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ON INFLUENCE OF BOUNDARY CONDITIONS AND TRANSVERSE SHEAR ON BUCKLING OF THIN LAMINATED CYLINDRICAL SHELLS UNDER EXTERNAL PRESSURE

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Abstract. *Buckling of a thin cylindrical sandwich shell composed of elastic isotropic layers with different elastic properties under normal external pressure is the subject of this investigation. Differential equations based on the assumptions of the generalized kinematic hypothesis for the whole sandwich are used as the governing ones. Two variants of the joint support conditions are considered at the shell edges: a) there are the infinite rigidity diaphragms inhibiting relative shears of layers along the shell edges, b) the diaphragms are absent. Using the asymptotic approach, the critical pressure and buckling modes are constructed in the form of the superposition of functions corresponding to the main stress-strain state and the edges integrals. As an example, a three-layered cylinder with the magnetorheological elastomer (MRE) embedded between elastic layers under different levels of magnetic field is studied. Physical properties of the magnetorheological (MR) layer are assumed to be functions of the magnetic field induction. Dependencies of the buckling pressure on the variant of boundary conditions and the intensity of applied magnetic field are analyzed.*

Key Words: *Sandwich Cylindrical Shell, Buckling Pressure, Diaphragms, Magnetorheological Elastomer*

1. INTRODUCTION

Thin multi-layered shells are used in many engineering structures, such as airborn/ spaceborne vehicles, underwater objects and cars [1]. Application of new materials with different physical properties allows one to design sandwich structures fulfilling up-to-date requirements such as good buckling resistance and noiselessness. Buckling as well

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as vibroprotection of thin–walled structures are of great practical interest for mechanical engineers who develop and model similar structures.

It is generally known that the critical value of an external load resulting in buckling of a thin shell depends on a great number of factors such as geometrical characteristics of a structure (thickness, radius, and length), physical properties of a material (Young's and shear module, Poisson's ratio), the boundary conditions, the way of loading (if it is combined) [2, 3]. If a laminated shell is assembled from layers having different properties, this dependence becomes more complex. One of the principal characteristics of the sandwich shell is shear compliance (or the reduced shear rigidity for a whole sandwich). It is influenced by a correlation between thicknesses and mechanical properties of layers composing the sandwich. As shown in [4, 5], taking into account the transverse shears results in an appreciable decrease of both the natural frequencies and the critical axial load of thin laminated shells. A less noticeable effect of transverse shears on the buckling external pressure of a three-layered cylindrical shell has been revealed in paper [6]. In these and other papers, the authors consider as a rule the simplest variant of boundary conditions - the joint support conditions when the edges have infinite rigidity diaphragms inhibiting relative shears of layers along the shell edges. Within the scope of the accepted kinematic hypothesis for laminated shells [4], this type of boundary conditions does not permit us to include the edge effects generated by shears, but gives us a possibility to find the buckling modes and the critical load in the explicit form.

The present paper mainly aims at studying the influence of shears on the critical pressure for a thin sandwich cylinder when its simply supported edges do not have diaphragms. Because a similar variant of the boundary conditions does not allow us to find an exact value of the critical pressure, the asymptotic approach is applied to predict the shell buckling.

The specific goal defined herein is to analyze the influence of a magnetic field on the buckling pressure of a three-layered cylinder with the magnetorheological elastomer (MRE) embedded between the elastic bearing layers. The MREs belong to the group of active materials whose physical properties (viscosity, Young's and shear module) can vary when subjected to different magnetic field levels [7-10]. The application of an external magnetic field is expected to permit us not only to suddenly change the rheological properties of the MRE-based sandwich and suppress its vibrations [11] but to considerably increase the total stiffness of the structure and prevent its buckling.

2. SETTING A PROBLEM

A thin middle length cylindrical sandwich shell consisting of N transversely isotropic elastic layers characterized by length L, thickness h_k, Young's modulus E_k, and Poisson's ratio ν_k is considered, where $k = 1,2,...N$. The middle surface of any fixed layer with radius R is taken as the original surface. Coordinate system α_1, α_2, α_3 is illustrated in Fig. 1, where α_1, α_2, are axial and circumferential coordinates, respectively. The sandwich shell is assumed to be under normal external pressure Q_n. Two variants of the joint support conditions are considered at shell edges $\alpha_1 = 0$, L: (a) there are infinite rigidity diaphragms inhibiting relative shears of layers along the shell edges, (b) diaphragms are absent at the edges.

The problem is to find buckling conservative pressure Q_n^* at different variants of the boundary conditions.

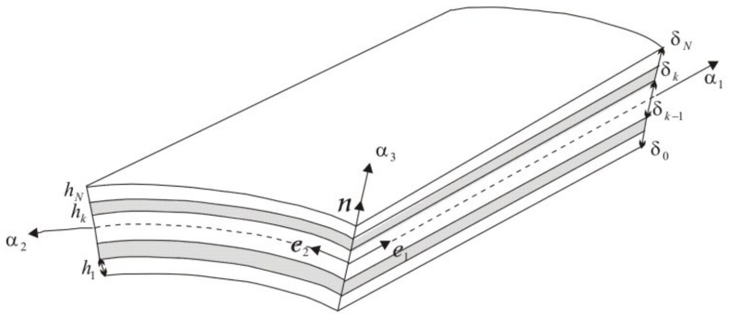

Fig. 1 Thin cylindrical sandwich shell and a curvilinear co-ordinate system

We accept here the unique kinematic hypotheses of Timoshenko for the whole package of the sandwich stated in [4]. From these hypotheses the principal ones relate to the distribution laws of the transverse shear stresses

$$\sigma_i^{(k)}(\alpha_1,\alpha_2,z) = f_0(z)\mu_i^{(0)}(\alpha_1,\alpha_2) + f_k(z)\mu_i^{(k)}(\alpha_1,\alpha_2)\,, \tag{1}$$

and tangential displacements

$$u_i^{(k)}(\alpha_1,\alpha_2,z) = u_i(\alpha_1,\alpha_2) + z\Theta_i(\alpha_1,\alpha_2) + g(z)\psi_i(\alpha_1,\alpha_2) \tag{2}$$

across the thickness of the kth layer, where f_0(z), f_k(z) and $g(z)$ are continuous functions of coordinate $\alpha_3 = z$ introduced as follows

$$f_0(z) = \frac{1}{h^2}(z-\delta_0)(\delta_N - z),\;\; f_k(z) = \frac{1}{h_k^2}(z-\delta_{k-1})(\delta_k - z)\,,\;\; g(z) = \int_0^z f_0(x)dx\,, \tag{3}$$

δ_k is the distance between the original surface and the upper bound of the kth layer, h is the total thickness of the sandwich, $u_i^{(k)}$ are the tangential displacements of points of the kth layer, Θ_i are the angles of rotation of the normal n about the vectors $\mathbf{e}_i$ (see Fig. 1), i = 1, 2; k = 1,2,..., N, and the functions $\mu_i^{(0)}$, $\mu_i^{(k)}$ may be found in paper [4].

The remaining two hypotheses concerning normal deflection w and normal stresses are the same as in the classic Kirchhoff–Love hypothesis. We note that at $g \equiv 0$ hypothesis (2) turns into the Kirchhoff–Love ones.

Based on the foregoing hypotheses, the system of five differential equations with respect to w, u_i, ψ_i have been derived in book [4]. If buckling occurs with formation of large number of hollows although in one direction at the shell surface, these equations may be essentially simplified. Introducing functions ψ_i appearing in (2) by

$$\psi_1 = a_{,1} + \phi_{,2}\,,\;\; \psi_2 = a_{,2} - \phi_{,1} \tag{4}$$

where a and ϕ are the shear functions defined from equations

$$a = -\frac{\eta_2}{\eta_1}\frac{h^2}{\beta}\Delta\chi,\;\; \phi = \frac{1-\nu}{2}\frac{h^2}{\beta}\Delta\phi\,, \tag{5}$$

the following compact system

$$\frac{Eh^3\eta_3}{12(1-\nu^2)}\left(1-\frac{\theta h^2}{\beta}\Delta\right)\Delta^2\chi+\frac{1}{R_2}\frac{\partial\Phi}{\partial\alpha_1^2}-T_2^0\frac{\partial^2 w}{\partial\alpha_2^2}=0\,,$$

$$\Delta^2\Phi-\frac{Eh}{R_2}\frac{\partial w}{\partial\alpha_1}=0\,,\qquad w=\left(1-\frac{h^2}{\beta}\Delta\right)\chi \tag{6}$$

is obtained [4] with respect to the displacement and force functions χ and Φ, respectively. In Eqs. (6), $T_2^0 = RQ_n$ is the hoop membrane stresses, Q_n is the external normal pressure. Other parameters appearing in (5) and (6) are introduced as follows:

$$E=\frac{1-\nu^2}{h}\sum_{k=1}^{3}\frac{E_k h_k}{1-\nu_k^2}\,,\quad \nu=\sum_{k=1}^{3}\nu_k\frac{E_k h_k \nu_k}{1-\nu_k^2}\left(\sum_{k=1}^{3}\frac{E_k h_k}{1-\nu_k^2}\right)^{-1},\quad h\varsigma_k=h_k\,,\quad h\zeta_n=\delta_n\,,$$

$$\eta_1=\sum_{k=1}^{3}\varsigma_k^{-1}\pi_{1k}\gamma_k-3c_{12}^2\,,\quad \eta_2=\sum_{k=1}^{3}\varsigma_k^{-1}\pi_{2k}\gamma_k-3c_{13}c_{12}\,,\quad \eta_3=4\sum_{k=1}^{3}(\varsigma_k^2+3\zeta_{k-1}\zeta_k)\gamma_k-3c_{13}^2\,,$$

$$\theta=1-\frac{\eta_2^2}{\eta_1\eta_3}\,,\quad \gamma_k=\frac{E_k h_k}{1-\nu_k^2}\left(\sum_{k=1}^{3}\frac{E_k h_k}{1-\nu_k^2}\right)^{-1},\quad c_{12}=\sum_{k=1}^{3}\varsigma_k^{-1}\pi_{3k}\gamma_k\,,\quad c_{13}=\sum_{k=1}^{3}(\zeta_{k-1}+\zeta_k)\gamma_k\,,$$

$$q_{44}=\frac{\left[\sum_{k=1}^{3}\left(\lambda_k-\frac{\lambda_{k0}^2}{\lambda_{kk}}\right)\right]^2}{\sum_{k=1}^{3}\left(\lambda_k-\frac{\lambda_{k0}^2}{\lambda_{kk}}\right)G_k^{-1}}+\sum_{k=1}^{3}\frac{\lambda_{k0}^2}{\lambda_{kk}}G_k\,,\quad \beta=\frac{12(1-\nu^2)}{Eh\eta_1}q_{44}\,,\quad G_k=\frac{E_k}{2(1+\nu_k)}\,, \tag{7}$$

$$\lambda_k=\int_{\delta_{k-1}}^{\delta_k}f_0(z)dz\,,\ \lambda_{kn}=\int_{\delta_{k-1}}^{\delta_k}f_k(z)f_n(z)dz\,,\ \frac{h^3\pi_{1k}}{12}=\int_{\delta_{k-1}}^{\delta_k}g^2(z)dz\,,\ \frac{h^3\pi_{2k}}{12}=\int_{\delta_{k-1}}^{\delta_k}zg(z)dz\,.$$

In Eqs. (5) - (7), the magnitudes E, ν are the reduced Young's modules and Poisson's ratio of the sandwich, and parameters θ, β, η_i characterize the reduced shear stiffness of the whole packet. At $\beta^{-1}\to 0$, Eqs. (6) degenerate into the well-known equations of the semi-moment theory of isotropic shells (see, for instance in book [3]).

In terms of the functions χ, Φ, the joint support conditions at the shell edges are written as

$$\chi=\Delta\chi=\Delta^2\chi=0,\quad \frac{\partial\phi}{\partial\alpha_1}=0\quad \text{at}\quad \alpha_1=0, L\,, \tag{8}$$

$$\Phi=\Delta\Phi=0\quad \text{at}\quad \alpha_1=0, L \tag{9}$$

for the case when there are the infinite rigidity diaphragms inhibiting relative shears of layers along the shell edges. If these diaphragms are absent, then the boundary conditions for χ become more complicated:

$$\left(1-\frac{h^2}{\beta}\Delta\right)\chi=0,\ \frac{\partial^2}{\partial\alpha_1^2}\left(1-\frac{h^2}{\beta}\Delta\right)\chi=0\quad \text{at}\quad \alpha_1=0,\ L \tag{10}$$

$$2\frac{\partial^2\chi}{\partial\alpha_1\partial\alpha_2}+\frac{\partial^2\phi}{\partial\alpha_1^2}-\frac{\partial^2\phi}{\partial\alpha_2^2}=0,\ \left(\frac{\partial^2}{\partial\alpha_1^2}+\nu\frac{\partial^2}{\partial\alpha_2^2}\right)\chi-(1-\nu)\frac{\partial^2\phi}{\partial\alpha_1\partial\alpha_2}=0 \text{ at } \alpha_1=0,\ L,\tag{11}$$

while the boundary conditions for force function Φ are the same as in Eqs. (9).

We call the boundary-value problems (5), (6), (8), (9) and (5), (6), (9) - (11) as the BV problems "A" and "B", respectively. The problem is to find the minimum absolute value of hoop stress T_2^0 for which every from these problems has a nontrivial solution. Note that the second Eq. (5) has a solution like the edge effect being a rapidly damped function from the shell edges. When considering the BV problem "A", the shear function ϕ is found from the second Eq. (5) and the last boundary condition (8). To estimate the influence of this function on the critical value of T_2^0, it is necessary to consider the complete system of five differential equations with respect to w, u_i, ψ_i [4]. So, solving the BV problem "A" based on the simplified equations (5), (6), we can assume $\phi = 0$. As concerns the BV problem "B", the boundary conditions (11) show that functions ϕ and χ are linked.

3. Solution of the BV problem "A"

The form of a solution of Eqs. (5), (6) depends on the shell sizes. Here we consider a middle length sandwich cylinder for which $L\sim R$. If the shell edges have diaphragms (the BV problem "A"), then the buckling mode is the same as for an isotropic one-layered simply supported shell (without taking into account transverse shears) [3] and may be found in the explicit form [4-6]

$$(\chi,\ \Phi)=(\chi_0,\ \Phi_0)\ \sin(\pi n\alpha_1/L)\sin(m\alpha_2/R),\tag{12}$$

where n and m are natural numbers, and χ_0, Φ_0 are constants. Substituting (12) into (6) results in the following equation for hoop stresses T_2^0:

$$T_2^0=T_2^0(n,m)=-\varepsilon^8\pi^4 hEm^{-2}\Delta_{nm},\tag{13}$$

$$\Delta_{nm}=\left(\frac{1+\theta K\delta_{nm}}{1+K\delta_{nm}}\right)\delta_{nm}^2+\frac{n^4}{\lambda^4\pi^4\varepsilon^4\delta_{nm}^2},\ K=\frac{\pi^2h^2}{\beta R^2},$$

$$\delta_{nm}=\left(\frac{n^2}{m^2}+\frac{m^2}{\pi^2}\right),\quad \varepsilon^8=\frac{h^2\eta_3}{12(1-\nu^2)R^2},\ \lambda=\frac{L}{R}.$$

The minimization of $\left|T_2^0\right|$ over n and m gives the buckling pressure

$$Q_n^*=T_2^*/R,\ \text{where}\ T_2^*=\min_{n,m}\left|T_2^0(n,m)\right|=\min_m\left|T_2^0(1,\ m)\right|=\left|T_2^0(1,\ m^*)\right|.\tag{14}$$

4. Solution of the BV problem "B"

If the simply supported edges do not have diaphragms, Eqs. (5), (6) do not admit a solution in the form (12). However, the introduction of additional assumptions for parameters appearing in the governing equations permits us to apply the asymptotic approach. Let ε be a small parameter. As shown in [12], for a three layered cylinder with the core made of the MRE, parameters K/π^2, $K\theta/\pi^2$ are also small and their orders depend on the correlation between thicknesses of the layers composed a cylindrical sandwich and its radius. In this study, it is assumed that

$$K/\pi^2 = \varepsilon^4\kappa, \quad K\theta/\pi^2 = \varepsilon^4\tau \text{ at } \varepsilon \to 0, \tag{15}$$

where κ, $\tau \sim 1$. Let us introduce dimensionless coordinates x, φ and load parameter Λ as follows:

$$\alpha_1 = Rx, \ \alpha_2 = R\varphi, \ T_2^0 = -\varepsilon^6 Eh\Lambda . \tag{16}$$

Functions χ, Φ and ϕ are sought in the form:

$$\chi = RX(x)\sin(\varepsilon^{-1}p\varphi), \ \Phi = \varepsilon^4 EhR^2 F(x)\sin(\varepsilon^{-1}p\varphi), \ \phi = RS(x)\sin(\varepsilon^{-1}p\varphi), \tag{17}$$

where p is a wave number, and $X(x)$, $F(x)$, $S(x)$ are unknown functions of x. The substitution of (15) – (17) into Eqs. (5), (6) gives the governing equations written in the dimensionless form:

$$\varepsilon^4(1-\varepsilon^4\tau\Delta_\varepsilon)\Delta_\varepsilon^2 X + \frac{d^2F}{dx^2} - \Lambda p^2(1-\varepsilon^4\kappa\Delta_\varepsilon)X = 0,$$

$$\varepsilon^4\Delta_\varepsilon^2 F - \frac{d^2}{dx^2}(1-\varepsilon^4\kappa\Delta_\varepsilon)X = 0, \tag{18}$$

$$\frac{1-\nu}{2}\kappa\varepsilon^4\Delta_\varepsilon S = S, \tag{19}$$

where $\Delta_\varepsilon = d^2/dx^2 - \varepsilon^{-2}p^2$ is the differential operator. The boundary conditions (9)-(11) at $x = 0$, $l = L/R$ are rewritten as follows:

$$(1-\varepsilon^4\kappa\Delta_\varepsilon)X = 0, \ \frac{d^2}{dx^2}(1-\varepsilon^4\kappa\Delta_\varepsilon)X = 0, \tag{20}$$

$$\left(\varepsilon^2\frac{d^2}{dx^2} - \nu p^2\right)X + \varepsilon(1-\nu)p\frac{dS}{dx} = 0, \quad 2\varepsilon p\frac{dX}{dx} + \varepsilon^2\frac{d^2S}{dx^2} + p^2 S = 0, \tag{21}$$

$$F = 0, \quad \varepsilon^2\frac{d^2F}{dx^2} - p^2 F = 0. \tag{22}$$

As seen, Eqs. (18), (19) are singularly perturbed equations. The solution of the boundary-value problem (18)-(22) may be presented in the form [3]:

$$X = X^{(bs)} + X^{(ed)}, \quad F = F^{(bs)} + F^{(ed)}, \tag{23}$$

where the superscripts (bs) and (ed) denote functions describing the main stress-strain state and the integrals of the edge effects, respectively. It is well known that under buckling of shells of zero Gaussian curvature under external pressure the following asymptotic estimates are valid [3]: $\partial X^{(bs)}/\partial x \sim X^{(bs)}$, $\partial F^{(bs)}/\partial x \sim F^{(bs)}$ at $\varepsilon \to 0$. From Eq.

(19), it may be seen that $\partial S/\partial x \sim \varepsilon^{-2} S$ at $\varepsilon \to 0$. Let $X^{(bs)}, F^{(bs)} \sim 1$. Then the asymptotic analysis of Eqs. (18), (19) permits us to determine the estimates

$$X^{(ed)},\ F^{(ed)} \sim \varepsilon^2,\ S \sim \varepsilon^4,\ \partial X^{(ed)}/\partial x \sim \varepsilon^{-2} X^{(ed)},\ \partial F^{(ed)}/\partial x \sim \varepsilon^{-2} F^{(ed)} \text{ at } \varepsilon \to 0. \quad (24)$$

4.1. Edge effect solutions

We assume that $(X^{(ed)}, F^{(ed)}) = \varepsilon^2 (\hat{X}, \hat{F})$, $S = \varepsilon^4 \hat{S}$, where $\hat{X}, \hat{F}, \hat{S} \sim 1$. When taking into account these correlations as well as above estimates for functions with the superscripts (bs), one obtains the following equations describing the edge effects:

$$\varepsilon^4 \frac{d^2 \hat{X}}{dx^2} - \tau \varepsilon^8 \frac{d^4 \hat{X}}{dx^4} + \hat{F} = 0,$$

$$\varepsilon^4 \frac{d^2 \hat{F}}{dx^2} - \hat{X} + \varepsilon^4 \kappa \frac{d^2 \hat{X}}{dx^2} = 0, \quad (25)$$

$$\varepsilon^4 \frac{d^2 \hat{S}}{dx^2} - \left[\frac{2}{\kappa(1-\nu)} + \varepsilon^2 m^2 \right] \hat{S} = 0. \quad (26)$$

Eqs. (25), (26) have the following solutions

$$\hat{X} = \sum_{i=1}^{3} \left(A_i e^{-\frac{\xi_i}{\varepsilon^2} x} + B_i e^{-\frac{\xi_i}{\varepsilon^2}(l-x)} \right),\ \hat{F} = \sum_{i=1}^{3} \xi_i^2 (\theta \kappa \xi_i^2 - 1) \left(A_i e^{-\frac{\xi_i}{\varepsilon^2} x} + B_i e^{-\frac{\xi_i}{\varepsilon^2}(l-x)} \right), \quad (27)$$

$$\hat{S} = C_1 e^{-\frac{\xi_4}{\varepsilon^2} x} + C_2 e^{-\frac{\xi_4}{\varepsilon^2}(l-x)},\ \xi_4 = \sqrt{\frac{2}{(1-\nu)\kappa} + m^2 \varepsilon^2} \quad (28)$$

decreasing far from the shell edges. Here A_i, B_i, C_1, C_2 are complex constants which will be determined from the boundary conditions below, and ξ_1, ξ_2, ξ_3 are complex roots (having positive real parts) of the algebraic equation

$$\tau \xi^6 - \xi^4 + \kappa \xi^2 - 1 = 0. \quad (29)$$

If κ; $\tau \to 0$, then the system of equations (25) degenerates into a simple edge effect equation, and Eq. (29) has two complex conjugate roots. So, transverse shears may distort the edge effect integrals.

4.2. Main stress state solution

We will construct functions $X^{(bs)}$, $F^{(bs)}$, and load parameter Λ in the form of series:

$$X^{(bs)} = X_0 + \varepsilon^2 X_2 + \ldots \qquad \ldots \quad (30)$$

$$\Lambda = \Lambda_0 + \varepsilon^2 \Lambda_2 + \ldots \quad (31)$$

The substitution of (23), (24), (30), (31) into Eqs. (18) and the boundary conditions (20)-(22) results in the consequence of the boundary-value problems.

In the first-order approximation, one gets the following boundary-value problem

$$\frac{d^4X_0}{dx^4}+(p^8-\Lambda_0 p^6)X_0=0\,,\quad F_0=\frac{1}{p^4}\frac{d^2X_0}{dx^2}\,, \tag{32}$$

$$X_0=F_0=0 \text{ at } x=0,\ l\,. \tag{33}$$

It has the simple solution

$$X_0=A\sin(\pi x/l) \tag{34}$$

if $\Lambda_0(p)=p^2+\delta^4/p^6$, where $\delta=\pi/l$. Minimizing this function, one obtains the zero-order approximation of the load parameter

$$\Lambda_0^0=\min_p \Lambda_0(p)=\Lambda_0(p^0)=4\cdot 3^{-3/4}\delta,\quad p^0=3^{1/8}\delta^{1/2}\,. \tag{35}$$

It should be noted that parameter Λ_0^0 does not take into account transverse shears in the shell. Eqs. (35) are the same as in the classical shell theory [3].

In the second-order approximation, one has the non-homogeneous differential equation

$$\frac{d^4X_2}{dx^4}+[(p^0)^8-\Lambda_0^0(p^0)^6]X_2=AZ(p^0,\ \Lambda_0,\ \Lambda_2) \tag{36}$$

with the non-homogeneous boundary conditions

$$X_2(0)=\beta_1,\ X_2(l)=\beta_2,\ X_2''(0)=\beta_3,\ X_2''(l)=\beta_4\,, \tag{37}$$

where A is an arbitrary constant, the prime denotes differentiation over variable x, and

$$Z=\Lambda_0\kappa(p^0)^8-\theta\kappa(p^0)^{10}-2\delta^2(p^0)^6+2(p^0)^{-2}\delta^6-\kappa(p^0)^2\delta^4+\Lambda_2(p^0)^6\,, \tag{38}$$

$$\beta_1=\sum_{i=1}^{3}(\kappa\xi_i^2-1)A_i,\ \beta_2=\sum_{i=1}^{3}(\kappa\xi_i^2-1)B_i,$$

$$\beta_3=(p^0)^4\sum_{i=1}^{3}\xi_i^2(1-\theta\kappa\xi_i^2)A_i,\ \beta_3=(p^0)^4\sum_{i=1}^{3}\xi_i^2(1-\theta\kappa\xi_i^2)B_i\,. \tag{39}$$

To determine constants A_i, B_i appeared in Eqs. (27), (39), we substitute (23), (27), (34) into the boundary conditions (21) and the second condition (20). Collecting coefficients at ε^2, one gets the following non-homogeneous system of six algebraic equations

$$\sum_{i=1}^{3}\xi_i^2A_i=0,\sum_{i=1}^{3}\xi_i^2B_i=0,\sum_{i=1}^{3}\xi_i^4A_i=0,\sum_{i=1}^{3}\xi_i^4B_i=0,$$

$$\sum_{i=1}^{3}\xi_iA_i=A\delta,\ \sum_{i=1}^{3}\xi_iB_i=A\delta \tag{40}$$

with respect to constants A_i, B_i (i = 1,2,3) It may be seen that A_i, B_i and β_i from (39) are directly proportional to A. Hence, without loss of generality, one can set $A=1$ in the boundary-value problem (36), (37).

For the non-homogeneous boundary-value problem (36), (37) to have a solution the following equation

$$Z(p^0,\ \Lambda_0^0,\ \Lambda_2)\int_0^l X_0^2 dx = \beta_1 X_0'''(0) - \beta_2 X_0'''(l) + \beta_3 X_0'(0) - \beta_4 X_0'(l) \quad (41)$$

is required to hold. From this equation, we obtain the correction for the load parameter

$$\Lambda_2 = \frac{[4\cdot 3^{-1/4} - 3\cdot 3^{1/4}(\kappa-\tau)]\delta^4 - 2\delta^2(\beta_1-\beta_2) + 2(\beta_3+\beta_4)}{3^{3/4}\delta^2}, \quad (42)$$

which takes into account the transverse shears (the parameters β, θ from Eqs. (5), (6)).

It should be noted that function $S = \varepsilon^4 \hat{S}$, with $\hat{S}$ defined by (28), does not affect parameter Λ_2. To find an appropriate correction for Λ introduced by function S it is required to consider the next approximation. However, the order of an expected correction equals h / R and are the same as the order of an error of the shell theory applied here. Thereby, if the shear parameters satisfy estimates (15), then the influence of shear function ϕ on the buckling pressure is negligibly small. So, in the BV problem "B" as well as in the problem "A", we can set $\phi = 0$.

5. Example: Buckling of Three-Layered Cylinder with MRE Core

As an example, we have considered a three-layered cylinder with MRE core of length L = 1m and radius R = 0,5m. We have performed calculations of Q_n^* vs. induction B of the magnetic field for the case when the external supporting surfaces having thicknesses $h_1 = h_3$ = 0,5m are made of the ABS-plastic SD-0170 with parameters $E_1 = E_3 = 15\cdot 10^9$ Pa, $\nu_1 = \nu_3$ = 0,4, and the internal layer of the thickness h_2 = 8mm is the MRE with Poisson's ratio ν_2 = 0,42 and Young's modules E_2 specified by equation [11]

$$E_2(B) = 13{,}230 + 45{,}040\, B \text{ kPa}. \quad (43)$$

This equation are valid at B < 200 mT. It approximates the experiment date obtained for MRE by means of the rheometer *Physica MCR 301* (Anton Paar) at the frequency of the external impact 10 Hz [9].

Now we can analyze the influence of the magnetic field induction on the buckling pressure for two variants of simply supported edges. If the edges have the diaphragms, critical pressure Q_n^* will be calculated by Eqs. (13), (14). If the diaphragms are absent, then the following asymptotic formula

$$Q_n^* = \varepsilon^6 EhR^{-1}[\Lambda_0^0 + \varepsilon^2\Lambda_2 + O(\varepsilon^4)] \quad (44)$$

with the parameters Λ_0^0, Λ_2 defined from (35), (42) will be applied. It should be noted that in our example parameters K, β, θ, κ, τ appearing in Eqs. (13), (29), (42) are functions of the magnetic field induction.

Table 1 demonstrates the dependence of dimensionless parameters Λ_2, Λ, κ and the critical pressure upon various factors: the variant of boundary conditions, transverse shears and the intensity of a magnetic field. It may be seen that assumption $\kappa \sim 1$ (see Eqs. (15)) holds weakly at a very low intensity of a magnetic field (at least for $B \le 20$ mT). But at B > 20 mT, the constructed solutions (17), (23), (30), (31) are asymptotically correct.

As follows from the asymptotic constructions, wave parameter p^0 and zero approximation Λ_0^0 are not influenced by the transverse shears and the magnetic field induction. For geometrical parameters and materials accepted above, one has $p^0 = 1{,}437$, $\Lambda^0_0 = 2{,}756$. The negative values of Λ_2 at $0 \le B \le 60$ mT point out that accounting transverse shears results in decreasing load parameter Λ (see Eq. (31)). But at a very high intensity of the magnetic field the influence of shears on correction Λ_2 decreases. However, in spite of the complex dependence of the load parameter on a magnetic field, the increase of its intensity results in an increase of both the reduced Young's modules for whole sandwich and the critical pressure.

Comparing data in the last two columns, one can conclude: the buckling pressure for the cylindrical sandwich shell having diaphragms on both edges is higher than the critical pressure for the shell without diaphragms. However, it is necessary to bear in mind that the data in the last column are obtained as the result of the exact solution of BV problem "A", whereas the critical pressures for the shell without diaphragms are the asymptotic estimates which are probably slightly understated.

Table 1 Dimensionless parameters Λ_2, Λ, κ and critical pressure Q_n^* vs. magnetic induction B for two variants of the boundary conditions

B, mT	Λ_2	Λ	κ	Critical pressure Q_n^*, Pa	
				Edges without a diaphragm	Edges with a diaphragm
0	−11,598	1,664	4,298	4174	7937
10	−7,462	2,054	3,260	5160	8935
20	−4,961	2,290	2,628	5762	9697
30	−3,286	2,448	2,203	6168	10300
40	−2,088	2,560	1,898	6463	10789
50	−1,188	2,645	1,668	6687	11195
60	−0,487	2,710	1,489	6865	11538
70	0,074	2,763	1,345	7009	11752
80	0,533	2,806	1,227	7129	11906
90	0,915	2,842	1,129	7232	12042
100	1,239	2,872	1,045	7320	12162

6. CONCLUSIONS

Governing equations, based on the assumptions of the generalized kinematic hypothesis of Timoshenko's, are used for the prediction of buckling of a thin composite sandwich cylindrical shell under external normal pressure. Two variants of the joint support conditions have been taken into consideration. If the shell edges have diaphragms preventing shears in the edge plane, the approximate solutions of the governing equations and the estimate for buckling pressure are found by using the asymptotic method; when the diaphragms are absent, the appropriate solution and the buckling pressure are determined in the explicit form.

As an example, the three-layered cylinder with MRE core has been considered. It is observed that the presence of the diaphragms at the shell edges leads to increasing the critical pressure. Other conclusions of this study concern the influence of the applied magnetic field and transverse shears on the shell buckling. When the induction of the magnetic field does not exceed 60 mT, accounting transverse shears results in decreasing dimensionless load parameter Λ. However, for both variants of the boundary conditions, applying the magnetic field with the induction varying in the interval from 0 to 100 mT gives an increase of both the reduced Young's modules and the buckling pressure.

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O UTICAJU GRANIČNIH USLOVA I POPREČNOG SMICANJA NA IZVIJANJE TANKIH LAMINIRANIH CILINDRIČNIH PANELA POD SPOLJNIM PRITISKOM

Predmet ovog istraživanja je izvijanje tankog cilindričnog sendvič panela koji se sastoji od elastičnih izotropskih slojeva sa različitim elastičnim svojstvima pod normalnim spoljnim pritiskom. Kao vodeće jednačine koriste se diferencijalne jednačine zasnovane na pretpostavkama o uopštenoj kinematskoj pretpostavki za ceo sendvič. Dve varijante zajedničkih uslova potpore se razmatraju na ivicama panela: a) da postoje dijafragme beskonačne krutosti u relativnim smicanjima *slojeva duž ivice panela, i b) da su dijafragme odsutne. Uz pomoć asimptotskog pristupa, kritični pritisak i modusi izvijanja se konstruišu u obliku superpozicija funkcija koje odgovaraju glavnom stanju naprezanja/deformacije i integrala ivica. Daje se primer troslojnog cilindra sa magnetoreološkim elastomerom (MRE) umetnutim između elastičnih slojeva pod različitim nivoima magnetnog polja. Fizička svojstva magnetorološkog sloja (MR) se uzimaju kao da su funkcije indukcije magnetnog polja. Zavisnosti pritiska izvijanja na varijantu graničnih uslova se proučavaju kao i intenzitet primenjenog magnetnog polja.*

Ključne reči: *sendvični cilindrični panel, pritisak izvijanja, dijafragme, magnetoreološki elastomer*