# Relative improved differential box-counting approach to compute fractal dimension of gray-scale images

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**ABSTARCT-** Fractal theory is used in image processing. The dimension of complex objects in nature is calculated by Fractal Dimension. Fractal Dimension is used in shape classification, graphic analysis in many fields, texture segmentation. FD's can be used to aid in several data mining tasks. Mainly box counting method is used to calculate the FD of an image. In this paper various methods used to calculate the FD of gray scale images are emphasized. Key problems involved in the computation of Differential box count method is presented. Experiments are conducted using the methods and FD is compared.

Keywords- Blocks, Boxes, Box count, Differential box-counting, Fractal Dimension, Gray scale images, Linear scale

## INTRODUCTION:

Fractal geometry was put forwarded by Mendelbrot in 1983 [1] to describe the self similar parts called fractals. FD provides many mathematical model for for coastlines, mountains, clouds. Fractal based analysis are great for digital image analysis[1-3]. It is followed by many other theories applicable to a wider class of fractals. Gangepain and Roques-carmes described the popular reticular cell counting method [10,11]. Keller et al. gave even more interesting theories [10,12]. Sarkar et. al. suggested a differential box-counting (DBC) method [4, 10, 13] that was considered to be an effective method [14,15]. The box-counting dimension is the most frequently used for measurements in various application fields. The reason for dominance lies in its simplicity and automatic computability [3].. It has an extremely wide area of applications such as finance, stock markets, medicine, quality of food analysis and even art[7]. Fractal dimension is the tool of fractal geometry is used to characterize, classify or to segment images or regions. Fractal dimension has many definitions, such as Hausdorff dimension, Self-similar dimension, Box-counting dimension, Correlation dimension, etc. Among these box-counting is most widely used dimension. Because it has usefulness for both linear and non-linear fractal images, its easy realization by computer, its effectiveness to describe image surface complexity and irregularity.

In this paper, the DBC and improved DBC are studied and the key problems involved are discussed like precise no.of boxes, range of linear scale etc, then the experiments are performed on various images. The results show that the improved DBC approach is more effective than DBC approach.

## METHODOLOGIES:

Here, in this section a review of Fractal Dimension estimation using DBC[2], improved DBC[2,4], are discussed. The basic principle to estimate FD is based on the concept of self-similarity. The FD of a bounded set A in Euclidian n-shape is defined as

$$D = \frac{\log(Nr)}{\log(1/r)} \quad \ldots\ldots(1)$$

Where $N_r$ is the least number of distinct copies of A in the scale r. The union of $N_r$ distinct copies must cover set A completely.

Fractal dimension estimation using DBC approach :

The DBC method was introduced as follows[2,4]. Let a 2-D gray sale image of size $\mathrm{M}*\mathrm{M}$ pixels scaled down to a size $s*s$ where $M/2 > l > 1$ and l is an integer. Then estimation of scaling ratio $\mathrm{r} = \mathrm{l/M}$. Let consider the image as a 3-D space with (x,y) denoting 2-D position and third co-ordinate (z) denoting gray level. The (x,y) space is divided in to grids of size $l*l$. On each grid there is a column of boxes of size $l*l*l'$. If the total no.of gray level is G then $l' = l*G/M$.

Let the minimum and maximum gray level of the image in the (i,j)th grid fall in box no. p & q respectively. Then

$$n_r(i,j) = q - p + 1 \qquad (2)$$

Is the contribution of $N_r$ in $(i,j)$ th grid. For example , in fig.1, $n_r(i,j) = 3-1+1$.(Although in this figure, for simplicity , smooth surface is taken, but in reality it will be digital image surface)[6,8]. Because of the differential nature of computing n, named as differential box-counting (DBC) approach. Taking contribution from all grids

$$N_r = \sum_{i,j} nr(i,j) \qquad (3)$$

$N_r$ is counted for different values of r, i.e, different values of l. Using eq$^n$ (1) we can estimate D, the fractal dimension from least square linear fit of $\log(N_r)$ versus $\log(1/r)$.

A typical plot of log (Nr) versus log (1 /r) of the image Lena is shown in Fig. 3. Let y = mx+c be the fitted straight line, where y denotes log (Nr) and x denotes log (1/r). Then error can be expressed as the root distance of the points from the fitted line. There are two major problems in the aforementioned procedure in the original DBC method (1) Box height selection, (2) Box number calculation [1,2,5,15]

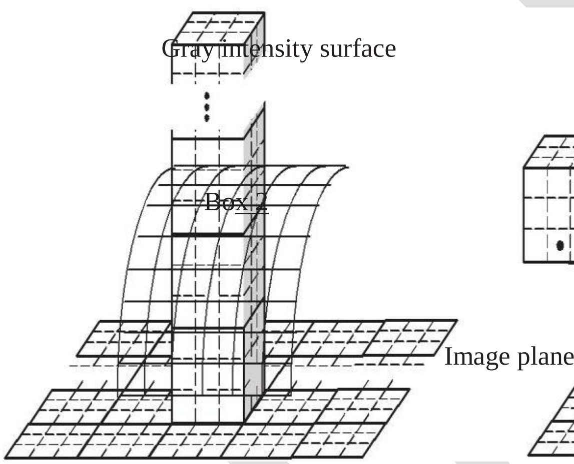


**Fig. 1.** Sketch of determination of the number of boxes by the DBC method

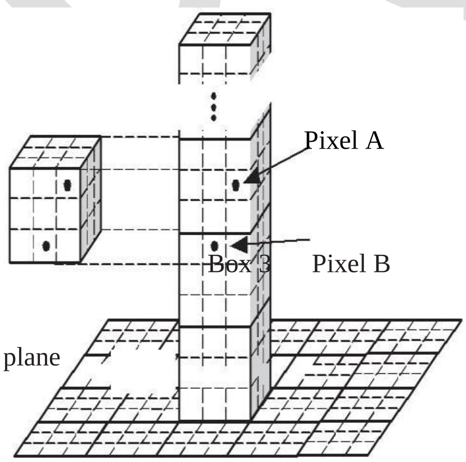


**Fig.2.** Two pixels fall in to 2 boxes

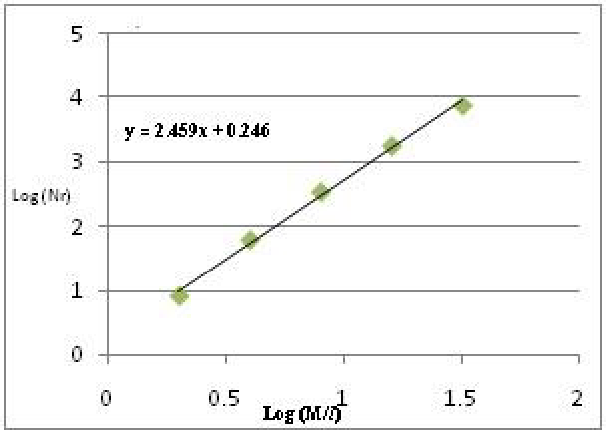


**Fig. 3** Plot of log(Nr) vs. log(1/r) of a gray image

1) **Box height Selection:**

If the height of boxes is selected as $l' = l * G / M$ , it may have a larger value when l is increased , then the box in the larger scale may result in greater computational error when counting the box numbers[9].

2) **Box Number Calculation:**

This method assigns a column of boxes on a block starting from the gray level zero. This problem is clearly visible in Fig.2 where pixels A and B represents the maximum and minimum gray levels of the block. The distance between the two pixels are less than 3 if a column of boxes of size 3×3×3 covers this block and the two pixels lie in boxes 2 and 3. Here the minimum box can be one but as the pixels fall in to different boxes so it is two which is not able to calculate least number of boxes.

**A. Fractal dimension estimation using improved DBC approach:**

Let $z = f(x, y)$ be an ideal fractal image with a 3-D continuous surface. It is divided in to distinct copies and a copy has a minimum denoted as $z_1 = f(x_1, y_1)$ and a maximum denoted as $z_2 = f(x_2, y_2)$ , when $x_1 \le x \le x_2$ and $y_1 \le y \le y_2$. Let dx and dy be the lengths of the copy in the directions of x and y respectively and dz be the height in the direction of z. Because $z = f(x, y)$ is continuous , then $d_x = x_2 - x_1$, $d_y = y_2 - y_1$, and $d_z = z_2 - z_1$. Suppose $d_x = d_y$ and define the box sales as $r = d_x = d_y$. Assume a column of boxes with the size of $d_x * d_x * d_x$ cover the copy of the surface completely[2]. The number of this column of boxes is equal to integer part of $(d_z / r + 1)$. Using Eq.(1) the FD o an ideal fractal can be determined via least squares linear fit[12,13]. Or Eq.(1) indicates there exists definitely a scale range [$r_1$,$r_2$] in which FD estimate of the ideal fractal surface an be calculated as

$$D = -\frac{\log N_{r1} - \log N_{r2}}{\log r_1 - \log r_2} \quad \text{…………………………………………………..(4)}$$

Where Nr1 and Nr2 are the number o boxes covering the ideal fractal surface when the box scales are r1 and r2, respectively . When r1 and r2 approach zero , the scale range can be found such that the FD estimate error approaches zero[13]. Any two values of r in $[r_1, r_2]$ and their corresponding $N_r$ can be used to substitute the parameters in (4) and obtain D since $N_r$ versus r is an exact straight line in log-log plot in the scale range. The negative range of the slope is equal to D[2].

1. Let a new scale $r' = r / c$ , where c is a positive real number greater than 1. The number of boxes covering a copy is calculated as the integer part of $(d_z / r' + 1)$ . Because $r' < r$ and $d_z / r' > d_z / r$ . Thus the errors introduced by the scale r' are smaller than r in the original DBC method[2-5].

2. $D_z$ is used to count the numbers of box covering a image surface instead of z , the gray levels of pixels being used for Eq.(2) in the original DBC method, resulting in more accurate box number counting[8,7].
3. The scale r for an ideal continuous surface is defined as $r = d_x = d_y$ , where $d_x = x_2 - x_1$ and $d_y = y_2 - y_1$. Therefore, for a digital image if the distance between any two neighboring pixels are 1, then the scale $r = l - 1$rather than r in the original DBC method, when $l * l$ pixels exists in a copy of a digital image surface[6-9].

***Box height selection:***

Let the mean and standard deviation of a digital image be μ and σ. Suppose most pixels fall in to the interval of gray level within$[\mu - a\sigma, \mu + a\sigma]$[2,6,12]. The box height r' is selected as

$$r' = \frac{r}{1+2a\sigma} \quad \text{......................................................................................................(5)}$$

Where a is a positive integer and 2aσ can indicate image roughness[2,4,6]. As a result a box with smaller height is chosen for an image surface with higher intensity variation. Compared with the height of boxes in the DBC method, the height o boxes in the improved DBC method is much smaller at different box sale for r. For an example if an image has the size of $256 * 256$ with 256 gray levels and he standard deviation σ being 15, while the box height is $r' = r = 3$ in the DB method and it becomes $r' = r / 91$ when $a = 3$ in the improved DBC method. So this method uses finer scales to count the number of boxes covering each block and the entire image surface through automatic adjustment based on image smoothness.

***Box number calculation:***

If the maximum and minimum gray levels of the (I,j)th block are p and q respectively , the number of boxes that cover the block surface can be calculated as

$$n_r(i, j) = \begin{cases} ceil\left(\dfrac{p-q}{r'}\right), p \neq q \\ 1, p = q \end{cases} \quad \text{..........................................................................(6)}$$

Where ceil(.) denotes the function to round a value to the nearest and greatest integer. The physical meaning of Eq.(3) is that the boxes are assigned from the minimum gray level of the block rather than gray level 0. It is expected that $n_r$(I,j) is the least number of boxes covering the surface of the (I,j)th block[2,8,13].

***Problem associated with box height selection in improved DBC method:***

In improved DBC method we are finding the standard deviation(σ) and mean(μ) of each window of an image in order to calculate very negligible height. FD have variation range computed by the improved DBC method for different values of 'a'. When a is chosen as 3 for a gray scale image, its FD is found to be greater than 3.

Due to this problem the accuracy of the improved DBC method is limited.

## I. NEW APPROACH TOWARDS BOX OUNTING METHOD

If M×M is the size of image and grids of size $l*l$ where $M/2 > l > 1$ and l is an integer, then we an cover up the entire image by boxes of sides $l*l*l'$ in the vertical direction. Here $l' = l*G/M$ can be multiple of the gray level units where G represents the total number of gray levels.

**a) Box height selection:**

When divide the image in to blocks of size $l*l$ ,are assign to cover the image surface, height of the $l*l$ size of window is defined as

$$\mathrm{H} = \mathrm{ceil}(\mathrm{MI}/\mathrm{l}')$$

Where MI=maximum intensity of particular window. H may not be same for each window because each window contains different gray level intensity.

**b) Box number calculation:**

Box interval of each window in between the minimum and maximum intensity gray level $=\frac{MI-MN}{Number\ \ of\ boxes\ present\ \ in\ this\ window}$

Where MN=Minimum gray level intensity of the particular window

Number of boxes present= Height of particular window

$$n_r(i,j) = \begin{cases} 1, & \text{MI = MN} \\ \text{Box interval having value, MI} \neq \text{MN} & \end{cases} \quad \ldots\ldots(7)$$

$n_r$= Number of boxes that can cover the block surface

$N_r$ is calculated by counting the total number of boxes that contains at least one gray level intensity surface. FD is calculated by various l values. $N_r$ is calculated and log-log plot of N versus (1/r) is drawn.

**SUMMARY OF ALGORITHM:**

1. If $\mathrm{M}*\mathrm{M}$ is the size of an image and blocks of size $l*l$ then the entire image size is covered up by the boxes of sides $l*l*l'$. Where $l' = l*G/M$ and G =total number of gray levels.
2. The boxes are assigned with a scale of $l*l*l'$ starting the pixel with minimum gray level in the block.
3. Number of boxes in each window is equal to $ceil(MI/l')$ where MI=maximum intensity of particular window.
4. Box interval of each window in between minimum and maximum intensity level is calculated and is equal to (MI-MN)/Number of boxes present in this window, where MN is the minimum intensity o the particular window. Then $n_r$ is calculated using Eq.(7).
5. For different scale l, the total number of boxes covering the full image surface is calculated.

The Fractal Dimension is calculated as the slope of the best fit line $\log(N_r)$ vs. $\log(1/r)$.

## EXPERIMENTS:

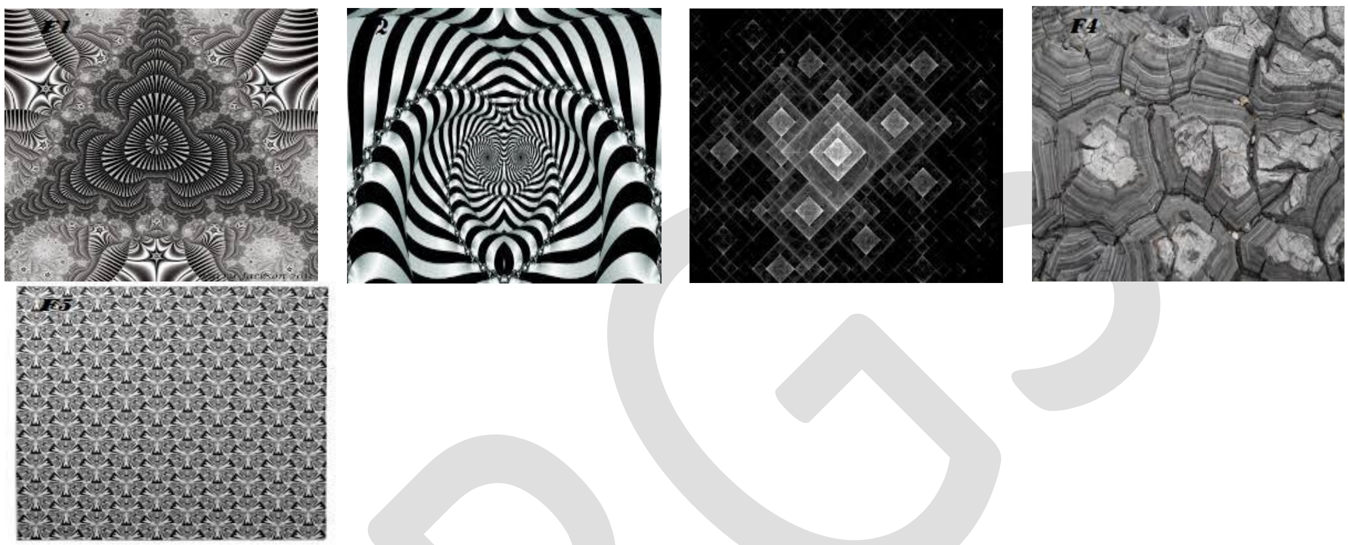


Fig. 4 Smooth gray scale image

For experiment we consider 5 gray scale images. The gray levels lie in the range 0-255 and size is equal to $256*256$. The algorithms are tested on gray images.

Table 1: Fractal dimensions of gray images obtained by various methods

| Gray Images | DBC method FD | Fit Error | Improved DBC FD | Fit Error | Proposed method FD | Fit Error |
|---|---|---|---|---|---|---|
| F1 | 2.2000 | 0.500 | 2.5500 | 0.083 | 2.5172 | 0.011 |
| F2 | 1.8024 | 0.187 | 2.0020 | 0.023 | 2.0617 | 0.053 |
| F3 | 2.2120 | 0.155 | 1.3520 | 0.059 | 2.4284 | 0.085 |
| F4 | 1.7248 | 0.255 | 2.5290 | 0.050 | 1.4534 | 0.002 |
| F5 | 2.0000 | 0.0641 | 2.1610 | 0.015 | 2.7372 | 0.004 |

Fit Error[12,13] is used to measure the least square linear fit of the $\log(N_r)\, versus \log(r)$ .The fit error E of points(x,y) from their fitted straight line satisfying $y = cx + d$ is defined as

$$E = \frac{1}{n}\sqrt{\sum_{i=1}^{n}\frac{cx_i + d - y_i}{1+c^2}}$$

where y and x denote $\log(N_r)$ and $\log(r)$

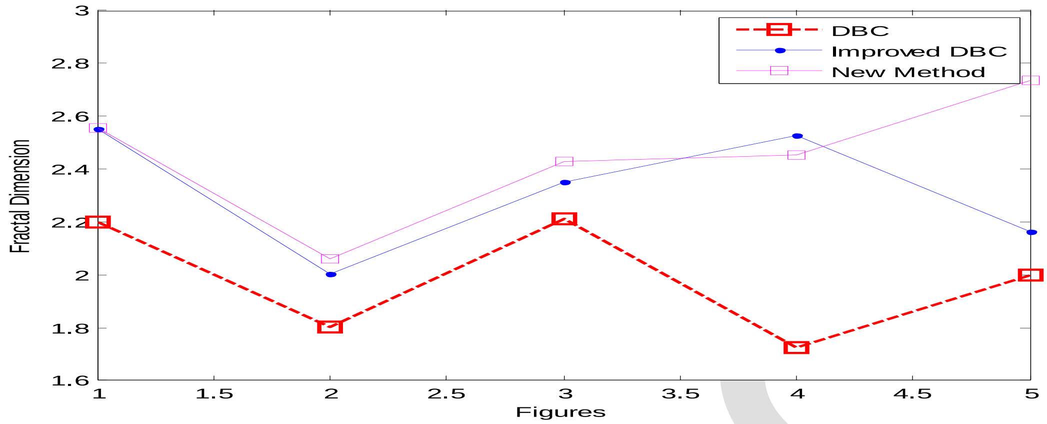


Fig. 5 Fractal dimension estimation of a gray image with various methods

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## CONCLUSION:

Due to rapid development of box counting method for calculating fractal dimension of an image, one new method is proposed to find more accurate FD results. The proposed technique was extensively tested with many types of images. FD of improved DBC is quite better than DBC and the result of new method gives promising result than improved DBC. This approach can also be extended to a 3-dimensional image as well

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