FACTA UNIVERSITATIS
Series: **Mechanical Engineering** Vol. 13, N° 1, 2015, pp. 11 - 20

NEW EXPERIMENTAL INVESTIGATIONS ON THE DIETERICH-RUINA FRICTION LAW

UDC 539.3

Grzemba Birthe

Berlin Institute of Technology, Germany

Abstract. *The Dieterich-Ruina friction law is widely used in rock friction and earthquake dynamical contexts. It has proven very powerful and versatile over the last decades, being able to reproduce stick-slip as well as fore- and aftershocks and healing of faults. This paper shows that the Dieterich-Ruina friction law can also reproduce the accelerated creep process preceding slip event in a stick-slip system, not only for steel contact, but also for rock contact. The friction law relies on three empirical parameters, which have to be identified for each application. The most common experimental set-up is the velocity step experiment. In this paper an alternative method to identify the parameters is proposed, which uses a stick-slip experiment and a numerical fitting approach.*

Key Words: *Dieterich-Ruina Friction, Stick-slip, Single Slider, Accelerated Creep, Parameter Fitting*

1. INTRODUCTION

The Dieterich-Ruina friction law is a special version of the rate-and-state friction law introduced by J.H. Dieterich in the 1970s [1, 2], which was proposed by A. Ruina [3, 4]. It was originally developed to describe earthquake dynamic processes, namely rock frictional contacts experiencing frictional instabilities and it is very widely used in this area [5-12]. The popularity of the Dieterich-Ruina friction law (DRF law) comes from its extreme versatility, being able to reproduce stick-slip, fore- and aftershocks as well as healing of faults [9, 10]. The DRF law states that the coefficient of friction μ depends on the relative velocity v and the contact history, represented by an internal state variable θ. Several examples for the use of internal variables in friction laws can be found in [13, 14]. A common form of the DRF law [15] is given as

Received: January 05, 2015
Corresponding author: Grzemba Birthe
Technische Universität Berlin, Strasse des 17. Juni 135, 10623 Berlin
E-mail: birthe.grzemba@tu-berlin.de

$$\mu = \mu_0 + a\ln\left(\frac{v}{v^*}\right) + b\ln\left(\frac{\theta}{\theta^*}\right) \tag{1}$$

where $(.)^*$represents a reference value. The reference state variable value is usually given as $\theta^* = D_C / v^*$. Parameters a, b and D_C are the empiric parameters of the friction law. Internal variable θ, representing the mean lifetime of a micro-contact, is connected with the relative velocity by a kinetic equation. Several forms of this kinetic equation have been proposed; the Dieterich-Ruina formulation is the one named 'slip law' in contrast e.g. to the also often used 'aging law', see [16] for comparison:

$$\dot{\theta} = 1 - \frac{v\theta}{D_C} \tag{2}$$

The use of the DRF law is not necessarily limited to earthquake dynamical processes. The DRF law has shown its applicability to a wide range of materials other than rock, including steel, glass, plastic and wood [17].

The DRF law is a laboratory based, empirical friction law, which means special attention has to be paid to the included parameters, which have to be found experimentally. Parameter D_C is a length parameter and is expected to be dependent on other system parameters and to show scaling behavior [1, 18, 19]. Dimensionless parameters a and b are usually found in the order of 10^{-2} to 10^{-3}. For characteristic length parameter D_C values around 0.5 to $50\,\mu\text{m}$ for laboratory set-ups and values around 1 to $10\,\text{mm}$ for real faults are identified [7, 18].

The most common experiment to identify the DRF law parameters is a velocity step experiment as introduced by Dieterich and Kilgore [17] and repeated later on by others. The experiment drives a frictional contact at steady velocity before introducing a sudden step in the velocity to a value about one order faster, then holds this faster velocity and steps down again. From a measurement of the coefficient of friction over slip distance parameter D_C and difference $(a - b)$ can be identified. This method is still used for the identification of DRF law parameters for specific material pairings under given load and temperatures [5 6, 20]. It is also used to decide if velocity weakening or velocity strengthening is present which can be read from the sign of $(a - b)$.

This method has a few disadvantages. Firstly, the velocity step experiment can be performed with any frictional contact, independent if the configuration behaves in a DRF law fashion or not. This means, even if the DRF law is not a suitable model, the velocity step experiment will provide values for its parameters. This can be seen e.g. in [21,22], where the found $(a - b)$ values depend on the sliding velocity, which should not be the case in a system obeying the DRF law. Secondly, the velocity step experiment does not contain stick-slip behavior. This can be advantageous if also possible velocity strengthening shall be addressed. But if the DRF law shall be used for the modeling of frictional instabilities, it would be desirable to include this dynamic in the process to identify the parameters. Lastly, the velocity step experiment only delivers difference $(a - b)$ but not both parameters.

This paper will present a new experimental method to deduce the DRF law parameters under consideration of both macroscopic and microscopic features of the stick-slip process. This will be done using a single block slider set-up. The elastically loaded single block slider has been excessively used as an analytical model to describe and investigate frictional instabilities and stick slip, mainly as a slider-on belt model for brake dynamics [23-25] and as loaded block on surface for earthquake dynamics [1, 4, 26, 27]. Experimental investigations of this system, on the other hand, are rare. Here, an extensive measurement series is performed using a single block slider set-up (section 2) combined with numerical modeling techniques (section 3) to show once more the eminent versatility and power of the DRF law and to identify the DRF law parameters of the system.

2. Experiments

2.1. Set-Up

The experimental set-up follows the concept of the single block slider model. The test stand and the measurement system are shown in Fig. 1.

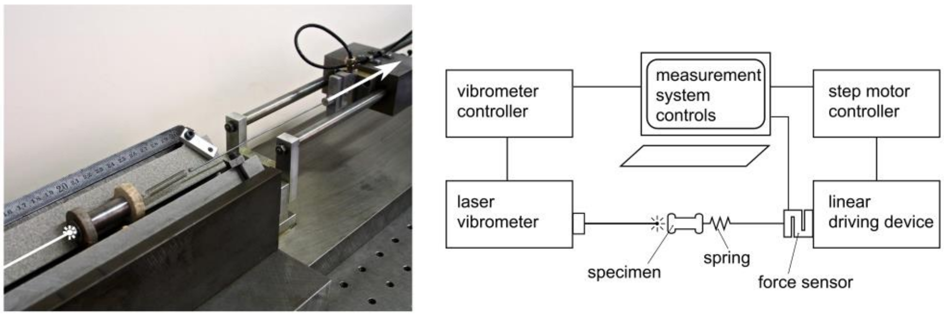

Fig. 1 Experimental set-up (left) and measurement system (right)

A specimen is pulled over exchangeable v-shaped base plates by a spring. The velocity of the spring's base point is constant and realized via a linear pulling device consisting of a linear guidance and a step motor driven spindle. If the specimen does not move, the tangential load on the specimen rises linearly with time. The displacement of the specimen is measured using a laser-Doppler vibrometer with 8 nm resolution. Additionally, the spring force is measured via a load cell.

Several parameters of this set-up are varied. For the results presented here, I have used four different specimens, two made of marble, two made of steel, with different contact geometries (see Fig. 2). Two different sets of base plates, one made of steel and one made of sandstone, are used. The stiffness of the pulling spring is varied as well as the driving speed. For each measured configuration at least thirty 10s measurements are performed, such that a total number of 932 experiments are evaluated for the presented results.

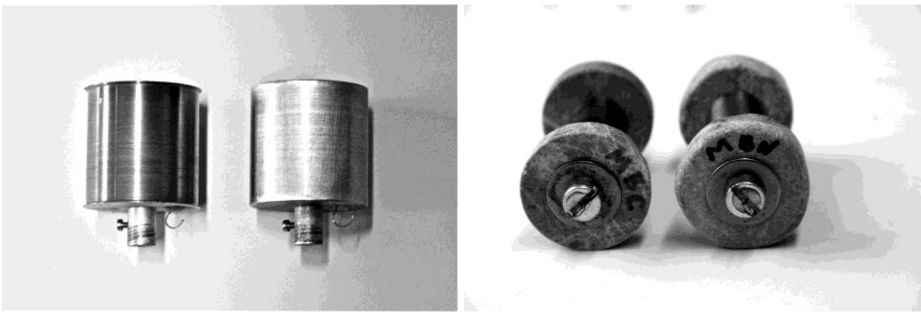

Fig. 2 Used specimens. Steel specimens (sharp edged and cylindric) on the left, marble specimens (regular circular and irregular) on the right

2.2. Results

All experimental configurations show a distinct stick-slip behavior. An example for a displacement and velocity curve over time can be seen in Fig. 3. We see comparably long stick phases that alternate with quick and sudden slip events. If one zooms in the stick phases to the micro meter scale, a regular accelerated creep process can be seen (Fig. 4). This preceding creep can even be used to predict the time of the next slip event [28, 29].

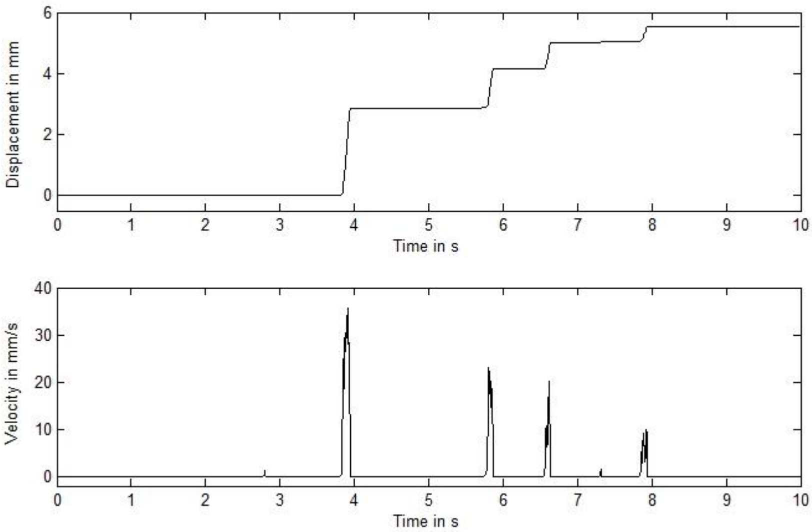

Fig. 3 Example of a typical displacement and velocity curve. The configuration of this data set is marble on sandstone with a driving velocity of 10mm/s

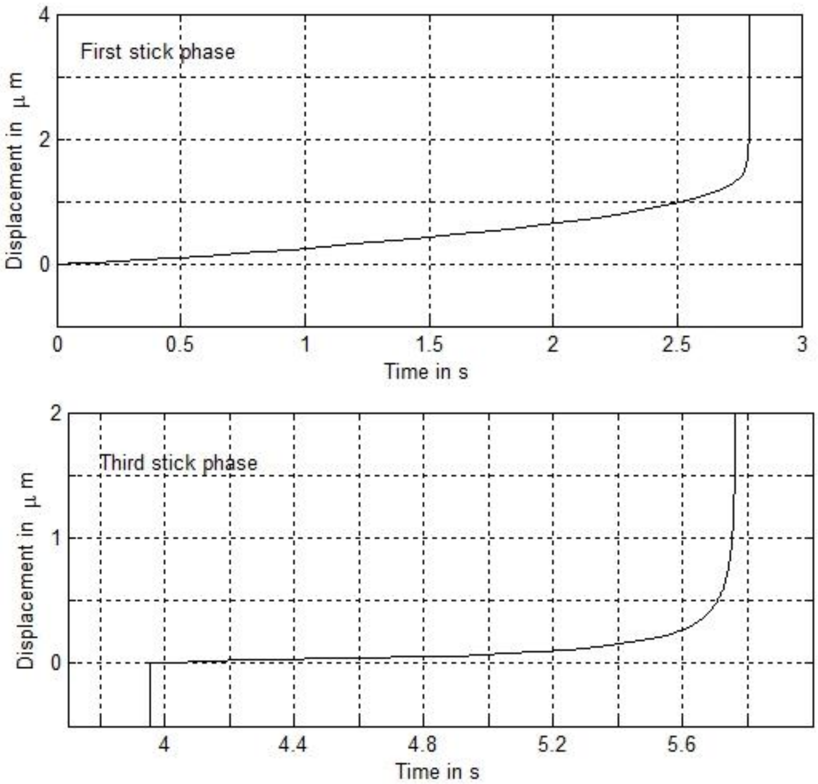

Fig. 4 Accelerated creep during stick phases. The shown data set is the same as in Fig.3, the first slip event is very small and can hardly be seen in the macroscopic view

In this paper, the experiments shall be used as a method to extract Dieterich-Ruina friction law parameters for the given contact. For this, first it must be shown how good the microscopic creep process and the fast slip can be described by a model using the DRF law.

3. Modeling and Numerical Fitting

The experimental set-up is modeled as a single block slider with Dieterich-Ruina friction (see Fig. 5). In this model the specimen is assumed to be rigid, which is a valid assumption because the longitudinal stiffness of the specimen is several orders of magnitude higher than the contact stiffness.

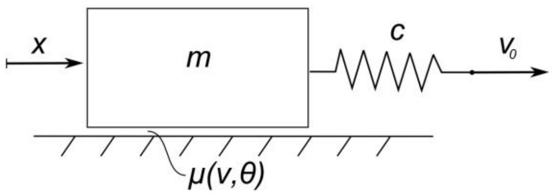

Fig. 5 Single block slider model with Dieterich-Ruina friction. The slider has mass m and the spring of stiffness c is driven with velocity v_0

Using the notation from Fig. 5 the equation of motion is given by

$$m\ddot{x} = c\left(v_0 t - x\right) - \mathrm{sgn}\left(\dot{x}\right) mg\left(\mu_0 + a\ln\left(\frac{\dot{x}}{v^*}\right) + b\ln\left(\frac{\theta v^*}{D_C}\right)\right) \tag{3}$$

where g is the gravity constant. Combined with the kinetic equation (2) a 2×2 nonlinear ordinary differential equation system is gained. This can be solved numerically, if some obstacles are overcome. I have attenuated the discontinuity of the friction law to make a numerical solution faster and more stable. To achieve this, I have approximated the signum function as a very steep hyperbolic tangent, which results in a stiff ode system, which again needs suitable solver. I have used a Matlab internal solver based on a semi-implicit Runge-Kutta method. Furthermore, for numerical stability, the arguments of the logarithms in (3) are completed with an additional $+1$ to avoid a sign change. This is allowed if reference velocity v^* is chosen much higher than all occurring relative velocities ($v^* >> \dot{x}$). The friction law then takes the form

$$\mu = \mu_0 - a\ln\left(\frac{v^*}{\dot{x}} + 1\right) + b\ln\left(\frac{\theta v^*}{D_C} + 1\right) \tag{4}$$

I have varied parameters a, b and D_C to achieve a best fit with the experimental data. During this process it has become clear that the model underestimates the very low creep velocities in the region of 10^{-7} m/s. This is assumed to be a result of the rigid body assumption: The frictional contact has a finite elasticity, which is reversible. This behavior is not included in the friction law. To compensate for this shortcoming, additional very slow creep velocity $\hat{v}$ is introduced, which can be estimated as

$$\hat{v} \approx 10^{-5}\cdot v_0 \quad \text{to} \quad 10^{-4}\cdot v_0 \tag{5}$$

This estimation is gained using a rough approximation of the tangential contact stiffness.

For the fitting, the numerical and experimental velocity curves are compared. I have found the velocity data to be more indulgent and therefore easier to fit than the displacement data. A logarithmic error definition is used to not only consider the fast slip events for the fitting, but also make the model meet the creep curves. This is especially important because the creep contains important information on the frictional behavior. The error definition reads

$$\Delta V_{log} = \frac{1}{T_{\text{fit}}} \sum_i \left(\log \dot{x}_{\text{num},i} - \log \dot{x}_{exp,i} \right)^2 \Delta t \tag{6}$$

As fitted time interval T_{fit} a value of approximately 16% of a characteristic stick time is used. The time resolution is $\Delta t = 2 \cdot 10^{-5}\text{s}$ as in the experiments. To find the optimal parameters, a gradient method is used. All fitting procedures use the same starting values (see Table 1). For the majority of all data sets the fitting works remarkably well and the experimental velocity curves are met over five to six orders of magnitude. A few examples are given in Fig. 6. 22% of the data sets could not be fitted satisfyingly with this model.

Table 1 Starting parameter values for fitting procedure

a	b	D_C
$6 \cdot 10^{-3}$	$1 \cdot 10^{-2}$	2μm

From the results of these fittings, mean values for the found optimal Dieterich-Ruina parameters can be gained. The found mean parameters are gathered in Table 2.

Table 2 Mean values of optimal parameters for all material pairings

Material pairing	Slider specimen	a in 10^{-3}	b in 10^{-3}	D_C in μm
Steel-steel	Sharp edged	4.16	10.66	1.26
	Cylindrical	3.48	12.16	0.74
Marble-sandstone	Regular circular	4.85	9.06	0.74
	Irregular circular	4.38	8.89	0.54

The found optimal parameters show a considerable statistical scatter, especially parameter D_C. This scatter is expected if recalling the physical meaning of D_C. Characteristic distance D_C describes the slip distance over which all micro-contacts are renewed [1]; this implies that is lies in the order of magnitude of the size of the micro-contacts. Since the microscopic contact configuration changes with every new contact realization, a statistical spread is anticipated. Parameters a and b do not possess such interpretation, but b has been shown to be closely related to the macroscopic slip length and the peak slip velocity [28]. These macroscopic entities should be affected less by changes in the microscopic configuration and are therefore expected to scatter less. The distributions of the found values for b and D_C follow the general shape of a Gaussian bell curve. An example for this can be seen in Fig. 7. The values found for parameter a build an exception. They are more uniformly distributed and also show a slight trend with the driving velocity, which is undesirable. I assume the reason for this in the definition of the length of the fitting interval, which depends on the characteristic stick time and therefore on the driving velocity. For future work, this definition should be revised.

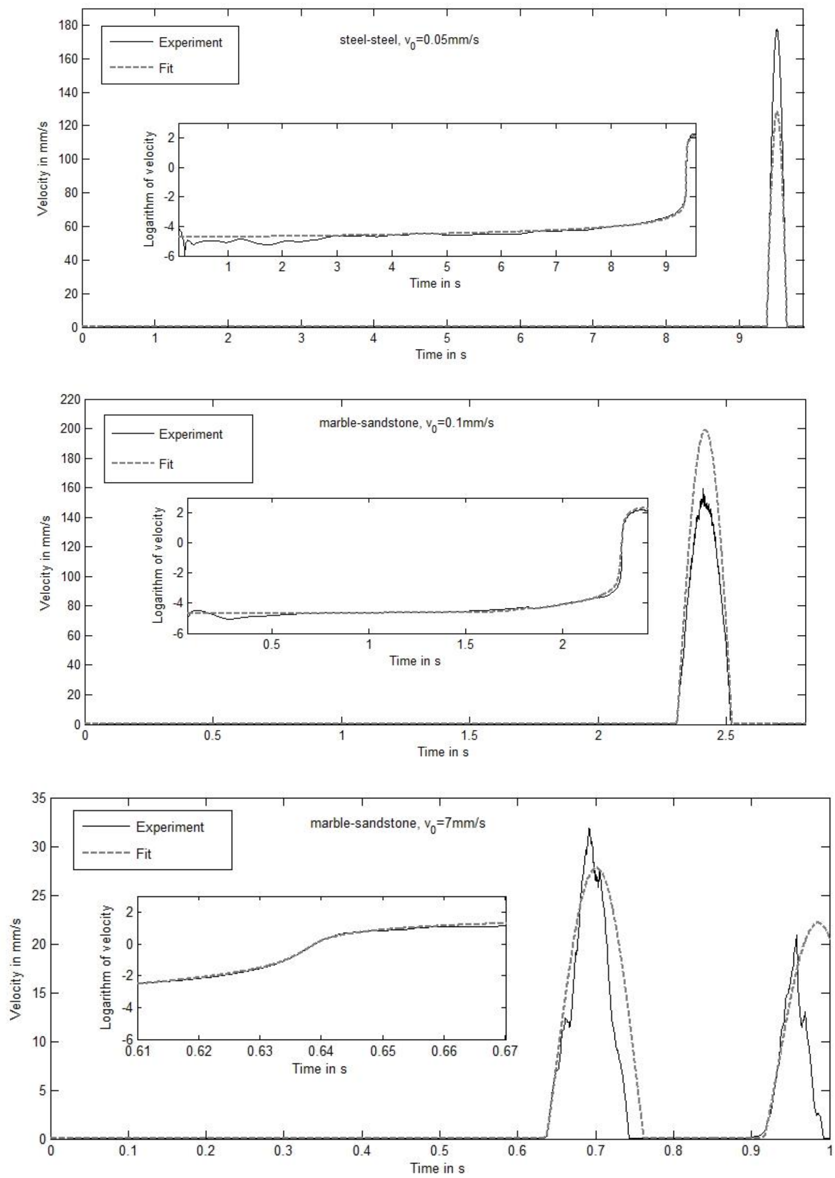

Fig. 6 Examples for successfully fitted data sets of different material pairings and driving velocities. The inserted smaller plots show the fitting region on a logarithmic scale

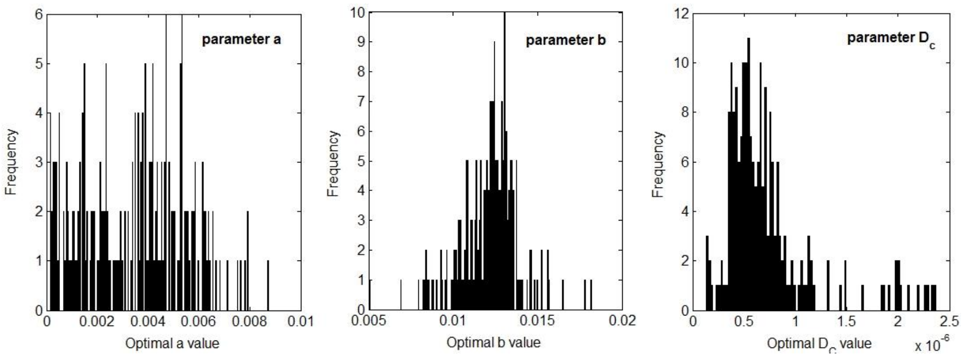

Fig. 7 Histograms showing the frequency of optimal found values for the three fitted parameters (a, b, D_C) for the configuration sharp edged steel specimen on steel base plates

A comparison of the found mean values with values from literature shows a good agreement. The values for D_C are relatively small, but this can be explained by the low normal forces and the very smooth contact surfaces in the experimental set-up. If one estimates the values for D_C using the recently proposed new scaling law [19], which takes into account the normal force and the surface roughness, value around 1 μm should be expected.

4. Conclusions

A fit of a Dieterich-Ruina friction law model to experimental stick-slip data is performed for different material pairings and contact geometries. Most data sets can be very well reproduced by the model; the velocity curves are met over five orders of magnitude, including the slow and accelerating creep as well as the fast slip events. The ability of the DRF law to reproduce the systems behavior over such a range of scales is impressive and underlines the importance and power of this friction law.

Additionally, the characteristic Dieterich-Ruina friction law parameters are gained from this fitting, which agree well with literature values. This procedure introduces a new experimental method to determine the correct empirical parameters for a given frictional contact. The experimental set-up is easier to construct and handle than the ones needed for the traditional velocity step experiments. Furthermore, this method directly uses the microscopic properties of the system, because the creep process is fitted. It can be an attractive alternative especially when the focus is on systems encountering frictional instabilities.

Acknowledgements: *I want to acknowledge the help of Dr. J. Starcevic with the execution of the experiments. Furthermore I am very thankful to Prof. V.L. Popov for the helpful conversations and his advice.*

REFERENCES

1. Dieterich, J.H. 1978, *Time Dependent Friction and the Mechanics of Stick—Slip,* Pure and Applied Geophysics, 116, pp. 790-806.
2. Dieterich, J.H., 1979, *Modeling of Rock Friction, Part 1: Experimental Results and Constitutive Equations,* Journal of Geophysical Research, 84(B5), pp. 2161-2168.
3. Ruina, A.L., 1980, *Friction Laws and Instabilities: A Quasistatic Analysis of Some Dry Frictional Behavior,* Brown University, Ph.D. dissertation.
4. Ruina, A.L., 1983, *Slip Instability and State Variable Friction Law.* Journal of Geophysical Research, 88(B12), pp. 10359-10370.
5. Tse, S.T., Rice, J.R., 1986, *Crustal Earthquake Instability in Relation to the Depth Variation on Frictional Slip Properties.* Journal of Geophysical Research, 91(B9), pp. 9452-9472
6. Marone, C., Scholz, C.H., 1988, *The depth of seismic faulting and the upper transition from stable to unstable slip regimes,* Geophysical Research Letters, 15, pp. 621-624.
7. Dieterich, J.H., 1992, *Earthquake Nucleation on faults with rate and state-dependent strength,* Tectonophysics, 211, pp. 115-134.
8. Beeler, N.M., Tullis, T.E., Weeks, J.D., 1994, *The Role of Time and Displacement in the Evolution Effect of Rock Frictio,* Geophysical Research Letters, 21, pp. 1987-1990.
9. Dieterich, J.H., Kilgore, B.D., 1996, *Implications of fault constitutive properties for earthquake prediction,* Proceedings of the National Academy of Sciences, 93, pp. 3787-3794.
10. Scholz, C.H. 1998, *Earthquakes and Friction Law,* Nature, 391, pp. 37-42.
11. Ampuero, J.-P., Rubin, A.M., 2008, *Earthquake nucleation on rate and state faults - Aging and slip laws,* Journal of Geophysical Research: Solid Earth, 113, p. B01302.
12. Bizzarri, A., Crupi, P., de Lorenzo, S., Loddo, M., 2011, *Time Occurence of Earthquake Instabilities in Rate-and-State Dependent Friction Model,* INGV, UNIVERSITÀ DEGLI STUDI DI BARI, Rapporti Technici.
13. Persson, B., 2000, *Sliding friction: Principles and Applications*, Springer Verlag, Berlin, 2nd edition.
14. Popov, V.L., 2010, *Contact Mechanics and Friction*, Springer Verlag, Berlin.
15. Dieterich, J.H. 2007, *Applications of Rate- and State-Dependent Friction to Models of Fault Slip and Earthquake Occurrence,* Treatise on Geophysics, 4, pp. 107-129.
16. Marone, C., 1998, *Laboratory--Derived Friction Laws and their Application to Seismic Faulting,* Annual Review of Earth and Planetary Sciences, 26, pp. 643-696.
17. Dieterich, J.H., Kilgore, B.D., 1994, *Direct Observation of Frictional Contacts: New Insights for State--dependent Properties,* Pure and Applied Geophysics, 143, pp. 283-302.
18. Scholz, C.H., 1988, *The Critical Slip Distance for Seismic Faulting,* Nature, 336, pp. 761-763.
19. Grzemba, B., Pohrt, R., Teidelt, E., Popov, V.L., 2014, *Maximum micro-slip in tangential contact of randomly rough self-affine surfaces,* Wear, 309, pp. 256-258.
20. den Hartog, S.A.M., Niemeijer, A.R., Spiers, C.J. 2012, *New constraints on megathrust slip stability under subduction zone P-T conditions,* Earth and Planetary Science Letters, 353-354, pp. 240-252.
21. Ikari, M.J., Saffer, D.M., 2011, *Comparison of fricitonal strength and velocity dependence between fault zones in the Nankai accretionary complex,* G3: Geochemistry, Geophysics, Geosystems, 12, p. Q0AD11.
22. den Hartog, S.A.M., Spiers, C.J. 2013, *Influence of subduction zone conditions and gouge composition on frictional slip stability of megathrust faults,* Tectonophysics, 600, pp. 75-90.
23. Blok, H. 1940, *Fundamental Mechanical Aspects of Boundary Lubrication,* SAE Journal Transactions, 46, pp. 54-68.
24. Derjaguin, B.V., Push, V.E., Tolstoi, D.M., 1957, *A theory of stick--slip sliding of solids,* The institution of mechanical engineers, Proceedings of The Conference on Lubrication and Wear. pp. 257-268.
25. Rabinowicz, E. 1958, *The Intrinsic Variable affecting the Stick--Slip Process,* Proceedings of the Physical Society, 71, pp. 668-675.
26. Rice, J.R., Ruina, A.L. 1983, *Stability of Steady Frictional Slipping,* Journal of Applied Mechaincs, 50, pp. 343-349.
27. Gu, J.-C., Rice, J.R., Ruina, A.L., Tse, S.T., 1984, *Slip Motion and Stability of a Single Degree of Freedom Elastic System with Rate and State Dependent Friction,* Journal of the Mechanics and Physics of Solids, 32, pp. 167-196.
28. Popov, V.L., Grzemba, B., Starcevic, J., Popov, M., 2012, *Rate and state dependent friction laws and the prediction of earthquakes: What can we learn from laboratory models?* Tectonophysics, 532-535, pp. 291-300.
29. Grzemba, B., 2014, *Predictability of Element Models for Earthquake Prediction,* Technische Universität Berlin, Ph.D. dissertation.