FACTA UNIVERSITATIS
Series: **Mechanical Engineering** Vol. 12, N° 2, 2014, pp. 137 - 148

A GENERAL APPROACH TO MODELING NONLINEAR AMPLITUDE AND FREQUENCY DEPENDANT HYSTERESIS EFFECTS BASED ON EXPERIMENTS WITH RUBBER PARTS

UDC 621.8

Christopher Heine[1,2], **Markus Plagemann**[2]

[1]Technical University Berlin, Germany
[2]BSH Bosch and Siemens Home Appliances GmbH, Berlin, Germany

Abstract. *A detailed description of the rubber parts' properties is gaining in importance in the current simulation models of multi-body simulation. One application example is a multi-body simulation of the washing machine movement. Inside the washing machine, there are different force transmission elements, which consist completely or partly of rubber. Rubber parts or, generally, elastomers usually have amplitude-dependant and frequency-dependent force transmission properties. Rheological models are used to describe these properties. A method for characterization of the amplitude and frequency dependence of such a rheological model is presented within this paper. Within this method, the used rheological model can be reduced or expanded in order to illustrate various non-linear effects. An original result is given with the automated parameter identification. It is fully implemented in Matlab. Such identified rheological models are intended for subsequent implementation in a multi-body model. This allows a significant enhancement of the overall model quality.*

Key Words: *Experiment, Rubber, Hysteresis, Nonlinear Viscoelastic Behavior, Frequency, Amplitude*

1. INTRODUCTION

Within the current studies, it is important to characterize nonlinear behavior of rubber parts or generally elastomers in view of the fact that they and their impact gain more and more in importance in modern simulation models. For this reason, NVH-researchers from various technical application fields deal with the description and characterization of rubber

---

Received: May 05, 2014
Corresponding author: Christopher Heine
Technical University Berlin, Germany; BSH Bosch and Siemens Home Appliances GmbH, Wohlrabedamm15, 13629 Berlin, Germany
E-mail: christopher.heine@bshg.com

parts and their properties. Puel et al. [5] have dealt with a linear viscoelastic rubber model with non-linear time dependence with the aim of integrating it into a multi one.

A common method [2, 4, 6] for describing rubber parts implies the division of their properties into three parts, briefly described in the following. The first part deals with non-linear dependence of the force response on the displacement amplitude - the amplitude response. The second part includes the force response dependence on the excitation frequency - the frequency dependence. The third part comprises modeling the temperature dependence. In the present work, the temperature dependence is not taken into account. The two first types of dependence are generally shown in Fig. 1. In order to describe these curves, rheological models are established. These models consist of different ways of coupling the basic elements of mechanics, e.g. spring, damper and friction elements. The amplitude dependence represents the static or quasi-static dependence of the force response on the excitation amplitude. These static hystereses are measured under quasi-static excitation with the excitation frequency ≤ 0.03 Hz [6]. Generally, elastomers behave increasingly stiffer for smaller amplitudes [6]. The hystereses for smaller amplitudes are applied to the reversal points of the larger angle amplitudes. With regard to the frequency dependence, the dynamic hysteresis becomes stiffer with increasing frequency. All graphs in this work were normalized and thus presented without units.

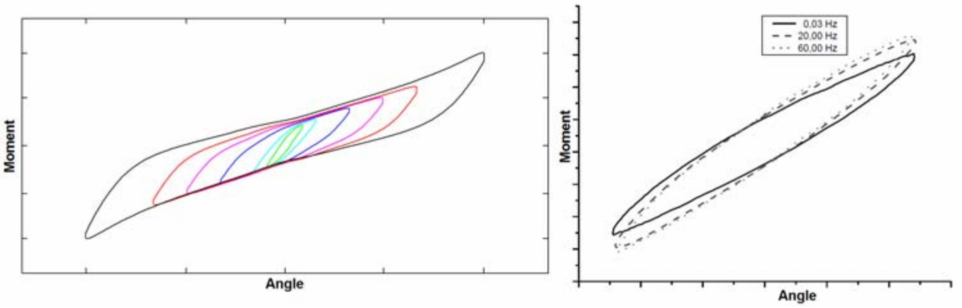

Fig. 1 Representation of the amplitude dependence (left, internal test) and the frequency dependence (right, [7])

This splitting of the dependencies is also performed in the work of Lou et al. [4] and Jrad et al. [2]. In both works, rheological models are used to describe amplitude and frequency-dependence of the respective rubber parts. These tested rubber parts have shown only linear viscoelastic properties. The models are characterized on the basis of measurements.

This paper is about rubber parts which are mounted inside of a washing appliance. The characteristics of these components will be characterized for use within a multi-body model. For this, the properties are split into amplitude and frequency dependence, which are individually detected by measurements, and are described below by means of rheological models. As a differentiation relative to the cited works, a special feature in this work is the occurrence of non-linear viscoelastic behavior, which is also characterized by means of advanced rheological models. This is an original feature of the conducted research.

2. MOTIVATION

The reason for this work is to optimize the multi-body predictive modeling performance. This has been undertaken by characterizing the force elements which are included in a model of a front loading washing machine. Fig. 2 depicts a front loading washing machine. The individual parts are numbered. The washing machine can be divided in two assemblies. The first assembly is the oscillating system and the second is the housing. The front panel (Fig. 2, No. 1; following part assignments refer to Fig. 2, too), the side and back panel (No. 10) and the feet (No. 11) are parts of the housing. The drum (No.3), the drum support (No. 4), the pulley (No. 5), the tub (No. 7), the electrical drive (No. 12), the front counterweight (No. 6) and the top counterweight (No. 9) are parts of the oscillating system. The oscillating system is hooked with the aid of the force transmission elements in the housing. These force transmission elements are the springs (No. 8), the damper (No. 13) and the gasket (No. 2).

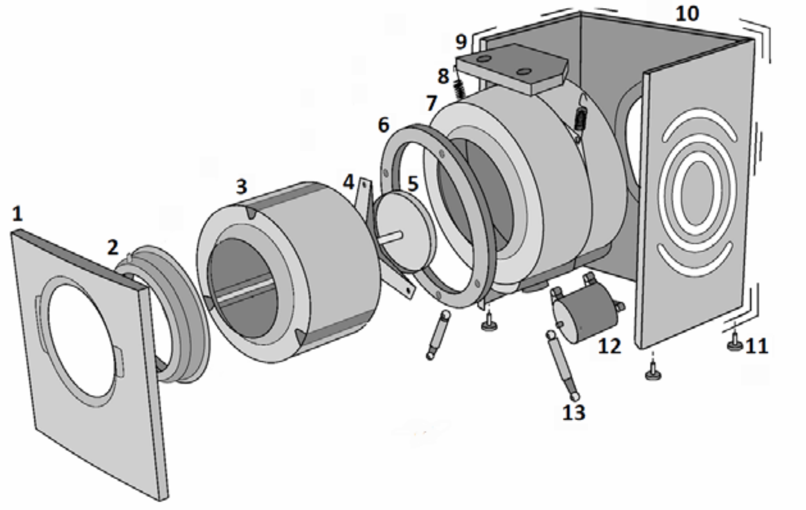

Fig. 2 Construction of a washing machine

In this work, models for characterizing the damper and their connections are to be found. Inside the damper connections are small rubber parts to effect an elastic connection. These connections are to be measured and subsequently characterized. The measurements of these connections are divided with respect to the two above named properties of the elastomers (amplitude- and frequency dependence). Both measurements are performed with an identical test setup. The using test setup is shown in Fig. 3 as an example. On the right side of the Fig. 2 the excitation (Fig. 3, **E**) is carried out with varying amplitudes or different frequencies. Using a displacement (Fig. 3, **DS**) and a force sensor (Fig. 3, **FS**), the response of the component to be tested is detected.

The results of these measurements are shown in the next section.

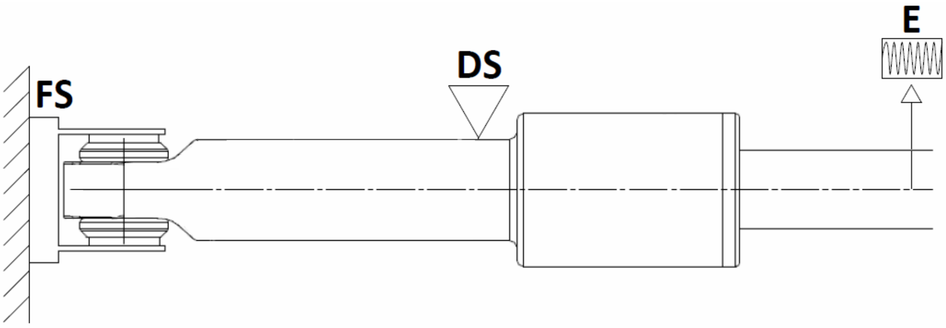

Fig. 3 Illustration of the test setup

3. RHEOLOGICAL MODEL FOR REPRESENTING THE AMPLITUDE DEPENDENCE

Within the quasi-static measurements, two cases of the amplitude dependence occur. The first type of hysteresis is shown in Fig. 4. All the given curves emanate from a series of characterization tests on a rubber specimen.

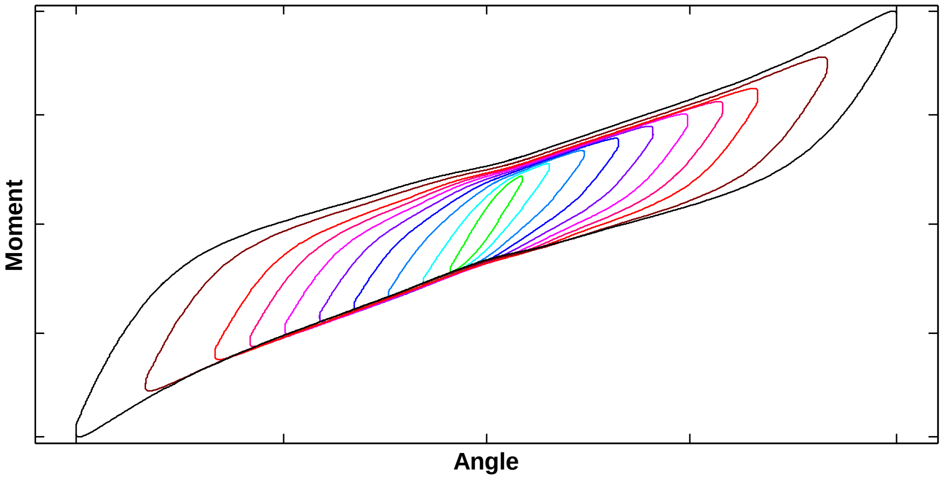

Fig. 4 Hysteresis curves for different excitation amplitudes (Type I)

On the other hand, static hystereses are measured, where the hysteresis demonstrates a constriction or a decrease of force magnitudes in the width toward the zero crossing. These are shown exemplarily in Fig. 5.

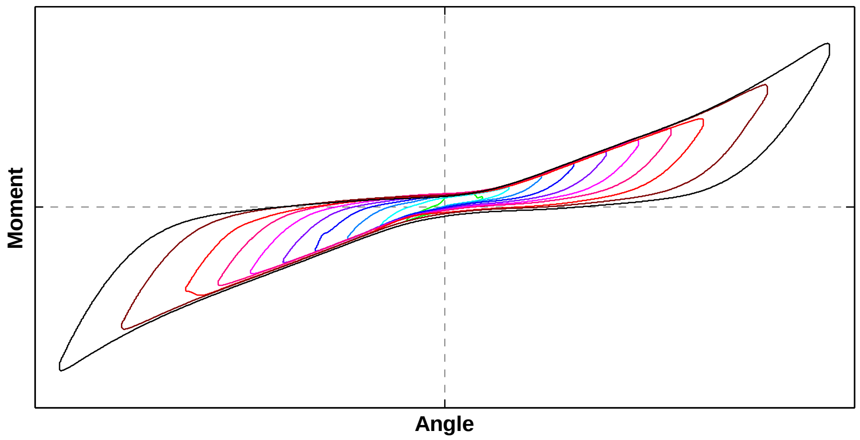

Fig. 5 Constriction of the trapezoidal hysteresis curves for different amplitudes (Type II)

As a basis for simulating the amplitude dependence, the so-called PRANDTL-element is used. This element consists of a serial arrangement of a spring and a friction element. The resultant force from this element is determined by using the following equation explained in Beerens [1], where x represents the displacement, k is the stiffness of the spring, R represents the frictional force of the friction element and F_{Pr} describes the resulting force of the PRANDTL-element.

$$\dot{F}_{\mathrm{Pr}} = \frac{1}{2} k\dot{x}(t)[1 - sign(F_{\mathrm{Pr}}^2 - R^2) - sign(F_{\mathrm{Pr}}\dot{x}(t))(1 + sign(F_{\mathrm{Pr}}^2 - R^2))] \tag{1}$$

The characteristic hysteresis of a PRANDTL-element is shown in Fig. 6.

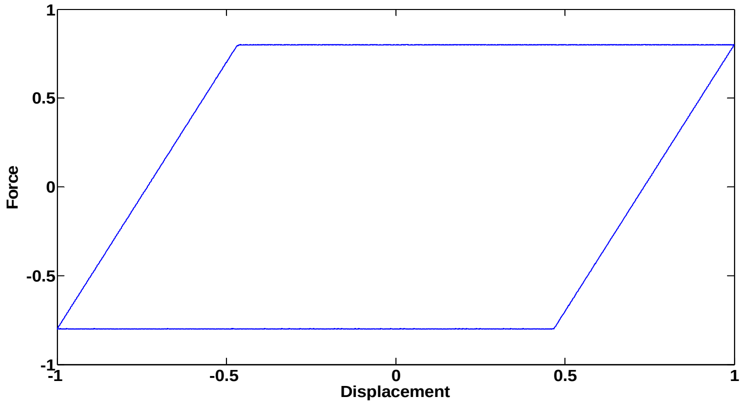

Fig. 6 Hysteresis of a PRANDTL-element

In order to display measured hysteresis curves *n* PRANDTL-elements are connected in parallel. In addition to the *n* PRANDTL-elements a non-linear spring is connected in parallel. This results in the so-called MASING model whose circuit diagram is shown in Fig. 7. In the paper by Beerens [1], the Masing model is discussed extensively.

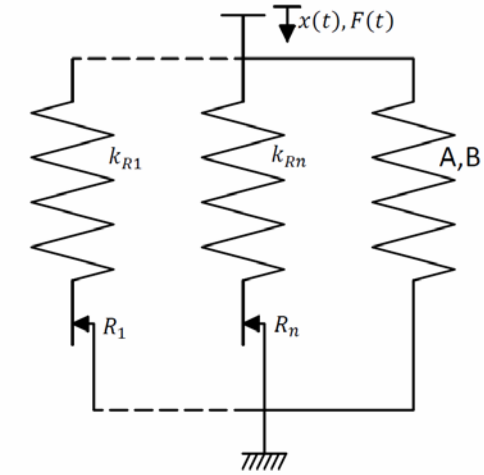

Fig. 7 MASING model with n PRANDTL-elements and a non-linear spring

For the description of the nonlinear spring force a modified third-order polynomial is used, in which the square shares are neglected.

$$F_F = x^3 A + xB \tag{2}$$

To make the number of parameter of the MASING model manageable, two further simplifications are made. The first simplification indicates that the stiffnesses of the springs inside the PRANDTL-elements are identical.

$$k_{R1} = k_{R2} = \ldots = k_{Rn} = k_R \tag{3}$$

The second simplification is concerned with the frictional forces of the friction elements of the PRANDTL-elements. These frictional forces are distributed with an exponential function.

$$R_i = \alpha e^{-\alpha\eta} \text{ with } \eta = \frac{i-1}{n-1} \text{ with } i = 0,\ldots,1 \text{ with } \eta \in [0,1] \tag{4}$$

As a result of both simplifications and the description of the non-linear spring via the said polynomial approach the used MASING model is completely described by five parameters. These five parameters comprise parameters *A* and *B* for the description of the polynomial, *k* is the stiffness of the PRANDTL-elements, the number *n* of PRANDTL-elements and parameter α to describe the distribution function of the frictional forces. The number *n* of PRANDTL-elements controls the smoothness of the modeled hysteresis and is set to seven within this work, identified by a short preliminary study.

4. RHEOLOGICAL MODEL FOR REPRESENTING THE FREQUENCY DEPENDENCE

This section is to explain the frequency dependence. In order to display frequency dependence the so-called MAXWELL-element is used. This element consists of a serial connection of a spring and a damper. The resulting force of this element can be calculated by using the following equation.

$$\dot{F}_{Max}(t) = C\dot{x}(t) - \frac{C}{D} F_{Max}(t) \tag{5}$$

In the present case, the frequency dependence is to be imaged from zero to 20 Hz. Pohl and Wahle [6] have demonstrated that the efficiency of a MAXWELL-element is limited in the damping characteristics to a narrow frequency band of about two frequency decades. For this reason, it is sufficient for the description here to use two MAXWELL-elements connecting in parallel to the MASING model. The complete model is shown in Fig. 8. A similar model has also been used in the work of Pohl and Wahle [6].

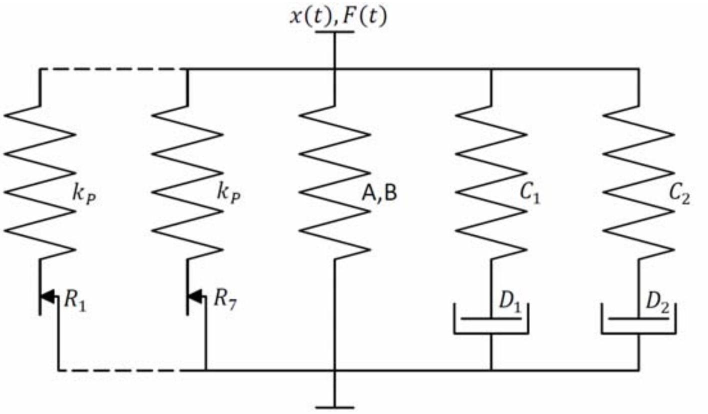

Fig. 8 The complete model used

5. RESULTS OF AMPLITUDE DEPENDENCE

For the calculation of the model parameters to be used in a Matlab script has been programmed. With the help of this script, an optimal parameter set for the assigned model is computed completely automatically.

Within the measurement ten measurements with different excitation amplitude are carried out for each rubber part. In the first calculation step, the model parameters are individually determined for each amplitude level.

The resulting data for various amplitudes are shown for four different amplitude levels in Fig. 9. It is clearly seen that the used model can represent different amplitude levels very well.

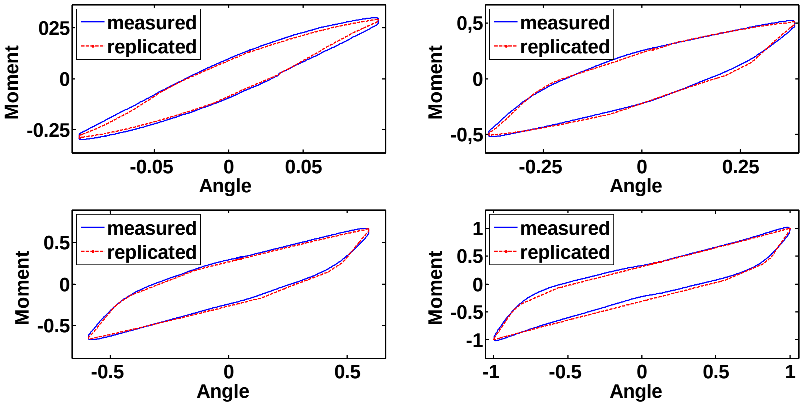

Fig. 9 Simulated hysteresis for different amplitude levels

Within a second step, the calculated values of the model parameters have been modeled as a function of the excitation amplitude. The functions to be used to describe the amplitude dependence of the model parameters could be reduced to three functions. It is a polynomial of degree 4 and degree 1 as well as the exponential function. The four model parameters and their characteristic functions of the excitation amplitude are shown in Fig. 10.

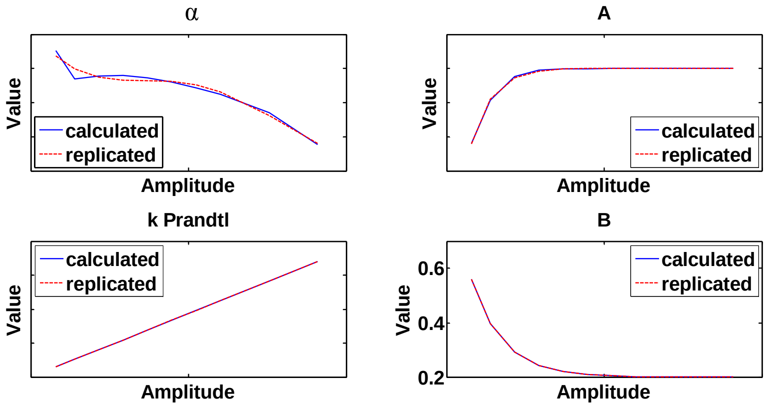

Fig. 10 Model parameters as a function of the amplitude

What follows is a special case of the constricted hysteresis. The total force of the PRANDTL-elements must be reduced down to the zero crossing of the path excitation. To realize this, the resultant force of the seven PRANDTL-elements is multiplied with a scaling function which depends on the path excitation.

$$SK_{F_{\text{Pr}}}(x) = \frac{(|\tanh(Px)|) + K}{(1+K)} \tag{6}$$

The used scaling function enhances the complete model with two additional parameters. On the one hand parameter P which describes the slope of the hyperbolic tangent and on the other hand parameter K which describes the minimum scaling at the zero crossing. An example of the profile of this scaling function is shown in Fig. 11.

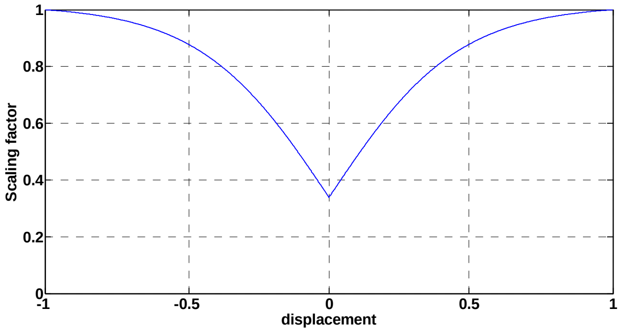

Fig. 11 Example of the profile of the used scaling function

This model with the described scaling function is implemented in Matlab. An optimal set of parameters is calculated by using Matlab for the model with implemented scaling function. Fig. 12 shows two different amplitude levels. On the left side of the figure, the hysteresis can be reproduced without using the scaling function. However, on the right side, the scaling function is used. It is clearly seen that the use of the scaling function leads to a high precision representation.

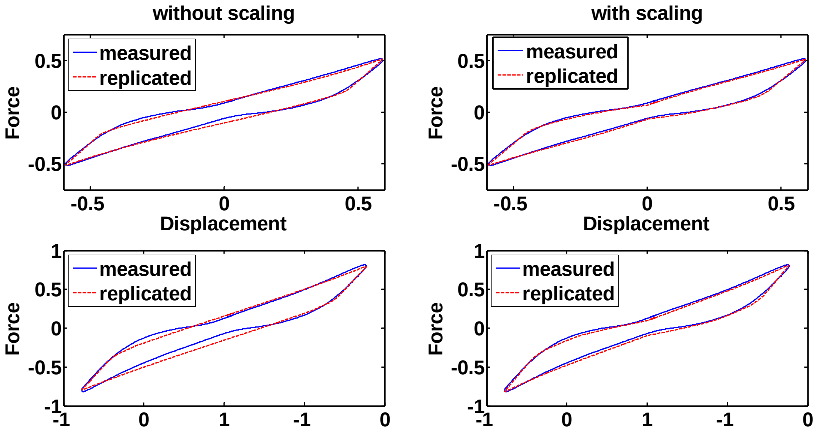

Fig. 12 Hysteresis with and without scaling function

Using the model with and without scaling function the amplitude dependence of the present elastomer could be clearly described and modeled.

6. RESULTS OF FREQUENCY DEPENDENCE

A Matlab script has been programmed for the second step, which now calculates the parameters for the frequency dependence. The calculation of the MAXWELL parameters can be performed by determining the loss and storage stiffness as described in the work of Lion [3]. It is not possible to use this method to calculate the MAXWELL parameters. The reason for this lies in the possibilities of measuring technology detection of the component properties. Therefore, parameters C1, C2, D1 and D2 need to be calculated individually. What results from this Matlab calculation is hysteresis for different excitation frequencies shown in Fig. 13.

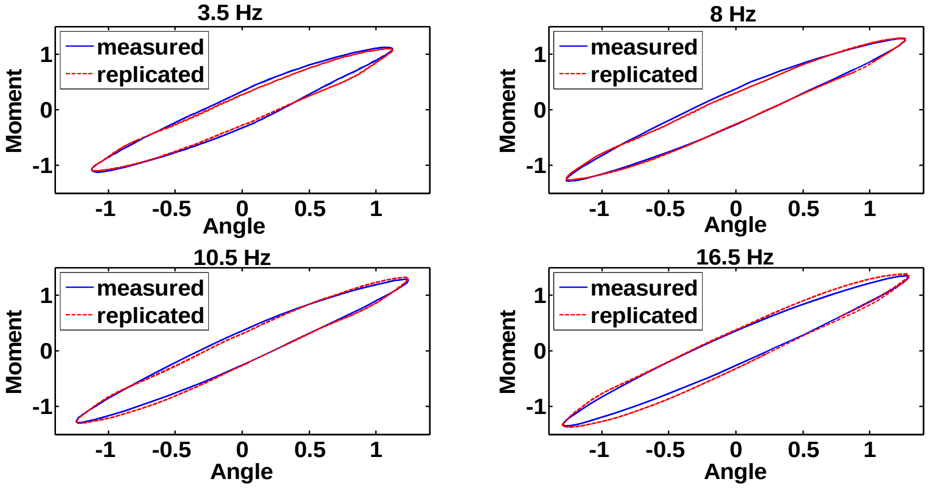

Fig. 13 Measured (straight) and simulated (dotted) hysteresis for different excitation frequencies

The calculation of the model parameters for describing the frequency dependence is carried out for a frequency range of two to 20Hz with a step size of 0,5Hz. From this calculation, the function of the model parameters of the excitation frequency could be calculated in a subsequent step. These dependencies could be described by linear functions and constant model parameters. The functions used to describe the frequency dependence of the model parameters are shown in Fig. 14.

7. CONCLUSION

In this work, the material properties of elastomers in terms of amplitude and frequency dependence have been measured. Then rheological mathematical models are identified to describe the dynamic behavior. In order to maximize the representation quality and to decrease modeling time a Matlab script is programmed. This Matlab script contain the possibility to expand or simplify the model to be calculated. By this fact various components could be described with the aid of this script.

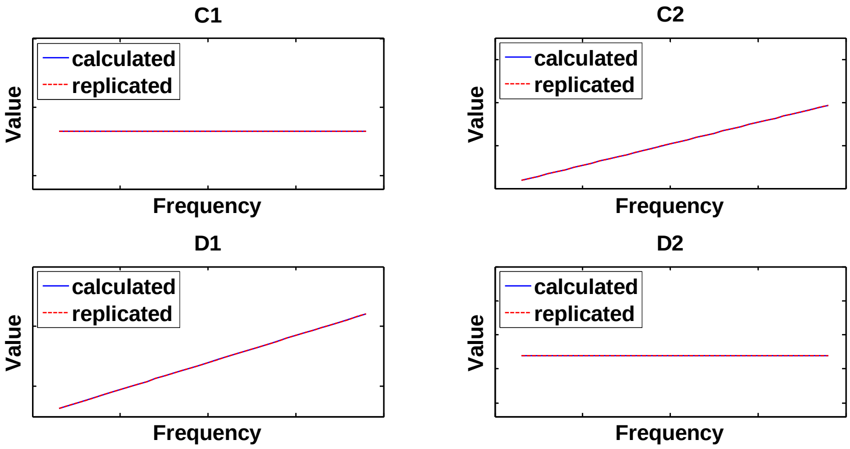

Fig. 14 Representation of the frequency dependence of the model parameters

The simulated hysteresis describes the measured hysteresis in terms of amplitude and frequency dependence. The identified model describes the material behavior of the measured elastomer within the work area completely automatically. Then the described models are successfully implemented in a multi-body simulation, leading to an improvement of the imaging quality of the multi-body simulation.

REFERENCES

1. Beerens, C., 1994, *Zur Modellierung nichtlinearer Dämpfungsphänomene in der Strukturmechanik*, Mitteiliungen aus dem Institut fuer Mechanik, Ruhr-Universität Bochum
2. Jrad, H., Renaud, F., Dion J.L., Tawfiq, I., Haddar, M., 2013, *Experimental characterization, modeling and parametric identification of hysteresis friction behavior of viscoelastic joints*, International Journal of Applied Mechanics, 5(2), pp. 1350018-1-21
3. Lion, A., 2005, *Phenomenological Modelling of Strain.Induced Structural Changes in Filler-Reinforced Elastomers*, KGK Kautschuk Gummi Kunststoffe, 58(2), pp.157-162
4. Lou, W., Hu, X., Wang, C., Li, Q., 2010, *Frequency- and strain-amplitude-dependent dynamical mechanical properties and hysteresis loss of CB-filled vulcanized natural rubber*, International Journal of Mechanical Sciences, 52(2), pp. 168-174
5. Puel, G., Bourgeteau, B., Aubry, D., 2013, *Parameter identification of nonlinear time-dependent rubber bushings models towards their integration in multibody simulations of a vehicle chassis*, Mechanical Systems and Signal Processing, 36(2), pp. 354-369.
6. Pohl, R., Wahle, M., 1999, *Entwicklung eines Rechenmodells zur Beschreibung von Gummibauteilen bei statischer und dynamischer Belastung (Simulationsmodell für Gummibauteile): Schlußbericht zum BMBF-Forschungsvorhaben*, Fachhochschule Aachen
7. Waltz, M., 2005, *Dynamisches Verhalten von gummigefederten Eisenbahnrädern*, PhD dissertation, Fachhochschule Aachen

OPŠTI PRISTUP MODELIRANJU NELINEARNIH HISTEREZISNIH EFEKATA ZAVISNIH OD AMPLITUDE I FREKVENCIJE NA OSNOVU EKSPERIMENATA SA GUMENIM DELOVIMA

Detaljni opis osobina gumenih delova dobija na značaju u sadašnjim modelima simulacije simulacija sa više tela. Jedan primer primene je multi body simulacija kretanja mašine za pranje. Unutar mašine za pranje javljaju se različiti elementi prenosa sile koji su potpuno ili delimično gumeni. Gumeni delovi ili, uopšte, elastomeri, obično imaju svojstva prenosa sile koji su zavisni od amplitude i frekvencije. Reološki modeli se koriste za opis ovih svojstava. Jedna metoda karakterizacije zavisnosti amplitude i frekvencije takvog reološkog modela je predstavljena u ovom radu. Po toj metodi, korišćeni reološki model se može svesti ili proširiti radi ilustracije raznih nelinearnih efekata. Originalan rezultat je dat sa automatizovanom identifikacijom parametra. U potpunosti je primenjen u Metlabu. Takvi identifikovani reološki modeli namenjeni su potonjoj implementaciji u modelu sa više tela. To nam dopušta značajno pojačanje ukupnog kvaliteta modela..

Ključne reči: *eksperiment, guma, histereta, nelinearno visko-elastično ponašanje, frekvencija, amplituda*