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An Inventory Model For Deteriorating Items With Two Parameter Weibull Deterioration And Price Dependent Demand

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Abstract - This paper presents an inventory model for deteriorating items with price dependent demand. Deterioration rate follows two parameter weibull distribution. Shortages are allowed and are completely backlogged.

Keywords: Deterioration items, holding cost, inventory, price-dependent demand time, shortages, weibull, completely backlogged.

INTRODUCTION:

Deterioration of items in an inventory is a common phenomenon in business situations. This is due to the fact that the items in the inventory become obsolete, devalued, decay or damaged depending on the type of goods. As a consequence of the deterioration shortages may occur. Hence deterioration factor has to be given importance while determining the optimal policy for an inventory model.

Ajanta Roy [1] presented an inventory model for time proportional deterioration rate and demand is function of selling price. The Author discussed the model without shortage and also with shortages in which the shortages are completely backlogged. Mukesh Kumar, Anand Chauhan, Rajat Kumar [9] extended Ajanta Roy models with trade credit. Tripathy C.K and L.M. Pradhan [14] gave a model in which the demand of the product decreases with the increase of time and sale price and deterioration rate follows a three parameter Weibull distribution functions. Tripathy C.K and L.M.Pradhan[15] gave a model in which the demand of the product decreases with the increase of time and sale price and deterioration follows a three parameter weibull distribution. Padmanabhan. G,Prem Vrat[10] formulated an EOQ model perishable items under stock dependent selling rate. Sahoo.N.K.,Sahoo.C.K. & Sahoo.S.K[12] described an inventory model for price dependent demand and time varying holding cost. Vikas Sharma and Rekha Rani Chaudhary [17] explained and inventory model for two parameter Weibull deterioration rate. They found profit for their model. Sanjay JAIN and Mukesh KUMAR [13] explained an inventory model with ramp type demand and three parameter Weibull deterioration rate. The Authors also analyzed and summarized economic order quantity models done by few researchers. There are some products which start deteriorate only after some interval of time. This was explained by taking three parameter Weibull distribution deterioration rate. Anil Kumar Sharma, Manoj Kumar Sharma and Nisha Ramani [3] described an inventory model for two – parameter Weibull distribution deterioration rate and demand rate is power pattern. Manoj Kumar Meher, Gobinda Chandra Panda, Sudhir Kumar Sahu [8] adopted a two – parameter Weibull distribution deterioration to develop an inventory model under permissible delay in payments. Kun – Shan Wu [7] made an attempt in his paper to obtain the optimal ordering quantity of deteriorating items for two – parameter Weibull distribution deterioration under shortages and permissible delay in payments. P.K.Tripathy and S.Pradham [16] also define an inventory model with two – parameter Weibull distribution as demand rate and deterioration rate increases with time. Recently R.Amutha and E.Chandrasekaran [2] developed an inventory model for deteriorating items with three-parameter weibull deterioration and price dependent demand.

In this present paper, we have developed an inventory model for two-parameter Weibull deterioration rate and price dependent demand. Shortages are allowed and are completely backlogged. Holding cost is assumed to be constant. Our aim is to increase the profit.

ASSUMPTIONS AND NOTATIONS

(i) The demand rate is a function of selling price.

(ii) Shortages are allowed and are completely backlogged

(iii) Lead time is zero.

(iv) Replenishment is instantaneous

(v) A is the set up cost

(vi) C is the unit cost of an item

(vii) p is the selling price

(viii) Demand D (t) = f (p) = a-p, where a > p.

(ix) C_2 is the shortage cost per unit time.

(x) $\theta(t) = \alpha\beta t^{\beta-1}$, $0 \le \alpha < 1$, $\beta > 0$ and $-\infty < \gamma < \infty$ is the deterioration rate. At time T_1 the Inventory becomes Zero and shortages start occurring.

(xi) h is the constant holding cost.

(xii) T is the length of the cycle.

MATHEMATICAL FORMULATION AND SOLUTION

Let I (t) be the inventory at time T ($0 \le t \le T$) the differential equation for the instantaneous state over (0,T) are given by

$$\frac{dI(t)}{dt} + \alpha\beta t^{\beta-1} I(t) = -(a-p) \qquad 0 \le t \le t_1 \qquad (1)$$

$$\frac{dI(t)}{dt} = -(a-p) \qquad t_1 \le t \le T \qquad (2)$$

Solving equations (1) and (2) with boundary condition $I(t_1)$ we get

$$I(t) = -(a-p)[\ (t-t_1) + \frac{\propto}{\beta+1}[t^{\beta+1} - t_1^{\beta+1}\ ] + \frac{\propto^2}{2(2\beta+1)}[t^{2\beta+1} - t_1^{2\beta+1}]\ ] \qquad (3)$$

$$I(t) = -(a-p)(t-t_1)$$

Shortage cost (4)

$$SC = \frac{-C_2}{T}\int_{t_1}^{T} -(a-p)(t-t_1)dt$$

$$= \frac{C_2}{2T}(a-p)\,(t-t_1)^2 \qquad (5)$$

Holding cost

$$HC = \frac{h}{T}\int_0^{t_1} I(t)dt$$

$$= \frac{-h}{T}(a-p)\left(-\frac{t_1^2}{2} + \frac{\propto}{\beta+1}\left[\ \frac{t_1^{\beta+2}}{\beta+2} - t_1^{\beta+2}\ \right] + \frac{\propto^2}{2(2\beta+1)}\left[\ \frac{t_1^{2\beta+2}}{2\beta+2} - t_1^{2\beta+1}\right]\right) \qquad (6)$$

Stock loss due deterioration

$$D = (a-p)\int_0^{t_1} e^{\propto t^{\beta}}\,dt - (a-p)\int_0^{t_1} dt$$

$$= (a-p)\left[\ \frac{\propto t_1^{\beta+1}}{\beta+1} + \frac{\propto^2 t_1^{2\beta+1}}{2(2\beta+1)}\ \right] \qquad (7)$$

Order quality

$Q = D + \int_0^T (a-p)dt$

$= (a-p)\,[\frac{\propto t_1^{\beta+1}}{\beta+1} + \frac{\propto^2 t_1^{2\beta+1}}{2(2\beta+1)} + T\,]$ (8)

Purchase cost $= \frac{CQ}{T}$

$= \frac{(a-p)C}{T}\,[\,\frac{\propto t_1^{\beta+1}}{\beta+1} + \frac{\propto^2 t_1^{2\beta+1}}{2(2\beta+1)} + T\,]$ (9)

Total profit per unit is = p(a-p) - $\frac{1}{T}$ [OC + SC + HC + PC]

$K(p,T,T_1) = p(a-p) - \frac{1}{T}\,[\,A + \frac{C_2}{2T}(a-p)\,(t-t_1)^2 - \frac{h}{T}(a-p)(\frac{-t_1^2}{2} + \frac{\propto}{\beta+1}[\,\frac{t_1^{\beta+2}}{\beta+2} - t_1^{\beta+2}] +$

$\frac{\propto^2}{2(2\beta+1)}[\,\frac{t_1^{2\beta+2}}{2\beta+2} - t_1^{2\beta+1}]) + \frac{(a-p)C}{T}\,(\,\frac{\propto t_1^{\beta+1}}{\beta+1} + \frac{\propto^2 t_1^{2\beta+1}}{2(2\beta+1)} + T\,)$ (10)

Let $t_1 = vT$, $0 < v < 1$

Therefore we have profit function,

$K(p,T) = p(a-p) - \frac{1}{T}\,[\,A + \frac{C_2}{2}(a-p)(t-vT)^2 - h(a-p)(-\frac{(vT)^2}{2} + \frac{\propto}{\beta+1}[\,\frac{(vT)^{\beta+2}}{\beta+2} - (vT)^{\beta+2}] + \frac{\propto^2}{2(2\beta+1)}[\,\frac{(vT)^{2\beta+2}}{2\beta+2} - (vT)^{2\beta+2}]) +$

$c(a-p)\,(\,\frac{\propto (vT)^{\beta+1}}{\beta+1} + \frac{\propto^2 (vT)^{2\beta+1}}{2(2\beta+1)} + vT\,)\,]$ (11)

$\frac{\partial K(p,T)}{\partial T} = \frac{1}{T^2}\,[\,A + \frac{c_2}{2}(a-p)\,(t-vT)^2 + h\,(a-p)\,[\,\frac{(vT)^2}{2} + \frac{\propto}{\beta+2}\,(vT)^{\beta+2} + \frac{\propto^2}{2}\,\frac{(vT)^{2\beta+2}}{2\beta+2}\,]$

$+ (a-p)c\,[\,\frac{\propto (vT)^{\beta+1}}{\beta+1} + \frac{\propto^2 (vT)^{2\beta+1}}{2(2\beta+1)} + vT]] + -\frac{1}{T}\,[\,c_2\,(a-p)(vT-t)v + c(a-p)\,(\alpha v^{\beta+1} T^{\beta}$

$+ \frac{\propto^2}{2} v^{2\beta+1} T^{2\beta} + v) + h(a-p)\,(\,v^2 T + \propto T^{\beta+1} v^{\beta+2} + \frac{\propto^2}{2}\,v^{2\beta+2} T^{2\beta+1}\,]$ (12)

$\frac{\partial K(p,T)}{\partial p} = a - 2p + \frac{c_2}{2T}(t-vT)^2 - \frac{h}{T}\,[\,\frac{(vT)^2}{2} + \frac{\propto}{\beta+2}\,(vT)^{\beta+2} + \frac{\propto^2}{2(2\beta+2)}(vT)^{2\beta+2} - \frac{c}{T}\,[\,\frac{\propto (vT)^{\beta+1}}{\beta+1} + \frac{\propto^2 (vT)^{2\beta+1}}{2(2\beta+1)} + vT\,]$ (13)

For the maximization of profit we set

$\frac{\partial K(p,T)}{\partial T} = 0$ and $\frac{\partial K(p,T)}{\partial p} = 0$ provided $\frac{\partial^2 K(p,T)}{\partial T^2} < 0$, $\frac{\partial^2 K(p,T)}{\partial p^2} < 0$ and

$(\frac{\partial^2 K(p,T)}{\partial T^2})(\frac{\partial^2 K(p,T)}{\partial p^2})\,(\frac{\partial^2 K(p,T)}{\partial T \partial p}) > 0$.

CONCLUSION

A deterministic inventory model for deteriorating inventory model with two parameter Weibull distribution deterioration rate has been developed. Demand rate is function of selling price and holding cost is constant occurring shortages and completely backlogged.

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