the object.	3	5
9.Determining on virtuality and reality.	9.1. All of the images on the other side of the optical device according to the object are virtual. (Fig. 1b).	8	22
	9.2. The virtual image formed by an optical device in a system is always the virtual object of the other optical device. (Fig. 3;5a;6)	9	22
	9.3. Not able to recognize the virtuality of the image given in the question.	5	7
	9.4. The virtual image of a virtual object is on the same side with the object. (Fig. 5b;6).	2	19

The alternative ideas given in Table 12 are the alternative ideas which we encountered on the students' explanations done by both solution methods. It is understood from Table 12, that the students who use the SSM have more difficulties related to the category of "determining on virtuality and reality" than the students who use the FSM.

The students, having alternative idea of 9.4, during the solution shown in the Figure 5b explained the position of i_3 that "it is at the same side with the object" by obtaining the virtual image i_3 from the i_2 object which is virtual according to the lens. When its reason is asked, they indicated that both of them were virtual. On the other hand, in the Figure 4, the students viewed the image i_1 as virtual object for the convex mirror and used the idea of virtuallness being on the same side while determining the place of the virtual image (i_2) of i_1 in the convex mirror.

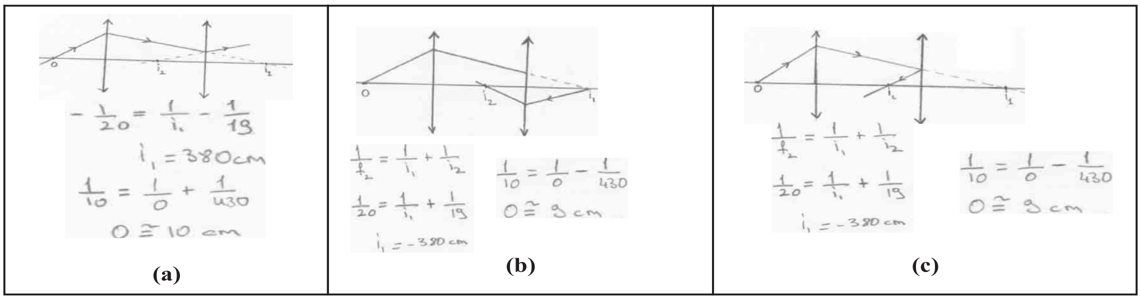

Figure 1: **Types of students' multiple representations in their responses to the problem 3: first solution method.**

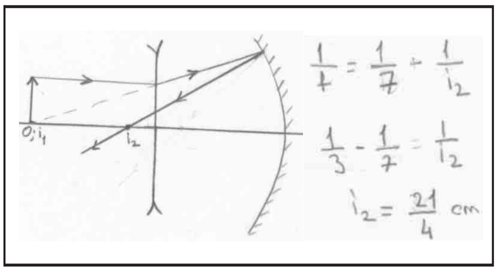

Figure 2: **A type of students' multiple representations in their responses to the problem 2: first solution method.**

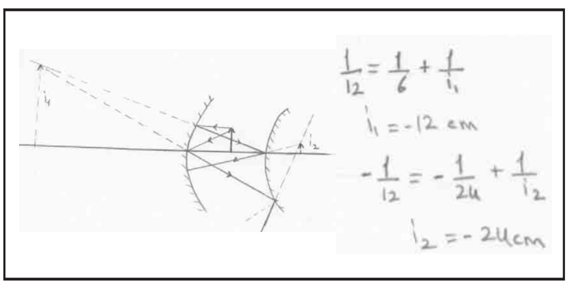

Figure 3: **A type of students' multiple representations in their responses to the problem 1: first solution method.**

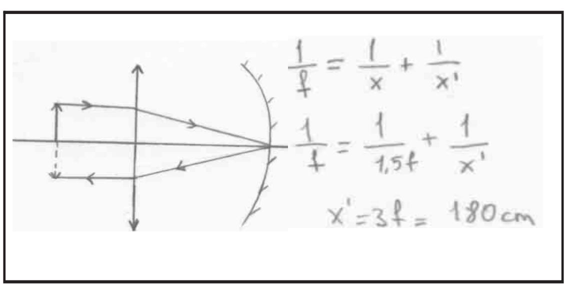

Figure 4: **A type of students' multiple representations in their responses to the problem 4: first solution method.**

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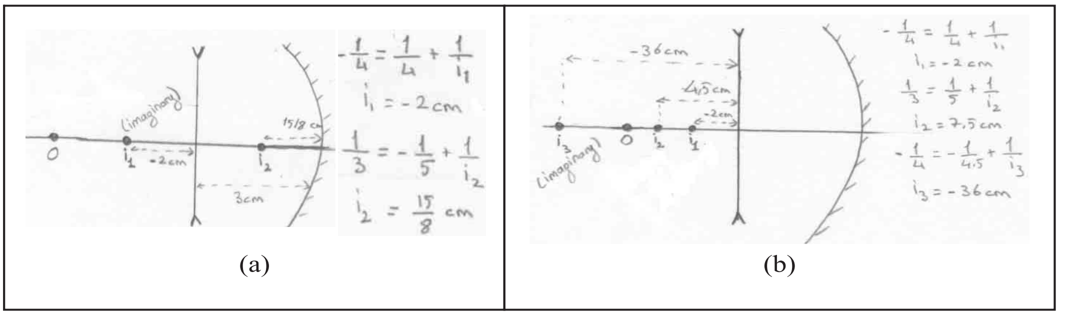

Figure 5: **Types of students' multiple representations in their responses to the problem 2: second solution method.**

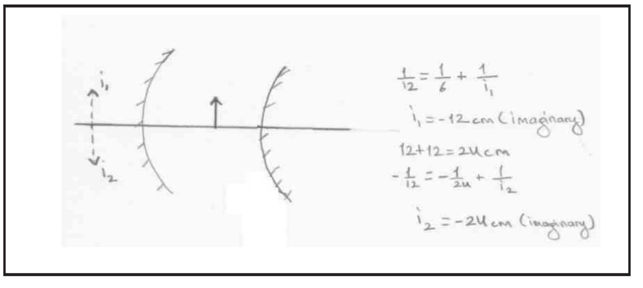

Figure 6: **A type of students' multiple representations in their responses to the problem 1: second solution method.**

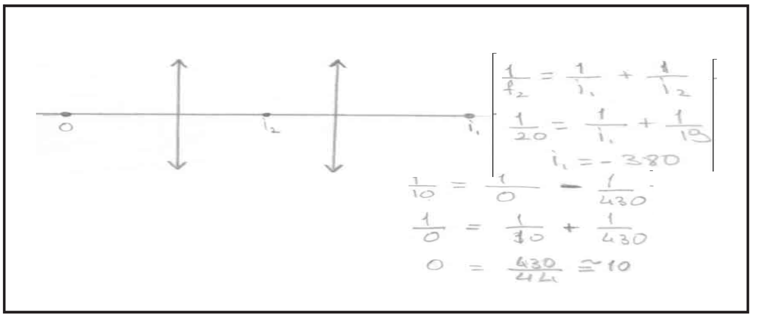

Figure 7: **A type of students' multiple representations in their responses to the problem 3: second solution method.**

Discussion

In this study, the role of the multiple representations (problem picture, ray diagram, mirror / lens equation) which are used as a tool in the solution of image formation problems containing two optical devices were investigated. In this way, it was trying to set forth how often the students use those representations in the solution of the problems mentioned, what kind of solution methods they used according to their representations, their problem solving success according to their methods of solutions, the quality of the representations they used during these solution methods and their alternative ideas towards the use of these representations during the image formation process.

In the study it is seen, that the students continuously used the problem picture and mirror / lens equation spontaneously while solving image projection problems (Table 3). When we think of the importance of mirror / lens equation in the solutions of such problems, it can be said that problem picture which is also used in the same frequency is also as important as the equations which help them solve the problem for the students. The importance of the problem picture was emphasized in many studies. In comprehending the problem, drawing the problem picture is quite important. In addition to this, it is seen that the students did not use the ray diagram in all of the solutions. And this situation shows us that the students used two types of solution methods in such types of problems; 1. first one is the problem solution by using the ray diagram, problem picture and the mirror / lens equation, that is the First Solution Method (FSM); and 2. second one is the problem solution by using the problem picture and the mirror / lens equation without drawing any ray diagram , that is Second Solution Method (SSM).

The results show that the students use the SSM more often than the FSM (see Table 4). It is thought that this situation is caused by their difficulties in drawing a light ray diagram, and not understanding the importance of light ray diagrams since they could not read the light ray diagrams correctly.

An important result is that the students who use the FSM, in other words, who use the ray diagrams are more successful than the students who use the SSM in all of the problem solutions (Table 5 ve Table 6). The reason of this is that correctly drawn and correctly read light ray diagrams mostly prevent the students from making mistakes during the solution.

When the qualities of the representations used by the students are examined, it is seen that there are inadequacies in the usage of all three representations (Table 7). It is remarkable that there is a high rate of inadequate usage, especially for the light ray diagram, and then the mirror / lens equation. The reason of light ray diagrams' being adequate in a quite low percentage ratio of 14% is that the students do not draw the ray diagrams completely. This situation is understood from both written answers and the interview easily. And the students' inability to draw the light ray diagrams completely is caused by their not knowing how the light rays would behave except the special light rays. And this generally causes a drawing which starts with a special ray not to be completed when the ray passed to the second optical device. And the problem pictures of the students are mostly in "needs improvement" stage (Table 7). The reason for having the great majority of this ratio in the SSM stage can be that the students, who use this solution method, did not need to draw the problem picture in detail (Table 9).

The most remarkable result is that the students who use the FSM have a quite higher adequacy of mirror / lens equations (53%) than the students who use the SSM (21%) (see Table 8; Table 9). The most important reason of this situation is that the ray diagrams help the students use the mirror / lens equations and especially the signs correctly as definitely seen from the interviews. Even inadequate ray diagrams drawn by a single light ray help the students differentiate between reality and virtuality and use the correct sign in the equation.

It appears in the analysis of interviews more clearly how students use the representations in problem solving, and what kind of alternative ideas they have (Table 10-11; Figure 1-7). For the students who use the first type of problem solution, their alternative knowledge related to usage of ray diagrams is collected under the following subfields such as "Image projection from image", "Image projection by means of the light rays", "Behaviour of spherical mirrors on light", "Behaviour of lenses on light", "Virtual image projection in lenses". In the alternative knowledge existing in the first category, the idea of "the image will behave as a light source" is accepted; this situation supports the holistic conceptualization model of the image in the studies of Galili and Hazan (2000) and Tao (2004). Whereas in the subcategory of "Image projection by means of the rays", it attracts attention that the students prefer a single ray for image projection as encountered in the studies of Galili and Hazan (2000) and Galili, Goldberg and Bendall (1993). As encountered in these studies, here the students also have the idea of "any point of the object can be represented by a light ray" and "a single light ray can represent whole of an object" (2.1.; 2.2.; 2.3. in Table 10). Moreover, the students' forming the image by intersecting a light ray coming to a system with a light ray going out from another system (2.4. in Table 10) shows us that they are not aware of the rules of image projection and they do not know how to use the light ray diagrams.

The most important difficulty in the subcategories of "Behaviour of the optical devices (spherical mirrors and lenses) on light" is the students' confusion of the behaviours of the mirror and lens as seen in

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some other studies (Cazorla and Cervantes, 1989; Kocaküllah 2002; Palacios et al. 1987). These confusions are caused by not knowing how and why the optical devices are used as Palacios et al. (1989) stated. It is thought that students', especially the great majority of the students using the reflection instead of refraction in the lenses is caused by their use of an arrow shape lens representation. The quantitative data of the study also support this situation which is observed in the interviews. When the students' drawings are examined for this purpose, it is seen that the great majority of the students who made this mistake drew the lenses in arrow shapes (82%), whereas it is seen that the students who drew both sides of a lens can be differentiated as they made a very few mistakes in this subject (18%). The representations drawn in arrow shape do not give the students feeling of the light's entering into a different media. Moreover, the expression of "the light ray coming parallel to the principal axis passes from the focal point" memorized as a jingle for the parallel light ray, causes mistakes in the drawings done in diffusing systems. The diffusion phenomenon in convex mirror and concave lens is still an unsettled knowledge. Moreover, the students think of the lenses as affecting only the light rays coming from in front of them as similar to the mirrors (4.3. in Table 10). These faulty "front" and "behind" concepts in the lenses cause the students find the number of images wrong.

For the students who solved according to the FSM, the difficulties in equation usage are mostly encountered in determining the distances (7.1. in Table 11). The most important reason for this is, that the students often try to display the images by a single light ray, and they think that the image will be formed on the point where the ray intersects with the principal axis. Moreover, although it is rarely encountered in the FSM solutions, in the SSM, it is seen that the students often have difficulties in commenting the meaning of the minus sign (-) or in deciding which distances they will display with a minus sign (-) (7.2.; 7.3. in Table 11). This situation is resulted from the students' alternative ideas related to the "virtuality" and "reality" (Table 12). It is also put forward in the studies of Palacios et al. (1989) that the students have difficulties in differentiating the virtuality and reality. It is understood from Table 12 that misconceptions related to the category of determining virtuality or reality are encountered more often among the students who use the SSM than the students who use the FSM. This situation shows that using the rays is a more practical way for the students to differentiate between the virtuality and the reality. While the students who use the FSM make correct decisions about the virtuality and the reality mostly by means of the light rays, the students who use the SSM make wrong decisions mostly regarding the place of the image with respect to the object (8.1.; 8.2.; 9.1 in Table 12). Especially the idea of the image in the *opposite direction* with respect to the real object indicating the "negativeness" is quite dominant (9.1. in Table 12).

In addition to this, the idea of thinking that these signs represent the directions causes the development of the idea that the images which have similar characteristics (virtual / real) will always be on the same side with respect to the optical device, in other words, there is an area where virtual images occur within the system (9.4. in Table 12).

Another important result is the alternative idea that the image of a virtual object should also be virtual which is also seen among the students who use the FSM, but not as much as among the students who use the SSM (9.2. in Table 12). As stated by Galili and Hazan (2000), the students think that the image is a copy of the object. For the students who use the FSM, this situation resulted from the students' accepting the rays as a physical substance responsible to transfer the image, as stated by Hubber (2005). This situation is also supported by the misconception of image projection by a single ray (Table 10). This idea was developed only for the images of the virtual image in both groups, the students could not imagine the transformation from virtuality to reality, and they perceived the virtual image as an abstract substance, and the real image as a concrete substance.

Conclusions

The results of the study have shown that the light ray diagram representation which is used in image formation problems reduces the mistakes in problem solutions. Drawing the light ray diagram results in the correct use of the other representations. Light ray diagrams are very important for the comprehension of the image formation process. Especially, light ray diagrams are important for comprehending the virtuality and reality concepts, and finding the number of images. On the other hand, students' assignment of a different meaning to the rays or their inability to draw or read the ray diagram correctly leads them to develop different misconceptions. As stated by Raftapolus et al. (2005), the light ray diagram is an important tool necessary to be used in teaching the optical phenomena as long as a geometrical model of it is developed in appropriate content according to the optics subject, and necessary precautions are taken. The precautions to be taken here are very important, especially, the students should have developed the idea of the light rays being only a tool displaying the way of light.

The results of the study have shown that some phenomena should be emphasized during the instruction of optics subject. It is necessary to explain the image projection process by means of ray diagrams, and the virtuality and reality concepts during this process, to develop the idea that the image of virtual object would be real as the real object might have virtual image by applying the light ray principle of returning back on the same way, and to infuse that light ray diagram is actually a technique to apply reflection and refraction laws into the students, thus to make them comprehend how to use the other rays except the special rays, and to explain the functions of lens and mirror by means of experiments.

It is thought that the results of the study would be helpful for teaching optics for secondary and tertiary levels. Moreover, it is obvious that more detailed ideas would occur by increasing the variety of question types in the study, for example by various questions where object and image properties were added to the distances. Also, important contributions can be made to pedagocigal content knowledge and MRs literature by comparing the usage of these MRs by teacher candidates with the usage of secondary school students.

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Appendıx A: Exam problems analysed

Question 1. A concave mirror and a convex mirror which has a focal length of 12 cm have been placed as their principal axes coincide with each other. The distance between the mirrors is 12 cm., and there is a candle in the middle of the mirrors. Find the distance of the 2nd image occurred in the system to the concave mirror after making the first reflection on the concave mirror.

Question 2. An object is placed 4 cm distance from the divergent (concave) lens which has a focal length of - 4cm on the left of the lens. Find the distance of the final image occurred in the system to the lens after a concave mirror with a radius of 6 cm is placed 3 cm distance from the lens on the right.

Question 3. Two convergent (convex) lenses which have focal lengths of 10 cm and 20 cm respectively are placed 50 cm distance from each other as their principal axes coincide with each other. It is expected that the final image will be between the lenses and 31 cm distance from the first lens. In this case, how much distance should the object be from the first lens on the left?

Question 4. In a system consisting of convex lens and concave mirror which have focal lengths of 60 cm and 40 cm respectively, how much should the distance between lens and mirror be for the object placed 90 cm distance from the lens on the left to have a final image occurred in the system which is on the same part with the object, inverse and in same length with the object?

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