Conductivity of Multi-Component Electron Gas

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Abstract **- The 2D semimetal consisting of heavy holes and light electrons is studied. The consideration is based on the assumption that electrons are quantized by magnetic field while holes remain classical. We assume also that the interaction between components is weak and the conversion between components is absent. The kinetic equation for holes colliding with quantized electrons is utilized. It has been stated that the inter-component friction and corresponding correction to the dissipative conductivity σ_{xx} do not vanish at zero temperature due to degeneracy of the Landau levels. This correction arises when the Fermi level crosses the Landau level. The limits of kinetic equation applicability were found. We also study the situation of kinetic memory when particles repeatedly return to their meeting points.**

Key words: **electron system, magnetic field, limit, kinetic equation, oscillator and distribution function.**

I. INTRODUCTION

Since the discovery of quantum Hall effect, the problem of 2D electron system in strong magnetic field has attracted big attention. It was generally accepted that the most interesting thing is the low-temperature limit when all inelastic processes are frozen out and the system can be treated as the electron-impurity one. Here we concentrate our consideration on the case of semimetal with coexist-ing electrons and holes. Such systems based on 2D layers were obtained and have been intensively studied for the recent years [1, 2]. The specificity of semimetal is the presence of electron-hole scattering. Due to large density of the second com-ponent this process can be comparable with the impurity scattering. Usually, in the Fermi system at T=0 the in-terparticle scattering disappears and the friction between components gives temperature additions T^2 to the transport coefficients [3]. This is not the case in the system with a degenerate ground state, in particular, caused by the Landau quantization. In such a system the scattering redistributes particles within the degenerate state that needs no energy transfer [4].

II. PROBLEM FORMULATION

We consider a 2D semimetal with q_e equivalent electron valleys and q_h equivalent hole valleys centered in points $\mathbf{p}_{e,i}$ and $\mathbf{p}_{h,i}$, correspondingly. The conduction bands with energy spectra $(\mathbf{p} - \mathbf{p}_{e,i})^2/2m_e$ overlaps with the valence bands $E_q - \varepsilon_{\mathbf{p}-\mathbf{p}h,i}$, $\varepsilon_{\mathbf{p}} = p^2/2m_h$ ($E_q > 0$). The hole mass m_h is assumed to be much larger than the electron mass m_e. The distances between electron and hole extrema $|\mathbf{p}_{h,i} - \mathbf{p}_{e,j}|$ are supposed to be large to suppress the electron-hole conversion. At the same time, the scattering between electrons and holes changing the momenta near extrema are permitted. Without the loss of generality, further we shall count the momenta from the band extrema and replace $\mathbf{p} - \mathbf{p}_{h,j} \to \mathbf{p}$, $\mathbf{p} - \mathbf{p}_{e,i} \to \mathbf{p}$.

The system is placed in a moderately strong magnetic field, such that the electrons are quantized, while holes stay classical. In other words, the number of filled hole Landau levels $N_h + 1$ is large, while the analogical electron number $N_e + 1$ has the order of unity. We shall consider the low-temperature limit when the electron transitions occur within the same Landau level and transitions between different electron Landau levels are forbidden. The energy conservation permits this process only when the Landau level is partially filled, i.e. in a state of compressible Landau liquid [5].

We shall neglect the rearrangement of the energy spectrum caused by the interaction between electrons and electrons with holes. The Landau levels widening will be also neglected. The interaction of quantized gas with a classical one is an unusual situation. The kinetics of holes can be described by a classical kinetic equation, while electrons need a quantum description. In accordance with above-mentioned, we shall use the states in crossed electric and magnetic fields $\psi_{n,k}(r) = e^{iky}\varphi_n((x - X_k)/a) / \sqrt{aL_y}$, where φ_n are normalized oscillator functions, L_y is the normalizing length of the system in y-direction, $X_k = X_k^{(0)} - v_d/\omega_e$, $X_k^{(0)} = -a^2k$ is the coordinate of the center of cyclotron motion, $v_d = c[\mathbf{E}, \mathbf{H}]/H^2$ is the drift velocity, $a = c/eH$ is the magnetic length, $\omega_e = eH/m_ec$ is the electron cyclotron frequency, $-e$ is the electron charge; we set $\hbar = 1$ and will restore the dimensionality in the final expressions. The corresponding energy is presented by $\varepsilon_{n,k} = \omega_e(n + 1/2) + eEX_k$.

III. DISSIPATIVE CONDUCTIVITY

The transmission of the momentum between electrons and holes is determined by the scattering processes. The collision term in the kinetic equation for holes in the Born approximation reads [6]

$$\hat{I}_{he}\{f_{\mathbf{p}}\} = \frac{2\pi}{S^2} 2q_e \sum_{\mathbf{p}',\mathbf{q},\gamma,\gamma'} |u_{\mathbf{q}}|^2 \delta_{\mathbf{p}',\mathbf{p}+\mathbf{q}} |J_{\gamma',\gamma}(\mathbf{q})|^2 \delta(\varepsilon_{\mathbf{p}'} - \varepsilon_{\mathbf{p}} + \varepsilon_{\gamma'} - \varepsilon_{\gamma}) \times$$
$$\times \left[f_{\mathbf{p}}(1 - f_{\mathbf{p}'})\varphi_{\gamma}(1 - \varphi_{\gamma'}) - f_{\mathbf{p}'}(1 - f_{\mathbf{p}})\varphi_{\gamma'}(1 - \varphi_{\gamma}) \right]. \tag{1}$$

Here $u_{\mathbf{q}}$ is the Fourier transform of potential of interaction between electron and hole, S is the system area

$$\begin{cases} J_{\gamma',\gamma}(\mathbf{q}) = \langle \gamma' | e^{iqr} | \gamma \rangle; \\ \gamma = (n, k); \gamma' = (n', k'); \\ \varepsilon_p = p^2/2m_h, \end{cases} \tag{2}$$

Manuscript received March 3, 2015.
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is the hole energy (m_h is the hole effective mass). Due to the uniformity of the space the quantity φ_γ does not depend on the wave vector and coincides with the equilibrium distribution function. At zero temperature all factors $\varphi_{n'}(1-\varphi_n)\equiv 0$, excluding the contribution with n = n⁻ = N_e, where N_e is the number of the last partially filled Landau level. The quantities N_e and $\varphi N_e \equiv \nu$ can be expressed via the electron density n_e as $N_e = [n_e \pi a^2/q_e]$, $\nu = \{n_e \pi a^2/q_e\}$ (square and figure brackets mean the integer and fractional parts). We shall expand the collision term with respect to weak non-equilibrium, assuming that the electric field and the deviation of distribution function from equilibrium are small [7].

$$\hat{I}_{he}\{f_{\mathbf{p}}\} = \frac{4\pi q_e}{S^2}\sum_{\mathbf{p}',n,k} R_n(|\mathbf{p}-\mathbf{p}'|)\varphi_n^{(0)}\left(1-\varphi_n^{(0)}\right)\times$$
$$\times\left[\delta(\varepsilon_{\mathbf{p}}-\varepsilon_{\mathbf{p}'})(\delta f_{\mathbf{p}}-\delta f_{\mathbf{p}'})+\delta'(\varepsilon_{\mathbf{p}}-\varepsilon_{\mathbf{p}'})\times\right. \tag{3}$$
$$\left.\times eEa^2(p'_y-p_y)\left(f_{\mathbf{p}}^{(0)}-f_{\mathbf{p}'}^{(0)}\right)\right].$$

Here $\delta f_{\mathbf{p}}$ is linear in E correction to the hole distribution function, $f_{\mathbf{p}}^{(0)}, \varphi_n^{(0)}$ are equilibrium distribution functions of holes and electrons,

$$R_n(q) = |u_q|^2 L_n^2(q^2a^2/2)e^{-q^2a^2/2}, \tag{4}$$

L_n are the Laguerre polynomials. The function $R_n(q)$ has a characteristic size in q-space 1/s. The parameter S is determined by the largest of sizes of potential L and wave functions of electrons a 2(n + 1). In the coordinate space, S corresponds to the typical impact parameter for scattering. Let us consider the hole transport. The kinetic equation for the nonequlibrium correction to the distribution function $\delta f_{\mathbf{p}}$ reads

$$\mathbf{eE}\frac{\partial f_{\mathbf{p}}^{(0)}}{\partial \mathbf{p}} + \omega_h[\mathbf{p},\mathbf{h}]\frac{\partial \delta f_{\mathbf{p}}}{\partial \mathbf{p}} = \hat{I}_{he}\{f_{\mathbf{p}}\}, \tag{5}$$

were $\omega_h = eH/m_h c, \mathbf{h} = \mathbf{H}/H$.

IV. Finite Landau level width

The crucial point for the previous consideration, in particular, for temperature-independent contribution of e-h scattering to the dissipative conductivity is the presence of the Landau levels degeneracy. There are different sources of the Landau levels broadening. One source is the scattering of electrons on holes. The rate of this scattering γ_{eh} can be calculated summing the probability of scattering $W_{\gamma'}$ over the finite states. The result is [8]

$$\gamma_{eh} = \frac{m_h^2 q_h T}{\pi^2}\int_0^{2pF,h} dq \frac{R_N^2(q)}{\sqrt{4p_{F,h}^2 - q^2}}. \tag{6}$$

Like D_e the width γ_{eh} vanishes with the temperature, while the hole damping γ_{eh} vanishes with the temperature, while the hole damping γ_{he} stays finite at $T\to 0$.

The smallness of the Landau level width as compared with the temperature is the sufficient condition for the neglect of width. Estimations give $\gamma_{eh}/T \sim m_h e^4/(\hbar^2\chi E_{F,h})$. The latter parameter is small if the holes are weakly interacting; this demand is supposed in the present study. In particular, the condition $m_h e^4/(\hbar^2\chi^2 E_{F,h}) << 1$ does not permit to consider the limit $m_h \to \infty$ when the holes become equivalent to immobile impurities. On the other hand, γ_{eh} may be omitted at a low enough T as compared with temperature independent Landau level width γ_i caused by the potential fluctuations. For a long-range potential γ_i is proportional to the amplitude of potential. For developed fluctuations the self consistent Born approximation gives the width of the Landau levels proportional to $\sqrt{n_i}$ (n_i is the impurity concentration) and the potential of individual impurity. The exception is the short-range impurities with $\delta-$like potential for which the part of the Landau level states $1/\pi a^2 - n_i$ remains degenerate if $1/\pi a^2 > n_i$, while n_i states form a band of localized states with a finite width. In the case of the Landau level with a finite width γ_i, the interparticle scattering depends on the radio of the temperature T to the width. If $T << \gamma_i$, the e-h scattering is suppressed and if $T >> \gamma_i$, the scattering does not notice the width. Thus, all the previous consideration of e-h scattering remains valid for intermediate temperature $\hbar\omega > T$. In the dissipative conductivity the scattering processes affect quantized electrons in a parallel manner. One can sum up the contributions to electron σ_{xx} conductivity caused by impurity scattering and electron-hole processes. According to [9], the contribution to electron dissipative current caused by short-range impurity scattering is

$$(\sigma_{xx})_{ei} = \frac{q_e e^2}{\pi^2\hbar}(N_e + 1/2)(1-\mu^2). \tag{7}$$

The reduced distance between the Fermi level and the Landau level with number N_e, $\mu = \left(\varepsilon_{F,e} - (N_e + 1/2)\,\omega_e\right)/\gamma N$, is connected with the quantity ν by the equation

$$\nu = \frac{1}{2\pi}\left(\pi + 2\mu\sqrt{1-\mu^2} + 2\arcsin\mu\right). \tag{8}$$

In the case of short-range impurities with small concentration $n_i << 1/\pi\alpha^2$, the absence of widening leads to the validity of the results obtained in the previous section up to zero temperature. In a wider range $n_i < 1/\pi\alpha^2$ the scattering rate $1/\tau_{he}$ should be corrected by the factor $1 - n_i\pi\alpha^2$ reflecting the fraction of degenerate states. If the long-range potential fluctuation case is realized the degeneracy disappears. In the absence of e-h scattering, the model of adiabatic transport is valid when electron cyclotron centers are drifting along the lines of constant potential. Without the external field only one infinite fractal level line of the fluctuating potential corresponding to the percolation threshold exists. In the presence of the finite electric field, this level line decays to independent infinite entangled lines going across the external electric field. The drift does not depend on the charge of particles: the velocities and trajectories of the cyclotron centers of quantized electrons and classical holes are same [10]. The dissipative conductivity of electrons vanishes, while the Hall conductivity changes stepwise between quantized Ne^2/h values. In the lack of degeneracy, the temperature independent e-h scattering also disappears.

V. Conclusion

We have studied the influence of electron-hole interaction on transport in the system where electrons are quantized and holes are not. In these conditions, the second type of carriers plays its role as an additional (or exceptional) channel of scattering. Weak electron-hole interaction can be considered in the Born approximation, despite the degeneracy of the Landau levels, in contrast to the impurity mechanism which is not perturbative in the quantizing magnetic field, even for a weak potential. The scattering of holes on quantized non-interacting electrons occurs if, and only if, the Landau level is partially filled. The chaotization results from the random distribution of electrons in the momentum space, and corresponding entropy at zero temperature remains finite. The scattering of holes can be considered by means of kinetic equation approxima-tion when the Fermi level is near the center of the Lan-dau levels; the kinetic approximation loses applicability apart from the center; on the far wings the holes become localized.

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