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ОБ ОДНОЙ МОДЕЛИ РАЗМЫВА СВЯЗНОГО ГРУНТА И ДВИЖЕНИЯ ПОВЕРХНОСТНЫХ ВОЛН

Х. Милошевич, Ю. Н. Захаров, Н. Контрец, А. И. Зимин, И. С. Нуднер, В. В. Рагулин

MODEL OF COHESIVE SOIL EROSION AND SURFACE WAVE MOTION

H. Miloshevich, Y. N. Zaharov, N. Kontrec, A. I. Zimin, I. S. Nudner, V. V. Ragulin

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В данной работе для моделирования процессов размыва связного грунта и распространения поверхностных волн используется нестационарная неоднородная система уравнений Навье-Стокса с переменной вязкостью, зависящей от плотности. Значение плотности определяется при помощи уравнения конвекции-диффузии. Для решения полученной системы уравнений используется алгоритм, состоящий из схемы расщепления по физическим факторам для системы уравнений Навье-Стокса и метода предиктора-корректора для уравнения переноса. Система решается на разнесенной сетке методом сеток. Представлены результаты двух- и трехмерных расчетов.

Non-stationary inhomogeneous system of Navier–Stokes equations with variable viscosity depending on the density for modeling the processes of cohesive soil erosion and surface wave propagation has been used. Value of the density has been determined by the convection-diffusion equation. For solving the obtained system we have used an algorithm consisting of the splitting scheme on physical factors and the predictor-corrector method. The system has been solved on the staggered grid by the grid method. The results of calculations for two-dimensional and three-dimensional problems are presented.

Ключевые слова: уравнения Навье-Стокса, размыв связного грунта, распространение поверхностных волн, переменная плотность, переменная вязкость, неоднородная жидкость, двухкомпонентная жидкость.

Keywords: Navier-Stokes, cohesive soil erosion, surface wave propagation, variable viscosity, variable density, inhomogeneous fluid, two-component fluid.

Introduction

Many problems of modern hydrodynamics consider more complicated medium and conditions in which they are moving. In particular, they include the problem for inhomogeneous fluid. Inhomogeneity can be caused by inconstancy of density or viscosity due to the dependence of these properties on temperature or by interaction of liquids with different hydrodynamic parameters (multi-component or multiphase medium). The movement of these fluids occurs in many areas of applied fluid dynamics: meteorology, aquatic ecology, oceanography and hydrology (filtration of immiscible liquids, transferring sand and clay sediments).

In this paper, we used the motion model of the two-component viscous incompressible fluid with variable hydrodynamic parameters (viscosity, density) to calculate the problems of cohesive soil erosion and surface wave propagation.

Mathematical model

We consider the motion of the two-component incompressible viscous fluid, its viscosity and density depending on the concentration of the components. Then the model of the fluid is described by the non-stationary Navier-Stokes equations [2].

$$\begin{cases} \dfrac{\partial v_i}{\partial t} + \sum_j v_j \dfrac{\partial v_i}{\partial x_j} = \dfrac{1}{\rho}\left(-\dfrac{\partial p}{\partial x_i} + \dfrac{\partial}{\partial x_i}\left(2\mu \dfrac{\partial v_i}{\partial x_i} \right) + \sum_{j \neq i} \dfrac{\partial}{\partial x_j}\left(\mu \left(\dfrac{\partial v_i}{\partial x_j} + \dfrac{\partial v_j}{\partial x_i} \right) \right) \right) + f_i, \quad i = 1,2,3, \\ \sum_{i=1}^{3} \dfrac{\partial v_i}{\partial x_i} = 0, \end{cases} \tag{1}$$

and by the convection-diffusion equation

$$\frac{\partial C}{\partial t} + \sum_{i=1}^{3} v_i \frac{\partial C}{\partial x_i} = D\Delta C. \tag{2}$$

Here the dependence of the viscosity and density on the concentration is expressed by the following equations:

$$\begin{cases} \mu = C(\mu_2 - \mu_1) + \mu_1, \\ \rho = C(\rho_2 - \rho_1) + \rho_1, \end{cases} \tag{3}$$

where (v_1, v_2, v_3) – vector of velocity projections on the spatial axes (x_1, x_2, x_3), μ – dynamic viscosity, ρ – density, p – pressure, (f_1, f_2, f_3) – vector of mass forces,

C – component concentration, D – diffusion coefficient, $\mu_1, \mu_2, \rho_1, \rho_2$ – viscosities and densities of the first and second components respectively.

Pressure difference as the boundary conditions at the inlet and outlet is a set for motion equations. We use a no-slip condition on the solid wall and boundary conditions of the second kind for the concentration equation. Some initial distribution for concentration is also given.

Solution scheme

To solve the initial boundary problem (1) – (3) we used the following algorithm.

The time step for the Navier-Stokes equations (1) is done in the first stage, based on the known velocity and concentration distribution (and hence the density and viscosity). The scheme of splitting on physical factors [1] is used for this purpose. The time step for the convection-diffusion equation (2) is done in the second stage, using the values obtained for the velocity components. We use a predictor-corrector scheme with approximation of the convective terms against the flow [4] for thispurpose. The values of density and viscosity in the space are recalculated according to (3) in the third stage. Then the transition to the first stage of the next iteration of the algorithm follows.

It is worth noting that the system of equations (1) – (3) is solved numerically by the grid method on the staggered grid [3].

Cohesive soil erosion

In order to simulate the process of wetting substance which is cohesive soil, divide it into two parts (see Fig. 1).

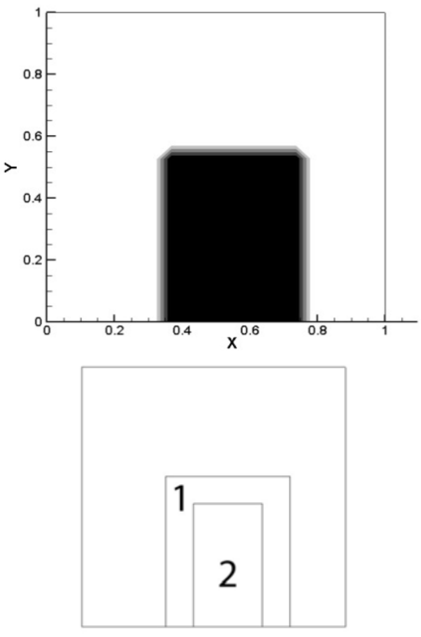

Figure 1. ***The initial position and separation scheme of substance***

Here, number 1 is the part of already soaked substance that behaves like some viscous impurity in the liquid, and number 2 is the part that is considered to be not soaked and behaves like a solid body. Respectively, region 1 is computational, and region 2 is not. The fluid is considered to have penetrated enough to make the abutting portion of region 2 computational when the concentration at some location of the boundary layer becomes smaller a predetermined value C^*. And so on. A similar approach is used for three-dimensional case.

Test calculation illustrating this approach was carried out for two-dimensional problem (see Fig. 2).

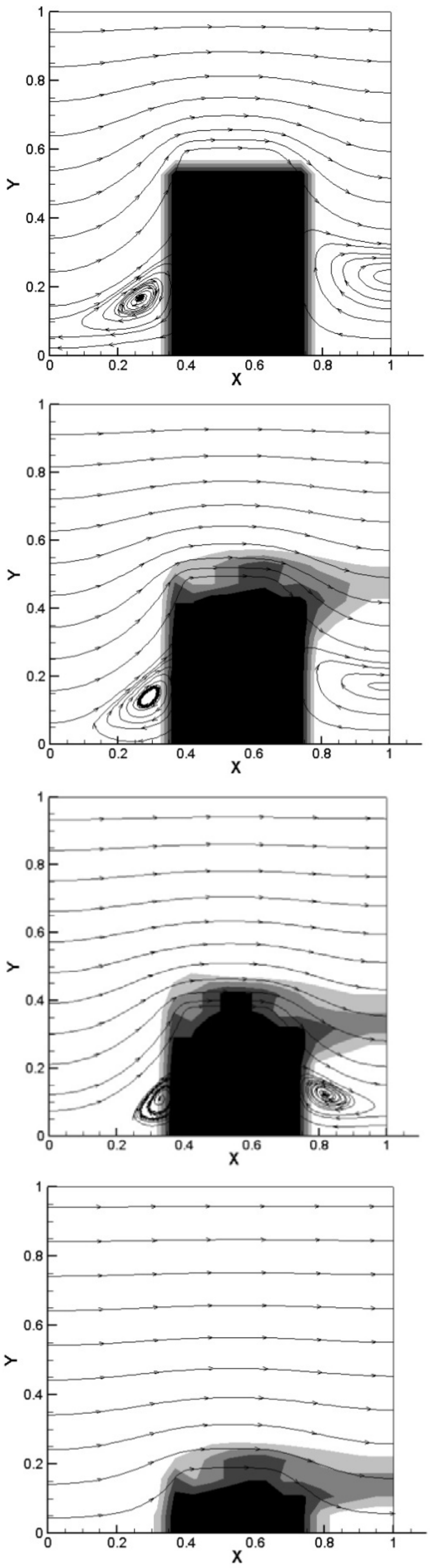

Figure 2. ***Smearing of substance under the conditions of the threshold value*** $C^* = 0.1$ ***for various time points t = 0, 1, 6, 13***

Obviously, the threshold concentration value affects not only dynamics and the process of substance erosion, but also the overall flow picture.

As part of the modeling process of the cohesive soil erosion we consider the following problem. Dense and quite viscous substance essentially differing in its hydrodynamic parameters from the surrounding liquid is located under stationary platform in the given area. Washing out the substances from under the platform occurs during the movement of the liquid. Fig. 3 shows the erosion for three-dimensional case.

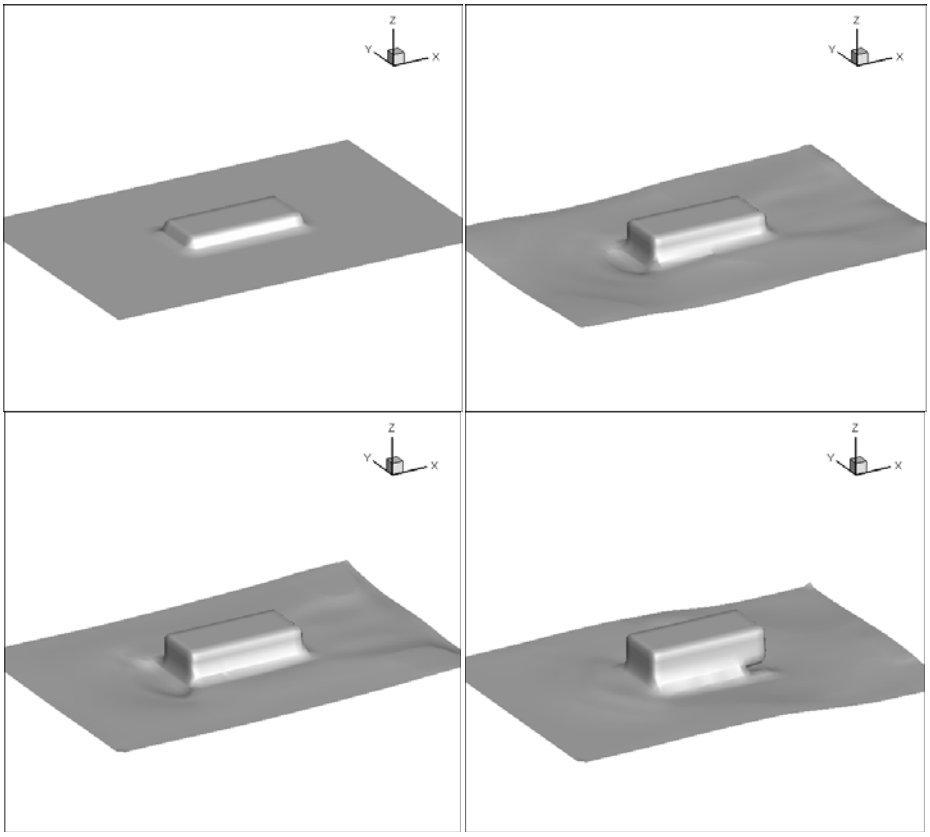

Figure 3. ***Smearing of the substance near solid platform under the conditions of the threshold value*** $C^* = 0.1$ ***for various time points t = 0, 208, 312, 1145***

Wave propagation on surface

Modeling waves on the surface of the viscous fluid is a difficult task. We propose the following method for simulating waves. The area is a two-phase medium described by equations (1) – (3), liquid phase being more dense and viscous than gaseous phase. We consider the boundary of the two components to take place at $C = 0.1$.

We considered the following problems to test the proposed method. The first one is the collapse of the liquid column. The liquid column is in the middle of the area at the initial time. Then column collapses under the influence of the gravity and movement of the entire medium takes place. The following hydrodynamic parameters were chosen here: $\nu_1 = 10^{-3}, \rho_1 = 10$ for liquid and $\nu_2 = 10^{-5}, \rho_2 = 1$ for gas. Fig. 4 and Fig. 5 show the appearance of the wave motion for two- and three-dimensional cases respectively.

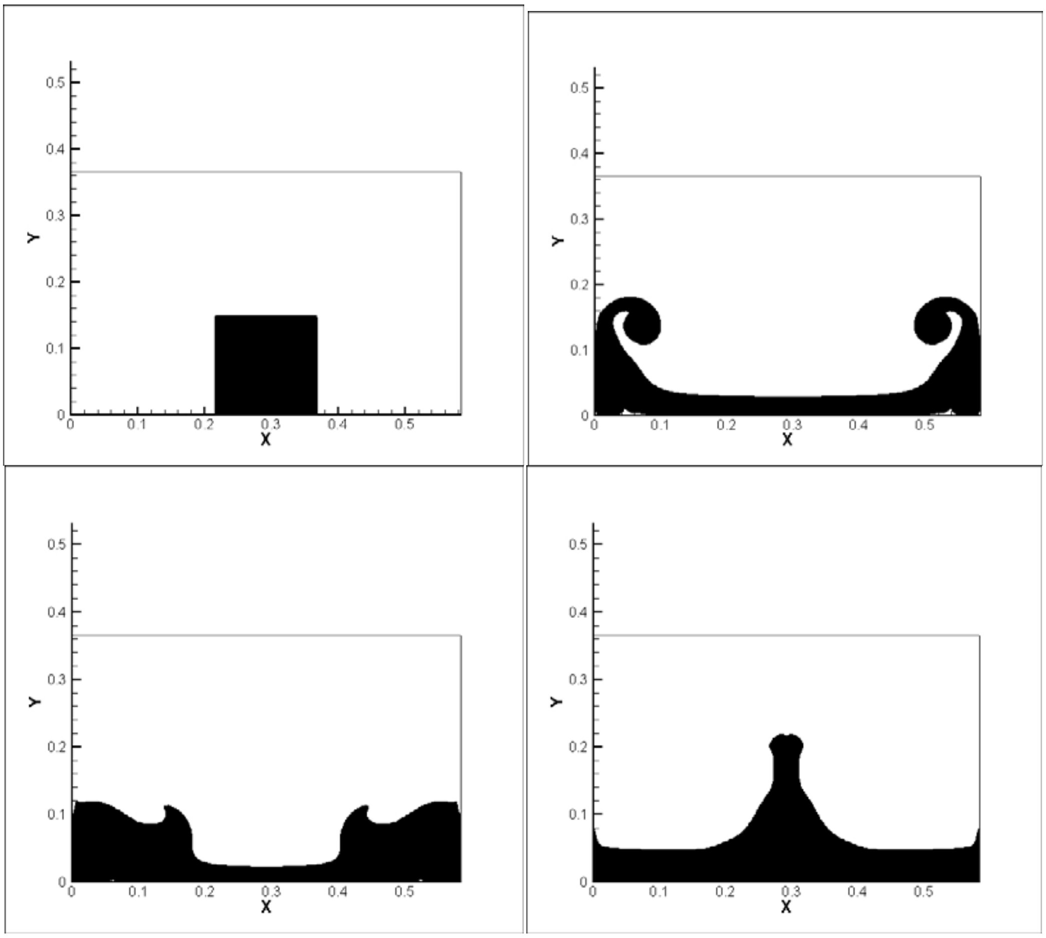

Figure 4. ***Picture of wave motion for various time points t = 0, 0.5, 0.7, 1.0***

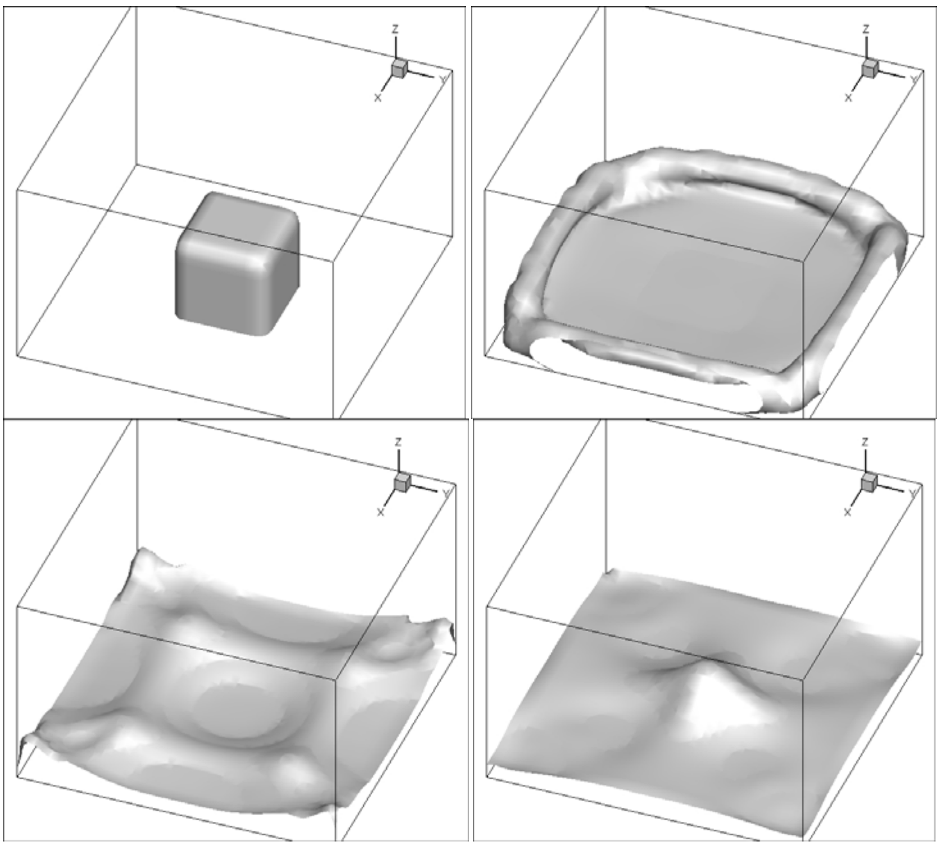

Figure 5. ***Picture of wave motion for various time points t = 0, 0.5, 0.9, 1.3***

The second one is the wave overrunning on the obstacle. Rectangle of the liquid substance is located above the general level in the left side of the area at the initial time. Then the collapse of the rectangle launches a wave in the direction of the obstacle. Here we used the same viscosities and densities as in the first problem. Fig. 6 and Fig. 7 show the wave overrunning on the obstacle for two- and three-dimensional cases respectively.

Thus, this method allows us to simulate wave formation on the heavy liquid surface using the uniform algorithm without isolation features of boundary motion between the components.

Conclusion

The carried out calculations demonstrate the possibility of the two-component incompressible fluid model described by the equations (1) – (3) to simulate some complicated processes such as cohesive soil erosion and surface wave propagation.

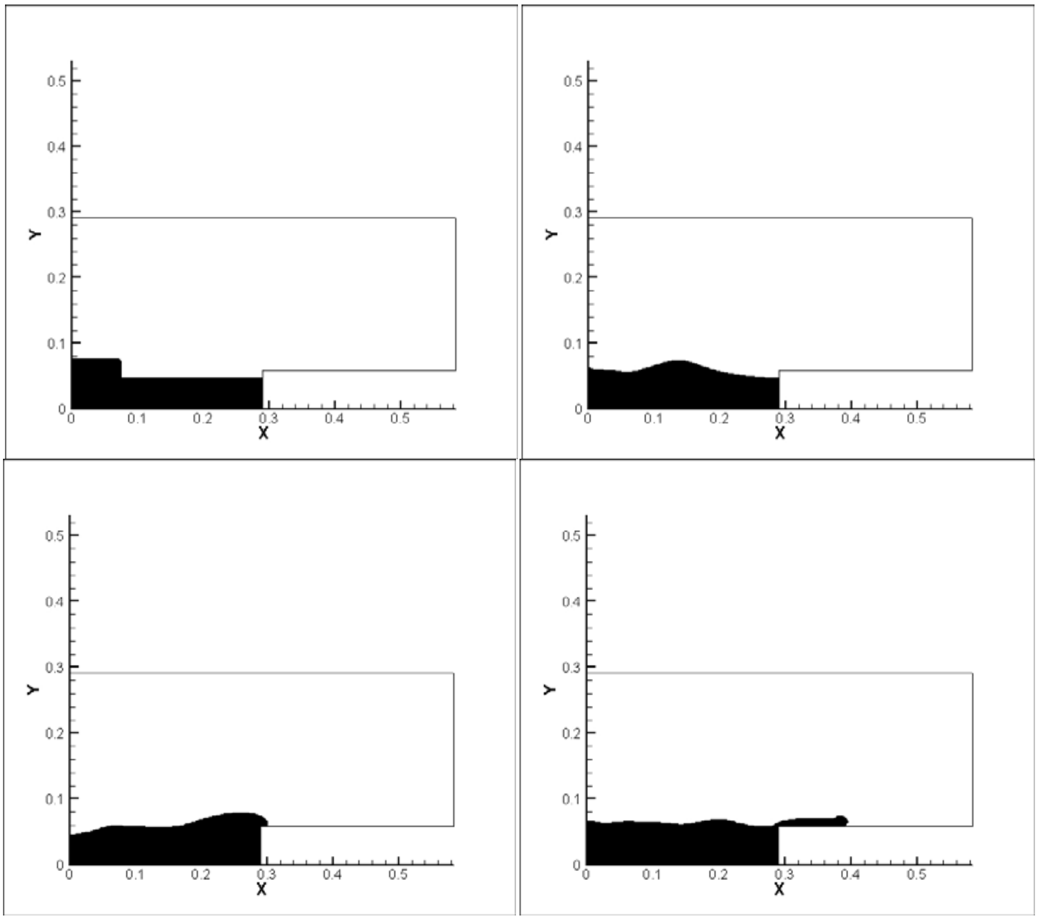

Figure 6. ***Picture of wave motion for various time points t = 0, 0.2, 0.4, 0.7***

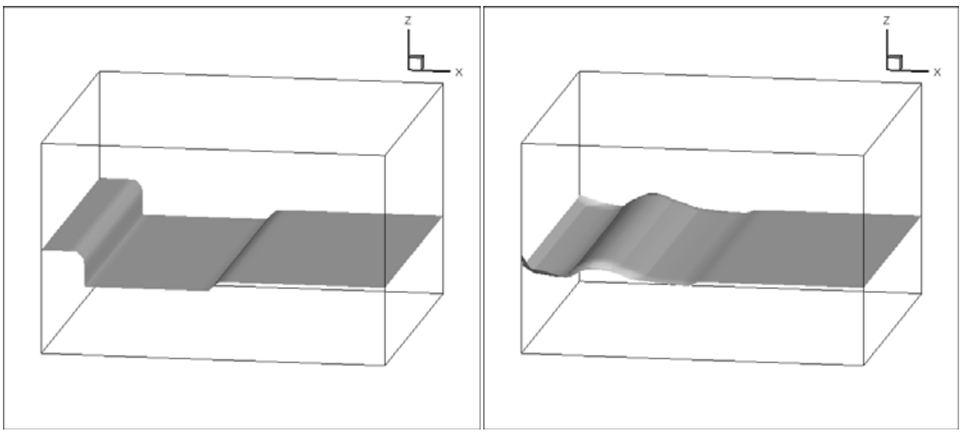

Figure 7. ***Picture of wave motion for various time points t = 0, 0.2, 0.5, 0.8***

Продолжение рис. 7

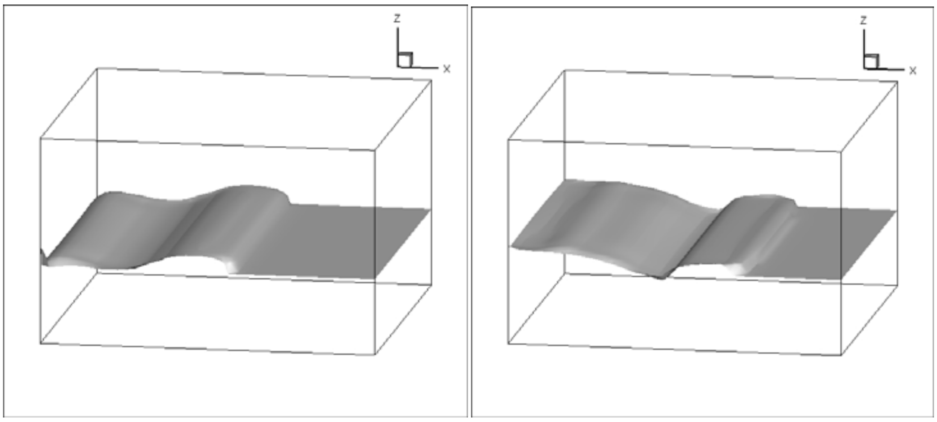

Figure 7. ***Picture of wave motion for various time points t = 0, 0.2, 0.5, 0.8***

References

1. Belotserkovskiy O. M. Chislennoye modelirovaniye v mekhanike sploshnykh sred [Numerical modeling in continuum mechanics]. 1994. 448 p.
2. Loytsyanskiy L. G. Mekhanika zhidkosti i gaza [Fluid and gas mechanics]. 2003. 846 p.
3. Patankar S. Numerical heat transfer and fluid flow. 1980. 197 p.
4. Yanenko N. N. Metod drobnykh shagov resheniya mnogomernykh zadach matematicheskoy fiziki [Method of fractional steps for solving multidimensional problems of mathematical physics]. 1967. 197 p.

Информация об авторах:

Милошевич Хранислав – доктор технических наук, профессор факультета математических наук и информационных технологий Сербского университета в Косовске Митровице, Сербия, mhrane@gmail.com.

Hranislav Milosevic – Doctor of Technical Sciences, Professor at the Faculty of Mathematical Science and Information Technology, Serbian University in Kosovska Mitrovica.

Захаров Юрий Николаевич – доктор физико-математических наук, профессор, заведующий кафедрой вычислительной математики КемГУ, zaxarovyn@rambler.ru

Yury N. Zakharov – Doctor of Physics and Mathematics, Professor at the Department of Computational Mathematics, Kemerovo State University.

Контрец Наташа – ассистент факультета математических наук и информационных технологий сербского университета в Косовске Митровице, Сербия.

Natasa Kontrec – Assistant Lecturer at the Faculty of Mathematical Science and Information Technology, Serbian University in Kosovska Mitrovica.

Зимин Антон Игоревич – аспирант Института вычислительных технологий СО РАН, Новосибирск, sliii@mail.ru.

Anton I. Zimin –post-graduate student at the Institute of Computational Technologies of the Siberian Branch of the RAS, Novosibirsk.

(***Научный руководитель – Ю. Н. Захаров***).

Нуднер Игорь Сергеевич – доктор технических наук, профессор Балтийского государственного технического университета "Военмех" им. Д. Ф. Устинова, Санкт-Петербург.

Igor S. Nudner – Doctor of Technical Sciences, Professor atBaltic State Technical University «Voenmeh» named after D. F. Ustinov, St.-Petersburg.

Рагулин Владимир Васильевич – кандидат физико-математических наук, доцент кафедры дифференциальных уравнений КемГУ.

Vladimir V. Ragulin – Candidate of Physics and Mathematics, Assistant Professor at the Department of Differential Equations, Kemerovo State University.

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