DESIGN AND OPTIMIZATION OF BOLTED JOINT SUBJECTED TO SHEAR AND BENDING LOAD

Khemchand M. Kapgate
Mechanical Engineering Department
K.D.K.C.E.
Nagpur, India
khemskapgatek2@gmail.com

Dr. C. C. Handa
Professor
Mechanical Engineering Department
K.D.K.C.E. Nagpur, India
chandrahashanda@rediffmail.com

V. D. Dhopte
Assistant Professor
Mechanical Engineering Department
K.D.K.C.E Nagpur, India.
vikrantdhopte@gmail.com

ABSTRACT

In this project a bolted joint, loaded by forces in it is studied. Finite Elements simulations are turned on ANSYS software in order to verify and validate results issued from analytical model. 3D FE simulations are used to show limits of application of developed model and also to study the ultimate stability of bolted joint under loading. APDL programming approach will be used for design optimization Due to flange opening, bending has been noticed in the bolt. Hence the bolts/studs should be designed to withstand against preload, internal pressure load and bending moment. Due to existence of Preload, internal pressure and bending moment at a time, the bolt behaviour is nonlinear which cannot not be evaluated by simple mathematical formulas. 3-Dimensional finite element analysis approach is only the technique which shows some satisfactory result.

Keywords :- Bolt, FEM, Pre-Load, Shear Load, Bending load

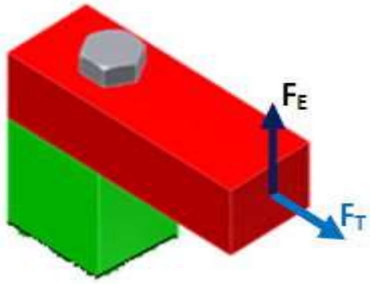
1.0 INTRODUCTION

Bolted joints are one of the most common elements in construction and machine design. They consist of fasteners that capture and join other parts, and are secured with the mating of screw threads. There are two main types of bolted joint designs: tension joints and shear joints.

In the tension joint, the bolt and clamped components of the joint are designed to transfer the external tension load through the joint by way of the clamped components through the design of a proper balance of joint and bolt stiffness. The joint should be designed such that the clamp load is never overcome by the external tension forces acting to separate the joint (and therefore the joined parts see no relative motion).

The second type of bolted joint transfers the applied load in shear on the bolt shank and relies on the shear strength of the bolt. Tension loads on such a joint are only incidental. A preload is still applied but is not as critical as in the case where loads are transmitted through the joint in tension. Other such shear joints do not employ a preload on the bolt as they allow rotation of the joint about the bolt, but use other methods of maintaining bolt/joint integrity. This may include clevis linkages, joints that can move, and joints that rely on a locking mechanism (like lock washers, thread adhesives, and lock nuts).

Typically, a bolt is tensioned (preloaded) by the application of a torque to either the bolt head or the nut. The preload developed in a bolt is due to the applied torque and is a function of the bolt diameter, length, the geometry of the threads and the coefficients of friction that exist in the threads and under the bolt head or nut. The stiffness of the components clamped by the bolt has no relation to the preload that is developed by the torque. The relative stiffness of the bolt and the clamped joint components do, however, determine the fraction of the external tension load that the bolt will carry and that in turn determines preload needed to prevent joint separation and by that means to reduce the range of stress the bolt experiences as the tension load is repeatedly applied. This determines the durability of the bolt when subjected to repeated tension loads. Maintaining a sufficient joint preload also prevents relative slippage of the joint components that would produce fretting wear that could result in a fatigue failure of those parts when subjected to in-plane shearing forces.



Bolt model with loads

A stacked heat exchanger is a piece of equipment built for efficient heat transfer from one medium to another. The media may be separated by a solid wall to prevent mixing or they may be in direct contact. They are widely used in space heating, refrigeration, air conditioning, power stations, chemical plants, petrochemical plants, petroleum refineries, natural-gas processing, and sewage treatment. The classic example of a heat exchanger is found in an internal combustion engine in which a circulating fluid known as engine coolant flows through radiator coils and air flows past the coils, which cools the coolant and heats the incoming air.

Double pipe heat exchangers are the simplest exchangers used in industries. On one hand, these heat exchangers are cheap for both design and maintenance, making them a good choice for small industries. On the other hand, their low efficiency coupled with the high space occupied in large scales, has led modern industries to use more efficient heat exchangers like shell and tube or plate. However, since double pipe heat exchangers are simple, they are used to teach heat exchanger design basics to students as the fundamental rules for all heat exchangers are the same. To start the design of a double pipe heat exchanger, the first step is to calculate the heat duty of the heat exchanger. It must be noted that for easier design, it's better to ignore heat loss to the environment for initial design.

Stacked Heat Exchanger

Literature review

Nomesh Kumar, P.V.G. Brahamanandam and B.V. Papa Rao "3-D Finite Element Analysis of Bolted Flange Joint of Pressure Vessel"

In this paper it was found that, the stresses in the bolts of the bolted flange joint of the pressure vessel so that bolts/studs should not be failed during proof pressure test. Bolted flange joints perform a very important structural role in the closure of flanges in a pressure vessel. It has two important functions: (a). to maintain the structural integrity of the joint itself, and (b). to prevent the leakage through the gasket preloaded by bolts. The preload on the bolts is extremely important for the successful performance of the joint. The preload must be sufficiently large to seat the gasket and at the same time not excessive enough to crush it. The flange stiffness in conjunction with the bolt preload provides the necessary surface and the compressive force to prevent the leakage of the gases contained in the pressure vessel. The gas pressure tends to reduce the bolt preload, which reduces gasket compression and tends to separate the flange faces. Due to flange opening , bending has been noticed in the bolt. Hence the bolts/studs should be designed to withstand against preload, internal pressure load and bending moment. Due to existence of Preload, internal pressure and bending moment at a time, the bolt behaviour is nonlinear which cannot not be evaluated by simple mathematical formulas. 3-Dimensional finite element analysis approach is only the technique which shows some satisfactory result.

Gowri Srinivasan & Terry F. Lehnhoff "Bolt Head Fillet Stress Concentration Factor Cylindrical Pressure Vessels"

In this paper it is found that, linear three-dimensional finite element analysis (FEA) was performed on bolted pressure vessel joints to determine maximum stresses and stress concentration factors in the bolt head fillet as a result of the prying action. The three-dimensional finite element models consisted of a segment of the flanges containing one bolt, using cyclic symmetry boundary conditions. The maximum stress in the bolt as well as the stress concentration factors in the bolt head fillet increase with an increase in bolt circle diameter for a given outer flange dimension. Keeping the bolt circle diameter constant, bolt stress and stress concentration factors inthe bolt head fillet decrease with increase in outer flange diameter. The maximum stresses in the bolt were also calculated according to the American Society of Mechanical Engineers (ASME) Boiler and Pressure Vessel Code and the Verein Deutscher Ingenieur (VDI) guidelines and compared to the results observed through finite element analysis. The stresses obtained through FEA were larger than those predicted by the ASME and VDI methods by a factor that ranged between 2.96 to 3.41 (ASME) and 2.76 to 3.63 (VDI).

S.H. Ju , C.Y. Fan, & G.H. Wub "3-dimensional finite elements of steel bolted connections"

In this paper it was found that, the three-dimensional (3D) elasto-plastic finite element method is used to study the structural behaviour of the butt-type steel bolted joint. The numerical results are compared with AISC specification data. The similarity was found to be satisfactory despite the complication of stress and strain fields during the loading stages. When the steel reaches the nonlinear behaviour, the bolt nominal forces obtained from the finite element analyses are almost linearly proportional to the bolt number arranged in the connection. Moreover, the bolt failure is marginally dependent on the plate thickness that dominates the magnitude of the bending effect. For the cracked plate in a bolted-joint structure, the relationship between KI and the applied load is near linear, in which the nonlinear part is only about one tenth of the total relationship. This means that the linear elastic fracture mechanics can still be applied to the bolted joint problem for the major part of the loading, even through this problem reveals highly nonlinear structural behaviour.

Iuliana PISCAN, Nicolae PREDINCEA & Nicolae POP "FINITE ELEMENT ANALYSIS OF BOLTED JOINT"

In this paper it was found that, this paper presents a theoretical model and a simulation analysis of bolted joint deformations. The bolt pretension force, friction coefficient and contact stiffness factor are considered as parameters which are influencing the joint deformation. The bolted joint is modelled using CATIA software and imported in ANSYS WORKBENCH. The finite element analysis procedure required in ANSYS WORKBENCH simulation is presented as a predefined process to obtain accurate results.

Ali Najafi, Mohit Garg and Frank Abdi "Failure Analysis of Composite Bolted Joints in Tension"

In this paper it is found that, the failure of preloaded cross-ply laminated composite has been studied through finite element simulation embedded in Progressive Failure Analysis (PFA). Two modelling strategies including low- and high-fidelity models have been considered for this investigation. The high-fidelity FE model consists of fixture components (bolts and washers). It has been shown that both low- and high-fidelity FE models are capable of predicting the experimentally observed failure modes of bolted joints that depends on the geometric parameters with reasonable accuracy. Two catastrophic failure loads, net-tension and shear out can be predicted using both low- and high-fidelity model while the failure load of bearing mode can only be predicted via high-fidelity model that considers the applied preload of the bolt. However, the overall stiffness in the actual experiment is lower than that of predicted via finite element simulation.

CONVENTIONAL DESIGN

Bolt Pretension

Bolt pretension, also called preload or prestress, comes from the installation torque T you apply when you install the bolt. The inclined plane of the bolt thread helix converts torque to bolt pretension. Bolt preload is computed as follows.

$P_i = T/(K\ D)$ (Eq. 1)

Where,

P_i =bolt preload (called F_i in Shigley).

T=bolt installation torque.

K = torque coefficient.

D = bolt nominal shank diameter (i.e., bolt nominal size).

Torque coefficient K is a function of thread geometry, thread coefficient of friction μ_t, and collar coefficient of friction μ_c. Look up K for your specific thread interface and collar (bolt head or nut annulus) interface materials, surface condition, and lubricant (if any). ("Torque specs for screws," Shigley, and various other sources discuss various K value estimates.) If you cannot find or obtain K from credible references or sources for your specific interfaces, then you would need to research to try to find the coefficients of friction for your specific interfaces, then calculate K yourself using one of the following two formulas listed below (Shigley, *Mechanical Engineering Design*, 5 ed., McGraw-Hill, 1989, p. 346, Eq. 8-19, and MIL-HDBK-60, 1990, Sect. 100.5.1, p. 26, Eq. 100.5.1, respectively), the latter being far simpler.

$$K = \{[(0.5\, d_p)(\tan\lambda + \mu_t \sec\beta)/(1 - \mu_t \tan\lambda \sec\beta)] + [0.625\mu_c D]\}/D \qquad \text{(Eq. 2)}$$

$$K = \{[0.5\, p/\pi] + [0.5\, \mu_t (D - 0.75\, p \sin\alpha)/\sin\alpha] + [0.625\, \mu_c D]\}/D \qquad \text{(Eq. 3)}$$

Where,
D = bolt nominal shank diameter.
p = thread pitch (bolt longitudinal distance per thread).
α= thread profile angle = 60° (for M, MJ, UN, UNR, and UNJ thread profiles).
β = thread profile half angle = 60°/2 = 30°.
$\tan\lambda$ = thread helix angle tan = $p/(\pi\, d_p)$.
d_p = bolt pitch diameter.
μ_t = thread coefficient of friction.
μ_c = collar coefficient of friction.
D and *p* can be obtained from bolt tables such as Standard Metric and USA Bolt Shank Dimensions.

The three terms in Eq. 3 are axial load component (coefficient) of torque resistance due to (1) thread helix inclined plane normal force, (2) thread helix inclined plane tangential (thread friction) force, and (3) bolt head or nut washer face friction force, respectively.

However, whether you look up K in references or calculate it yourself, the engineer must understand that using theoretical equations and typical values for K and coefficients of friction merely gives a preload *estimate*. Coefficient of friction data in published tables vary widely, are often tenuous, and are often not specific to your specific interface combinations and lubricants. Such things as unacknowledged surface condition variations and *ignored dirt* in the internal thread can skew the results and produce a false indication of preload.

The engineer and technician must understand that published K values apply to perfectly clean interfaces and lubricants (if any). If, for example, the threads of a steel, zinc-plated, K = 0.22, "dry" installation fastener were not clean, this might cause K to increase to a value of 0.32 or even higher. One should also note that published K values are intended to be used when applying the torque to the nut. The K values will change in relation to fastener length and assembly running torque if the torque is being read from the bolt head.

One should measure the nut or assembly "running" torque with an accurate, small-scale torque wrench. ("Running" torque, also called prevailing torque, is defined as the torque when all threads are fully engaged, fastener is in motion, and washer face has not yet made contact.) The only torque that generates bolt preload is the torque you apply *above* running torque.

A few more things to be aware of are as follows. Bolt proof strength S_p is the maximum tensile stress the bolt material can withstand without encountering permanent deformation. Published bolt yield strengths are determined at room temperature. Heat will lower the yield strength (and proof strength) of a fastener. Especially in critical situations, you should never reuse a fastener unless you are certain the fastener has never been yielded.

Bolt Preload Measurement

If a more accurate answer for bolt preload is needed than discussed above, the specific combination and lubricant would have to be *measured* instead of calculated. Measurement methods are generally involved, time-consuming, and expensive, and are beyond the scope of this article. But perhaps one of the simplest and least expensive methods, to test specific combinations and lubricants, is to measure the installed fastener with a micrometer, if possible, and compute torque coefficient K as follows, per Shigley, op. cit., p. 345, para. 2.

$$K = T\,L/(E\,A\,\text{delta}\,D) \qquad \text{(Eq. 4)}$$

Where,
T = bolt installation torque, L = bolt grip length, E = bolt modulus of elasticity, A = bolt cross-sectional area, D = bolt nominal shank diameter, and delta = measured bolt elongation in units of length.

FASTENER MODELING

A fastener is modeled by CBAR or CBEAM elements [2] with corresponding PBAR or PBEAM cards for properties definition. For the CBAR or CBEAM elements connectivity, a separate set of grid points coincidental with corresponding plate grid points (Figure 3) is created. This set also includes grid points located on intersection of the fastener axis and outer surfaces of the first and last connected plates.

All CBAR or CBEAM elements representing the same fastener reference the same PBAR or PBEAM card [2] with following properties:

· MID to reference the fastener material properties.

· Fastener cross-sectional area

$$A = \frac{\pi {d_f}^2}{4}$$

where d_f- fastener diameter.

· Moments of inertia of the fastener cross section

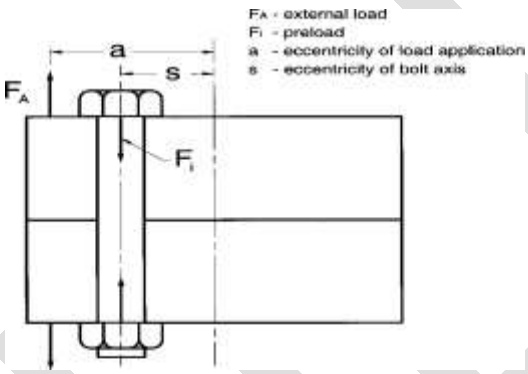
$$l_1 = l_2 = \frac{\pi {d_f}^4}{64}$$

Torsional constant

$$J = \frac{\pi {d_f}^4}{32}$$

Area factors for shear of circular section

$$K_1 = K_2 = 0.9$$



Conventional Bolt

DESIGN DATA

Table 1 Reference Drawings

SR NO.	Description	Drawing NO.
1	HES-R0353AB-E-001-RCModel _GA_Dwg	SDM-235ES-103R1/SDM-235ES-204-R1
2	HES-R0353ABE004RCModel _Nozzle_dwg	SDM-235ES-103R1/SDM-235ES-204-R1
3	HES-R0353ABE005RCModel _Saddle_Dwg	SDM-235ES-103R1/SDM-235ES-204-R1

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Table 2 Reference Codes

Sr No.	Description
1	ASME Boiler and Pressure Vessel Code, SectionVIII, Div. 1, Ed. 2013, ADD 2011
2	ASME Boiler and Pressure Vessel Code, SectionVIII, Div. 2, Ed. 2013, ADD 2011
3	ASME Boiler and Pressure Vessel Code, Section II, PartD, Ed. 2013, ADD 2011

Table 3 Design Parameters

COMPONENT CRITERIA	UNIT	CHANNEL SIDE	SHELL SIDE
Design Pressure	Mpa	1.65	5.66
Pretension	N	672475.000	49934.516
No of Bolts		16	8
Load on each Bolt	N	42029.6875	6241.8145
Design Temperature	^{0}c	279	313
Operating Pressure	Mpa	1.509	5.188
Fluid Handled		Hydrocarbons, H2	Hydrocarbons, H2

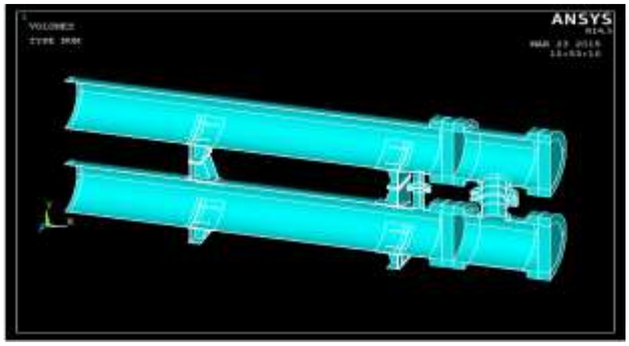
Table 4 Material Properties

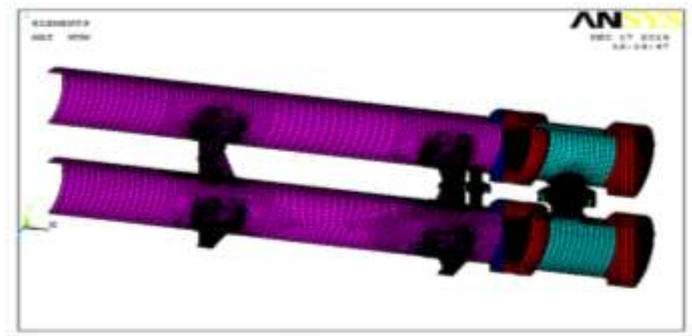
Material Properties has been taken from ASME Sec II Part D.

Component	Material	Design Temp.	Modulus of Elasticity (1)	Allowable Design Stress at Design Temp. (2)
		°C	N/mm^2	N/mm^2
Channel Shell	SA 387 Gr11	313	185698.96	148
Channel Flange & Cover	SA 516 Gr 70	313	185698.96	-
Tubesheet (Equivalent Solid Tubesheet)	SA 336 Gr F12	313	E*=55516	-
Main Shell Head	SA516 Gr70	279	185268.27	137
Main Shell Flange	SA 266 Gr2N	279	185268.27	-
Channel Side Nozzle Flanges	SA182 Gr F12, Cl2	313	185268.27	111
Channel Side Nozzle Neck	SA336 gr F12	313	185268.27	132
Shell Side Nozzles Flanges	SA105 N	279	186412.3	132
Shell Side Nozzle Neck	SA 106 GrB	279	186412.3	118
Saddles & Wrapper plate	SA516 Gr 70	279	186412.3	137

CAD MODELLING (SIMULATION)

MESHING





CONTACT AND PRETENSION MODELLING

STEP 1) The first phase is modeling the joint using CAD software. The model geometry was generated using ANSYS software and then opened as a neutral file in ANSYS APDL. Due to symmetry conditions the model is sectioned. Geometric details, such as chamfers, radii of connection have only a local influence on behaviour of the structure therefore those are neglected. In this analysis we neglect the bolt thread and surface roughness.

STEP 2) Next, the prepared geometric structure is reproduced by finite elements. The finite elements are connected by nodes that make up the complete finite element mesh. Each element type contains information on its degree-of-freedom set (e.g. translational, rotational, and thermal), its material properties and its spatial orientation (3D-element types). The mesh was controlled in order to obtain a fine and good quality mapped mesh. The assembly had 574086 nodes and 115534 elements.

STEP 3) In order to solve the resulting system equation, boundary and loaded conditions are specified to make the equation solvable. In our model, the interconnecting nozzle (i.e. N5 nozzle and N6 nozzle) with attached bolts axial load were applied. The pretension in the bolt was generated at the mid plane of the bolt using the pretension element PRETS179, which is contained in the ANSYS v14.5 element library. These elements allow direct specification of the pretension in bolt. For specifying the bolt pretension a local coordinate system was defined, with the Y axes along the bolt length. After the bolt pretension, an external load was applied to the bolted joint.

STEP 4) The last phase is interpreting the results. For contact analysis ANSYS supports three contact models: node to node, node to surface and surface to surface. In this case a surface to surface model was created and contacts elements were used. ANSYS provides several element types to include surface-to-surface contact and frictional sliding. One of these elements is the 3D 8-node surface-to-surface contact element CONTAC174. Contacts elements use a target surface and a contact surface to form a contact pair. According to stiffness behaviour the parts can be rigid or flexible. Our model is defined as flexible one.

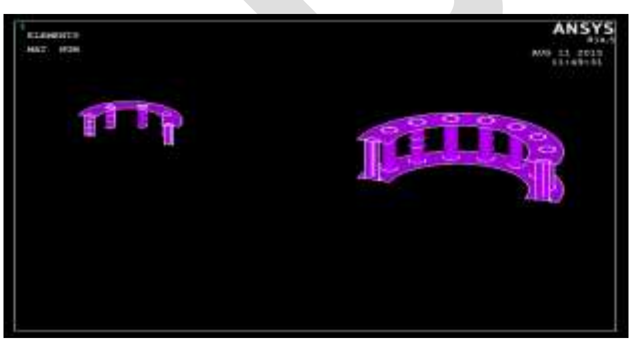


FIG 1. Contacts and Pretension

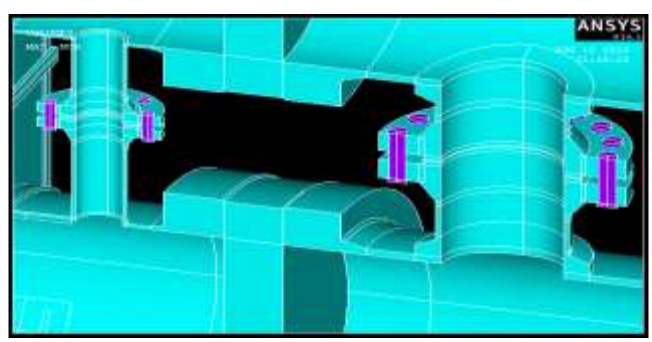


FIG 2. Bolt and Nozzle

BEAM AND PRETENSION MODELLING

STEP 1) to STEP 3) As per the previous technique i.e. contact and pretension the pretension is applied on bolt is as same as STEP 1) to STEP 3), but only difference is that the bolt is not designed but a beam element is introduced and line representing bolt is meshed and in the middle of its node pretension is applied.

STEP 4) The last phase is interpreting the results. A coupling is generated in the flange of the bolt. The top and bottom nodes of flange with centre node are selected where the bolt is to be situated. Then coupling force/moment is generated on top and bottom nodes of flange by selecting middle/master node and coupling is designed.

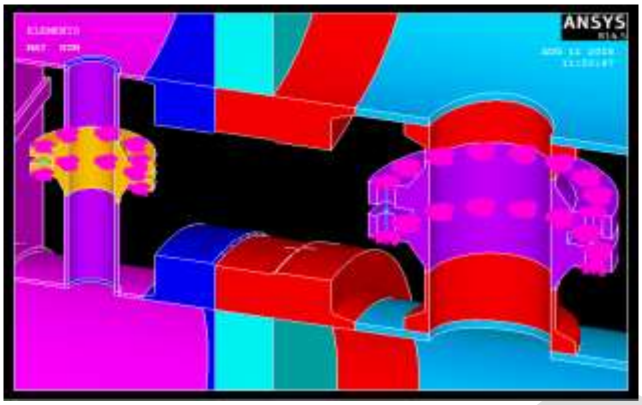


FIG 3. Beam and Nozzle

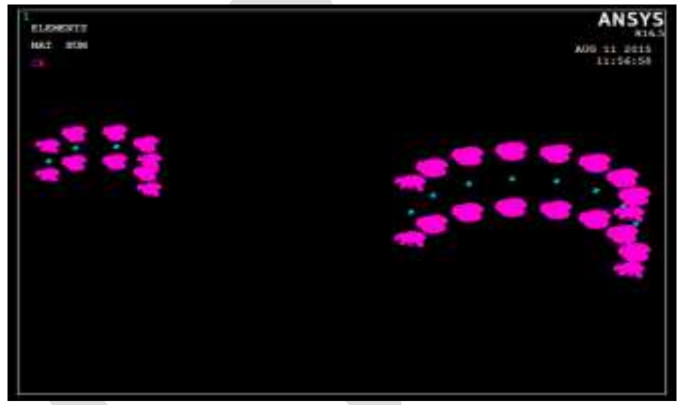


FIG 4. Coupled and Pretension nodes

OPTIMIZATION

Bolt size and the number of bolts are the most important parameters to consider when optimizing bolted joints. Because bolts come in discrete sizes (e.g. M4, M5, M6, etc. for metric bolts) there is only one optimum combination of bolt size and bolt number which will maximize the efficiency of joint under a given load. A procedure aimed at selecting optimum combinations of bolt size and bolt number has been implemented through ANSYS software.

CONCLUSION

The various parameters for design of bolt i.e. pretension, pressure, bolt geometry, deformation, sliding behaviour, friction, contact stiffness, analysis approach has been studied for modeling bolt which is used in stacked heat exchanger.

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