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# DETERMINAREA MULŢIMII NASH ÎN JOCUL DIADIC ÎN STRATEGII MIXTE

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În acest articol este cercetată determinarea mulţimii de echilibre Nash pentru jocul diadic în strategii mixte, fiind studiate proprietăţile şi structura mulţimii Nash şi elaborat un algoritm de calcul simbolic în baza metodei de intersecţie a graficelor aplicaţiilor de răspuns optim [2,3]. Scopul principal al lucrării este de a simplifica esenţial algoritmul în baza căruia este alcătuit programul de calcul în sistemul Wolfram Mathematica 9, care vine ca o perfecţionare a celui publicat în 2007 pe Wolfram Demonstration Project [3]. Rezultatul principal ţine de reducerea esenţială a numărului de cazuri cercetate la doar 38.
***Cuvinte-cheie:*** *joc strategic în strategii mixte, joc diadic, echilibru Nash, mulţime de echilibre Nash, grafic al aplicaţiei de răspuns optim, intersecţia graficelor aplicaţiilor de răspuns optim, algoritm.*

**NASH EQUILIBRIUM IN THE DYADIC PLAY IN MIXT STRATEGIES**
In this study it's investigated the determination of Nash equilibrium for the dyadic play with mixt strategies through studying the properties and structure of Nash equilibrium and elaboration of an algorithm of symbolic computation based on the method of intersection of application graphics of optimal answer[2, 3]. The basic purpose of this study is to simplify the essential algorithm in which base is made the program of calculation in Wolfram Mathematic 9 system, which comes as an improvement of that which was published in 2007 on Wolfram Demonstration Project [3]. The principal result keeps of the essential reduction of investigated cases to 38.
***Keywords:*** *strategic play in mixt strategies, dyatic play, Nash equilibrium, lot of Nash equilibriums, application graphics of optimal answer, intersection of application graphics of optimal answer.*

## Introducere

Jocul este definit de către John von Neumann şi Oscar Morgenstern ca „orice interacţiune între diverşi agenţi, guvernată de un set de reguli specifice care stabilesc mutările posibile ale fiecărui participant şi câştigurile pentru fiecare combinaţie de mutări" [1]. Această descriere poate fi aplicată aproape oricărui fenomen în care interacţionează diverşi subiecţi cu interese diferite.

Să examinăm jocul strategic de doi jucători în forma normală:
$$G = \langle N, X, Y, f_i(\boldsymbol{x}, \boldsymbol{y}), i = \overline{1,2}, (\boldsymbol{x}, \boldsymbol{y}) \epsilon X \times Y \rangle;$$
unde:
- ✓ N – mulţimea de jucători, $N = \{1,2\}$;
- ✓ X – mulţimea de strategii ale primului jucător,
  $X = \{\boldsymbol{x} = (x_1, x_2): x_1 + x_2 = 1, x_i \geq 0, i = \overline{1,2}\}$;
- ✓ $Y$ – mulţimea de strategii al jucătorului al doilea,
  $Y = \{\boldsymbol{y} = (y_1, y_2): y_1 + y_2 = 1, y_i \geq 0, i = \overline{1,2}\}$;
- ✓ $f_i(x_1, x_2)$ – funcţiile de câştig ale jucătorilor.

**Definiţie.** *Vectorul* $(\boldsymbol{x}^*, \boldsymbol{y}^*)$ *se numeşte* ***Echilibru Nash (EN)*** *dacă au loc condiţiile:*
$$f_1(\boldsymbol{x}^*, \boldsymbol{y}^*) \geq f_1(\boldsymbol{x}, \boldsymbol{y}^*), \forall \boldsymbol{x} \in X,$$
$$f_2(\boldsymbol{x}^*, \boldsymbol{y}^*) \geq f_2(\boldsymbol{x}^*, \boldsymbol{y}), \forall \boldsymbol{y} \in Y.$$

Ţinând cont de specificul mulţimilor de strategii, prin transformări evidente se trece de la patru variabile la doar două:
$$x_1 = x,\ x_2 = 1 - x,\ x \in [0,1];$$
$$y_1 = y,\ y_2 = 1 - y,\ y \in [0,1].$$

Funcţiile de câştig sunt biliniare (pentru strategia fixă a adversarului sunt liniare) şi iau forma:
$$f_1(x, y) = x^T A y,\ f_2(x, y) = y^T B x,$$
unde $x, y \in [0,1] \times [0,1]$. $A, B \in R^2 \times R^2$ sunt matricele de cost ale jucătorilor:
$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}; \quad B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}.$$

Funcţiile de câştig pot fi scrise şi în următoarea formă:
$$f_1(x, y) = [\alpha y + \alpha_0]x + (a_{21} - a_{22})y + a_{22};$$
$$f_2(x, y) = [\beta x + \beta_0]y + (b_{12} - b_{22})x + b_{22},$$
unde
$$\alpha = a_{11} - a_{21} + a_{22} - a_{12};\ \alpha_0 = a_{12} - a_{22} \text{ şi } \beta = b_{11} - b_{12} + b_{22} - b_{21};\ \beta_0 = b_{21} - b_{22}.$$

### Graficele aplicaţiilor de răspuns optim

Pentru a determina mulţimea de echilibre Nash, se construiesc graficele aplicaţiilor de tip cel mai bun răspuns:

$$Gr_1 = \begin{cases} 1, \text{ dacă } \alpha y + \alpha_0 > 0 \\ 0, \text{ dacă } \alpha y + \alpha_0 < 0 \\ (0,1), \text{ dacă } \alpha y + \alpha_0 = 0 \end{cases} \quad \text{şi} \quad Gr_2 = \begin{cases} 1, \text{ dacă } \beta x + \beta_0 > 0 \\ 0, \text{ dacă } \beta x + \beta_0 < 0 \\ (0,1), \text{ dacă } \beta x + \beta_0 = 0 \end{cases}.$$

La construirea graficelor în funcţie de valorile elementelor matricelor de câştig pot fi evidenţiate următoarele nouă cazuri [3]:

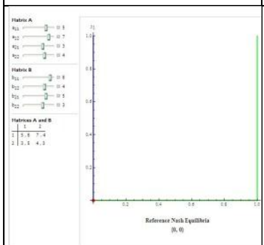
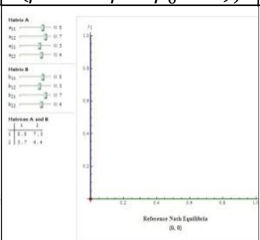
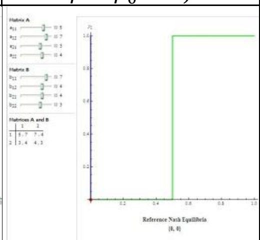
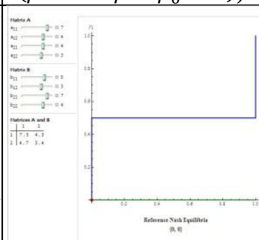
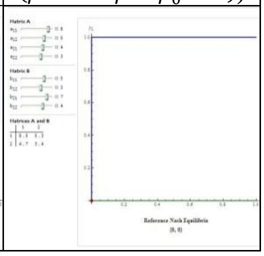
| Nr. | Primul jucător | | Al doilea jucător | |
|---|---|---|---|---|
| crt. | Condiţia | Graficul | Condiţia | Graficul |
| 1. | $\alpha = 0 \,\&\, \alpha_0 = 0$ | Dreptunghiul $(0,0);\ (1,1);$ | $\beta = 0 \,\&\, \beta_0 = 0$ | Dreptunghiul $(0,0);\ (1,1);$ |
| 2. | $(\alpha \geq 0 \,\&\, \alpha_0 > 0)$ sau $(\alpha < 0 \,\&\, \alpha + \alpha_0 > 0)$ | Segmentul $(1,0);\ (1,1);$ | $(\beta \geq 0 \,\&\, \beta_0 > 0)$ sau $(\beta < 0 \,\&\, \beta + \beta_0 > 0)$ | Segmentul $(0,1);\ (1,1);$ |
| 3. | $(\alpha \leq 0 \,\&\, \alpha_0 < 0)$ sau $(\alpha > 0 \,\&\, \alpha + \alpha_0 < 0)$ | Segmentul $(0,0);\ (0,1);$ | $(\beta \leq 0 \,\&\, \beta_0 < 0)$ sau $(\beta > 0 \,\&\, \beta + \beta_0 < 0)$ | Segmentul $(0,0);\ (1,0);$ |
| 4. | $\alpha > 0 \,\&\, \alpha_0 = 0$ | Linia frântă $(0,0);\ (1,0);(1,1);$ | $\beta > 0 \,\&\, \beta_0 = 0$ | Linia frântă $(0,0);\ (0,1);(1,1);$ |
| 5. | $\alpha > 0 \,\&\, \alpha_0 < 0 \,\&$ $\alpha + \alpha_0 > 0$ | Linia frântă $(0,0);\ \left(0,-\frac{\alpha_0}{\alpha}\right);$ $\left(1,-\frac{\alpha_0}{\alpha}\right);\ (1,1);$ | $\beta > 0 \,\&\, \beta_0 < 0 \,\&$ $\beta + \beta_0 > 0$ | Linia frântă $(0,0);\ \left(-\frac{\beta_0}{\beta},0\right);\left(-\frac{\beta_0}{\beta},1\right);\ (1,1);$ |
| 6. | $\alpha > 0 \,\&\, \alpha + \alpha_0 = 0$ | Linia frântă $(0,0);\ (0,1);(1,1);$ | $\beta > 0 \,\&\, \beta + \beta_0 = 0$ | Linia frântă $(0,0);\ (1,0);(1,1);$ |
| 7. | $\alpha < 0 \,\&\, \alpha_0 = 0$ | Linia frântă $(1,0);\ (0,0);(0,1);$ | $\beta < 0 \,\&\, \beta_0 = 0$ | Linia frântă $(0,1);\ (0,0);(1,0);$ |
| 8. | $\alpha < 0 \,\&\, \alpha_0 > 0 \,\&$ $\alpha + \alpha_0 < 0$ | Linia frântă $(1,0);\ \left(1,-\frac{\alpha_0}{\alpha}\right);$ $\left(0,-\frac{\alpha_0}{\alpha}\right);\ (0,1);$ | $\beta < 0 \,\&\, \beta_0 > 0 \,\&$ $\beta + \beta_0 < 0$ | Linia frântă $(0,1);\ \left(-\frac{\beta_0}{\beta},1\right);\left(-\frac{\beta_0}{\beta},0\right);\ (1,0);$ |
| 9. | $\alpha < 0 \,\&\, \alpha + \alpha_0 = 0$ | Linia frântă $(1,0);\ (1,1);(0,1);$ | $\beta < 0 \,\&\, \beta + \beta_0 = 0$ | Linia frântă $(0,1);\ (1,1);(1,0);$ |

### Mulţimea de echilibre Nash

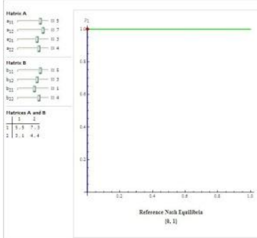
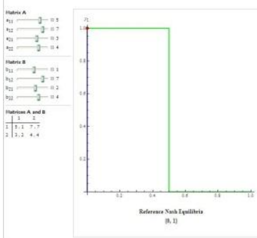
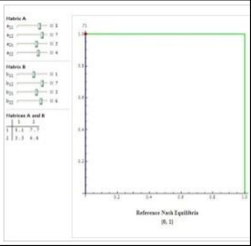
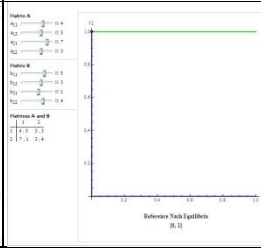
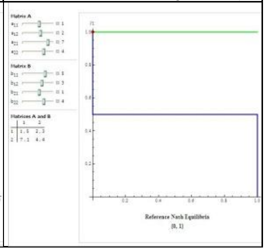
Deoarece $NE = Gr_1 \cap Gr_2$ [2 - 4], în general trebuie să cerceteze $9 \times 9 = 81$ cazuri. În continuare reducem numărul lor la doar 38, ceea ce înseamnă că pentru mulţimea Nash evidenţiem doar 38 de cazuri (forme) posibile.

### Un punct

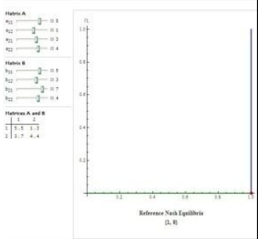
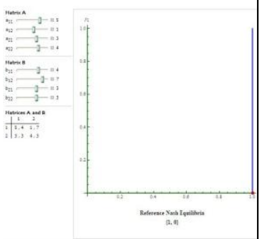
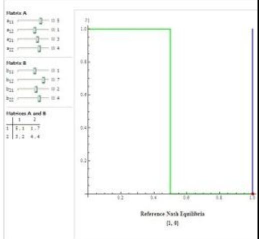
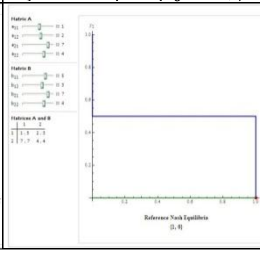
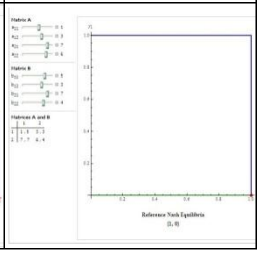
1. $NE = \{punctul\ (0,0)\}$:

| 1. | 2. | 3. | 4. | 5. |
|---|---|---|---|---|
| $((\alpha \leq 0 \,\&\, \alpha_0 < 0)$ sau $(\alpha > 0 \,\&\, \alpha + \alpha_0 < 0)) \cap$ $(\beta > 0 \,\&\, \beta + \beta_0 = 0)$ | $((\alpha \leq 0 \,\&\, \alpha_0 < 0)$ sau $(\alpha > 0 \,\&\, \alpha + \alpha_0 < 0)) \cap$ $((\beta \leq 0 \,\&\, \beta_0 < 0)$ sau $(\beta > 0 \,\&\beta + \beta_0 < 0))$ | $((\alpha \leq 0 \,\&\, \alpha_0 < 0)$ sau $(\alpha > 0 \,\&\, \alpha + \alpha_0 < 0)) \cap$ $(\beta > 0 \,\&\, \beta_0 < 0 \,\&$ $\beta + \beta_0 > 0)$ | $(\alpha > 0 \,\&\, \alpha_0 < 0 \,\&$ $\alpha + \alpha_0 > 0) \cap$ $((\beta \leq 0 \,\&\, \beta_0 < 0)$ sau $(\beta > 0 \,\&\, \beta + \beta_0 < 0))$ | $(\alpha > 0 \,\&\, \alpha +$ $\alpha$_0 $= 0) \cap$ $((\beta \leq 0 \,\&\, \beta_0 < 0)$ sau $(\beta > 0 \,\&\, \beta + \beta_0 < 0))$ |
|  |  |  |  |  |

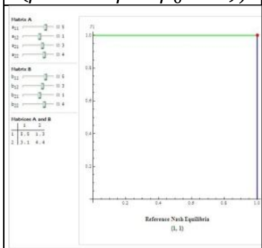
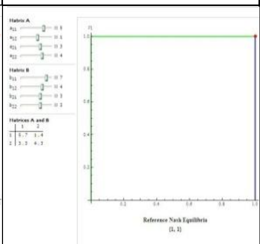
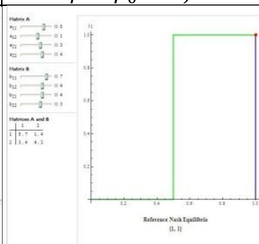
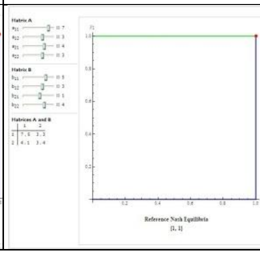
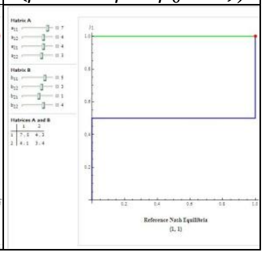
2. $NE = \{punctul\ (0,1)\}$:

| 1. | 2. | 3. | 4. | 5. |
|---|---|---|---|---|
| $((\alpha \le 0 \,\&\, \alpha_0 < 0)$ sau $(\alpha > 0 \,\&\, \alpha + \alpha_0 < 0)) \cap ((\beta \ge 0 \,\&\, \beta_0 > 0)$ sau $(\beta < 0 \,\&\, \beta + \beta_0 > 0))$ | $((\alpha \le 0 \,\&\, \alpha_0 < 0)$ sau $(\alpha > 0 \,\&\, \alpha + \alpha_0 < 0)) \cap (\beta < 0 \,\&\, \beta_0 > 0 \,\&\, \beta + \beta_0 < 0)$ | $((\alpha \le 0 \,\&\, \alpha_0 < 0)$ sau $(\alpha > 0 \,\&\, \alpha + \alpha_0 < 0)) \cap (\beta < 0 \,\&\, \beta + \beta_0 = 0)$ | $(\alpha < 0 \,\&\, \alpha_0 = 0) \cap ((\beta \ge 0 \,\&\, \beta_0 > 0)$ sau $(\beta < 0 \,\&\, \beta + \beta_0 > 0))$ | $(\alpha < 0 \,\&\, \alpha_0 > 0 \,\&\, \alpha + \alpha_0 < 0) \cap ((\beta \ge 0 \,\&\, \beta_0 > 0)$ sau $(\beta < 0 \,\&\, \beta + \beta_0 > 0))$ |
|  |  |  |  |  |

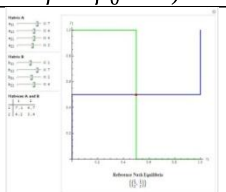
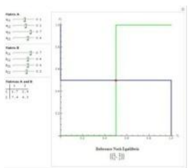
3. $NE = \{punctul\ (1,0)\}$:

| 1. | 2. | 3. | 4. | 5. |
|---|---|---|---|---|
| $((\alpha \ge 0 \,\&\, \alpha_0 > 0)$ sau $(\alpha < 0 \,\&\, \alpha + \alpha_0 > 0)) \cap ((\beta \le 0 \,\&\, \beta_0 < 0)$ sau $(\beta > 0 \,\&\, \beta + \beta_0 < 0))$ | $((\alpha \ge 0 \,\&\, \alpha_0 > 0)$ sau $(\alpha < 0 \,\&\, \alpha + \alpha_0 > 0)) \cap (\beta < 0 \,\&\, \beta_0 = 0)$ | $((\alpha \ge 0 \,\&\, \alpha_0 > 0)$ sau $(\alpha < 0 \,\&\, \alpha + \alpha_0 > 0)) \cap (\beta < 0 \,\&\, \beta_0 > 0 \,\&\, \beta + \beta_0 < 0)$ | $(\alpha < 0 \,\&\, \alpha_0 > 0 \,\&\, \alpha + \alpha_0 < 0) \cap ((\beta \le 0 \,\&\, \beta_0 < 0)$ sau $(\beta > 0 \,\&\, \beta + \beta_0 < 0))$ | $(\alpha < 0 \,\&\, \alpha + \alpha_0 = 0) \cap ((\beta \le 0 \,\&\, \beta_0 < 0)$ sau $(\beta > 0 \,\&\, \beta + \beta_0 < 0))$ |
|  |  |  |  |  |

4. $NE = \{punctul\ (1,1)\}$:

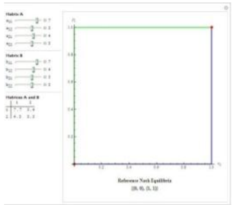
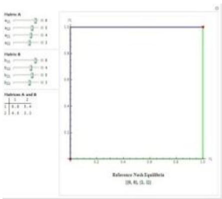
| 1. | 2. | 3. | 4. | 5. |
|---|---|---|---|---|
| $((\alpha \ge 0 \,\&\, \alpha_0 > 0)$ sau $(\alpha < 0 \,\&\, \alpha + \alpha_0 > 0)) \cap ((\beta \ge 0 \,\&\, \beta_0 > 0)$ sau $(\beta < 0 \,\&\, \beta + \beta_0 > 0))$ | $((\alpha \ge 0 \,\&\, \alpha_0 > 0)$ sau $(\alpha < 0 \,\&\, \alpha + \alpha_0 > 0)) \cap (\beta > 0 \,\&\, \beta_0 = 0)$ | $((\alpha \ge 0 \,\&\, \alpha_0 > 0)$ sau $(\alpha < 0 \,\&\, \alpha + \alpha_0 > 0)) \cap (\beta > 0 \,\&\, \beta_0 < 0 \,\&\, \beta + \beta_0 > 0)$ | $(\alpha > 0 \,\&\, \alpha + \alpha_0 = 0) \cap ((\beta \ge 0 \,\&\, \beta_0 > 0)$ sau $(\beta < 0 \,\&\, \beta + \beta_0 > 0))$ | $(\alpha > 0 \,\&\, \alpha_0 < 0 \,\&\, \alpha + \alpha_0 > 0) \cap ((\beta \ge 0 \,\&\, \beta_0 > 0)$ sau $(\beta < 0 \,\&\, \beta + \beta_0 > 0))$ |
|  |  |  |  |  |

5. $NE = \left\{punctul\ \left(-\frac{\beta_0}{\beta}, -\frac{\alpha_0}{\alpha}\right)\right\}$:

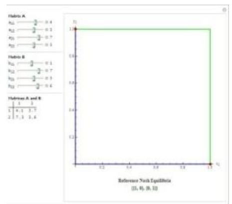
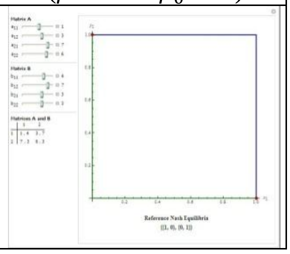
| 1. | 2. |
|---|---|
| $(\alpha > 0 \,\&\, \alpha_0 < 0 \,\&\, \alpha + \alpha_0 > 0) \cap (\beta < 0 \,\&\, \beta_0 > 0 \,\&\, \beta + \beta_0 < 0)$ | $(\alpha < 0 \,\&\, \alpha_0 > 0 \,\&\, \alpha + \alpha_0 < 0) \cap (\beta > 0 \,\&\, \beta_0 < 0 \,\&\, \beta + \beta_0 > 0)$ |
|  |  |

**Două puncte**

6. $NE = \{punctele\ (0{,}0); (1{,}1)\}$:

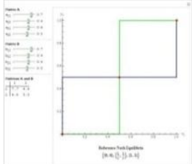
| 1. | 2. |
|---|---|
| $(\alpha > 0 \,\&\, \alpha_0 = 0) \cap (\beta > 0 \,\&\, \beta_0 = 0)$ | $(\alpha > 0 \,\&\, \alpha + \alpha_0 = 0) \cap (\beta > 0 \,\&\, \beta + \beta_0 = 0)$ |
|  |  |

7. $NE = \{punctele\ (0{,}1); (1{,}0)\}$:

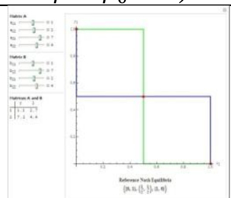
| 1. | 2. |
|---|---|
| $(\alpha < 0 \,\&\, \alpha_0 = 0) \cap (\beta < 0 \,\&\, \beta + \beta_0 = 0)$ | $(\alpha < 0 \,\&\, \alpha + \alpha_0 = 0) \cap (\beta > 0 \,\&\, \beta_0 = 0)$ |
|  |  |

**Trei puncte**

8. $NE = \left\{punctele\ (0{,}0); \left(-\frac{\beta_0}{\beta}, -\frac{\alpha_0}{\alpha}\right); (1{,}1)\right\}$:

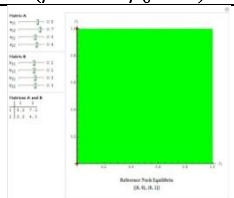
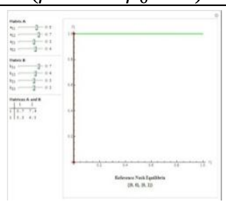
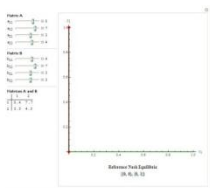
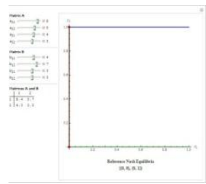
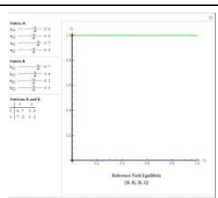
| 1. |
|---|
| $(\alpha > 0 \,\&\, \alpha_0 < 0 \,\&\, \alpha + \alpha_0 > 0) \cap (\beta > 0 \,\&\, \beta_0 < 0 \,\&\, \beta + \beta_0 > 0)$ |
|  |

$NE = \left\{punctele\ (1{,}0); \left(-\frac{\beta_0}{\beta}, -\frac{\alpha_0}{\alpha}\right); (0{,}1)\right\}$:

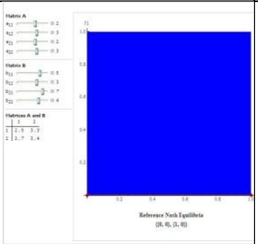
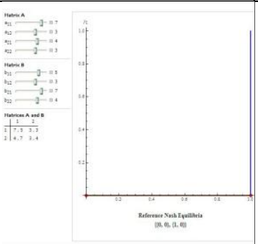
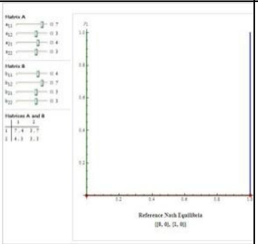
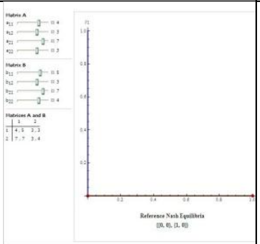
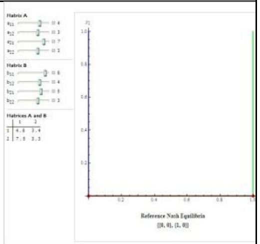
| 1. |
|---|
| $(\alpha < 0 \,\&\, \alpha_0 > 0 \,\&\, \alpha + \alpha_0 < 0) \cap (\beta < 0 \,\&\, \beta_0 > 0 \,\&\, \beta + \beta_0 < 0)$ |
|  |

**Un segment**

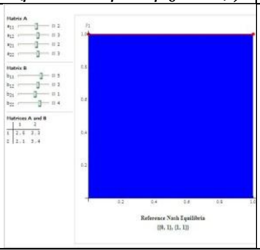
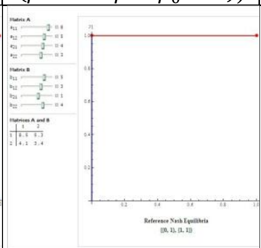
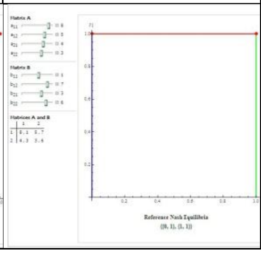
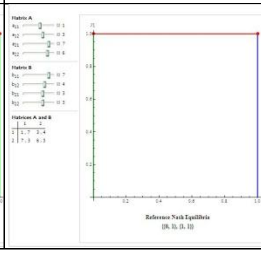
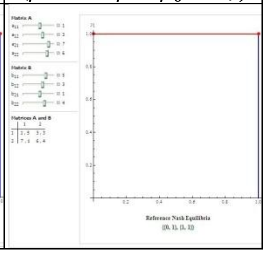
9. $NE = \{segmentul\ (0{,}0); (0{,}1)\}$:

| 1. | 2. | 3. | 4. | 5. |
|---|---|---|---|---|
| $((\alpha \le 0 \,\&\, \alpha_0 < 0)$ sau $(\alpha > 0 \,\&\, \alpha + \alpha_0 < 0)) \cap (\beta = 0 \,\&\, \beta_0 = 0)$ | $((\alpha \le 0 \,\&\, \alpha_0 < 0)$ sau $(\alpha > 0 \,\&\, \alpha + \alpha_0 < 0)) \cap (\beta > 0 \,\&\, \beta_0 = 0)$ | $((\alpha \le 0 \,\&\, \alpha_0 < 0)$ sau $(\alpha > 0 \,\&\, \alpha + \alpha_0 < 0)) \cap (\beta < 0 \,\&\, \beta_0 = 0)$ | $(\alpha > 0 \,\&\, \alpha + \alpha_0 = 0) \cap (\beta < 0 \,\&\, \beta_0 = 0)$ | $(\alpha < 0 \,\&\, \alpha_0 = 0) \cap (\beta > 0 \,\&\, \beta_0 = 0)$ |
|  |  |  |  |  |

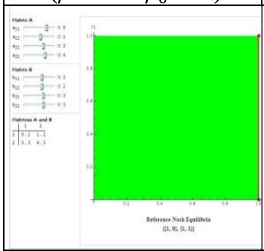
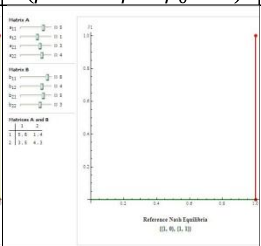
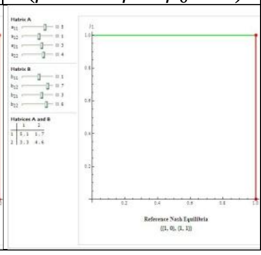
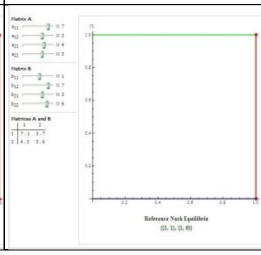
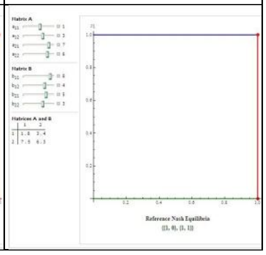
10. $NE = \{segmentul\ (0{,}0); (1{,}0)\}$:

| 1. | 2. | 3. | 4. | 5. |
|---|---|---|---|---|
| $(\alpha = 0 \,\&\, \alpha_0 = 0) \cap ((\beta \le 0 \,\&\, \beta_0 < 0)$ sau $(\beta > 0 \,\&\, \beta + \beta_0 < 0))$ | $(\alpha > 0 \,\&\, \alpha_0 = 0) \cap ((\beta \le 0 \,\&\, \beta_0 < 0)$ sau $(\beta > 0 \,\&\, \beta + \beta_0 < 0))$ | $(\alpha > 0 \,\&\, \alpha_0 = 0) \cap (\beta < 0 \,\&\, \beta_0 = 0)$ | $(\alpha < 0 \,\&\, \alpha_0 = 0) \cap ((\beta \le 0 \,\&\, \beta_0 < 0)$ sau $(\beta > 0 \,\&\, \beta + \beta_0 < 0))$ | $(\alpha < 0 \,\&\, \alpha_0 = 0) \cap (\beta > 0 \,\&\, \beta + \beta_0 = 0)$ |
|  |  |  |  |  |

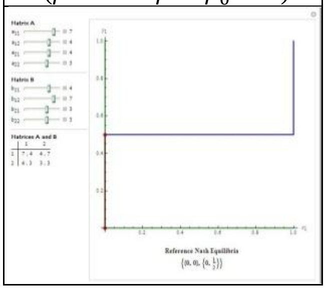
11. $NE = \{segmentul\ (0,1); (1,1)\}$:

| 1. | 2. | 3. | 4. | 5. |
|---|---|---|---|---|
| $(\alpha = 0\ \&\ \alpha_0 = 0) \cap ((\beta \geq 0\ \&\ \beta_0 > 0)$ sau $(\beta < 0\ \&\ \beta + \beta_0 > 0))$ | $(\alpha > 0\ \&\ \alpha_0 = 0) \cap ((\beta \geq 0\ \&\ \beta_0 > 0)$ sau $(\beta < 0\ \&\ \beta + \beta_0 > 0))$ | $(\alpha > 0\ \&\ \alpha + \alpha_0 = 0) \cap (\beta < 0\ \&\ \beta + \beta_0 = 0)$ | $(\alpha < 0\ \&\ \alpha + \alpha_0 = 0) \cap (\beta > 0\ \&\ \beta_0 = 0)$ | $(\alpha < 0\ \&\ \alpha + \alpha_0 = 0) \cap ((\beta \geq 0\ \&\ \beta_0 > 0)$ sau $(\beta < 0\ \&\ \beta + \beta_0 > 0))$ |
|  |  |  |  |  |

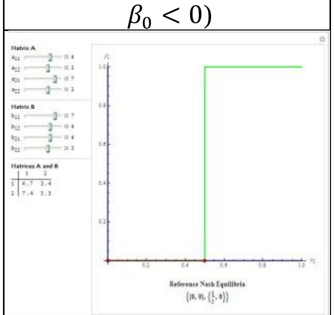
12. $NE = \{segmentul\ (1,0); (1,1)\}$:

| 1. | 2. | 3. | 4. | 5. |
|---|---|---|---|---|
| $((\alpha \geq 0\ \&\ \alpha_0 > 0)$ sau $(\alpha < 0\ \&\ \alpha + \alpha_0 > 0)) \cap (\beta = 0\ \&\ \beta_0 = 0)$ | $((\alpha \geq 0\ \&\ \alpha_0 > 0)$ sau $(\alpha < 0\ \&\ \alpha + \alpha_0 > 0)) \cap (\beta > 0\ \&\ \beta + \beta_0 = 0)$ | $((\alpha \geq 0\ \&\ \alpha_0 > 0)$ sau $(\alpha < 0\ \&\ \alpha + \alpha_0 > 0)) \cap (\beta < 0\ \&\ \beta + \beta_0 = 0)$ | $(\alpha > 0\ \&\ \alpha_0 = 0) \cap (\beta < 0\ \&\ \beta + \beta_0 = 0)$ | $(\alpha < 0\ \&\ \alpha_0 = 0) \cap (\beta > 0\ \&\ \beta + \beta_0 = 0)$ |
|  |  |  |  |  |

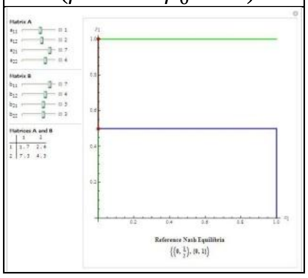
13. $NE = \left\{segmentul\ (0,0); \left(0, -\frac{\alpha_0}{\alpha}\right)\right\}$:

| 1. |
|---|
| $(\alpha > 0\ \&\ \alpha_0 < 0\ \&\ \alpha + \alpha_0 > 0) \cap (\beta < 0\ \&\ \beta + \beta_0 = 0)$ |
|  |

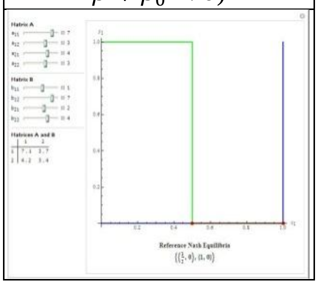
14. $NE = \left\{segmentul\ (0,0); \left(-\frac{\beta_0}{\beta}, 0\right)\right\}$:

| 1. |
|---|
| $(\alpha < 0\ \&\ \alpha_0 = 0) \cap (\beta < 0\ \&\ \beta_0 > 0\ \&\ \beta + \beta_0 < 0)$ |
|  |

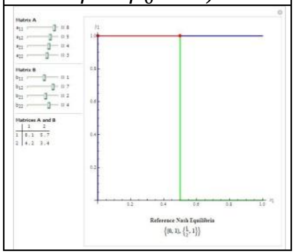
15. $NE = \left\{segmentul\ \left(0, -\frac{\alpha_0}{\alpha}\right);\ (0,1)\right\}$:

| 1. |
|---|
| $(\alpha < 0\ \&\ \alpha_0 > 0\ \&\ \alpha + \alpha_0 < 0) \cap (\beta > 0\ \&\ \beta_0 = 0)$ |
|  |

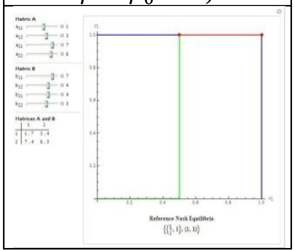
16. $NE = \left\{segmentul\ \left(-\frac{\beta_0}{\beta}, 0\right);\ (1,0)\right\}$:

| 1. |
|---|
| $(\alpha > 0\ \&\ \alpha_0 = 0) \cap (\beta < 0\ \&\ \beta_0 > 0\ \&\ \beta + \beta_0 < 0)$ |
|  |

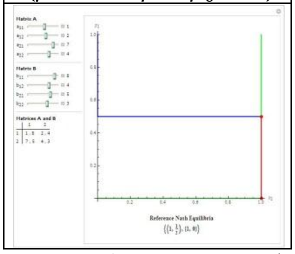
17. $NE = \left\{segmentul\ (0,1); \left(-\frac{\beta_0}{\beta}, 1\right)\right\}$:

| 1. |
|---|
| $(\alpha > 0 \,\&\, \alpha + \alpha_0 = 0) \cap (\beta > 0 \,\&\, \beta_0 < 0 \,\&\, \beta + \beta_0 > 0)$ |
|  |

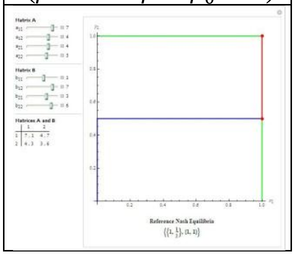
18. $NE = \left\{segmentul\ \left(-\frac{\beta_0}{\beta}, 1\right);\ (1,1)\right\}$:

| 1. |
|---|
| $(\alpha < 0 \,\&\, \alpha + \alpha_0 = 0) \cap (\beta > 0 \,\&\, \beta_0 < 0 \,\&\, \beta + \beta_0 > 0)$ |
|  |

19. $NE = \left\{segmentul\ (1,0); \left(1, -\frac{\alpha_0}{\alpha}\right)\right\}$:

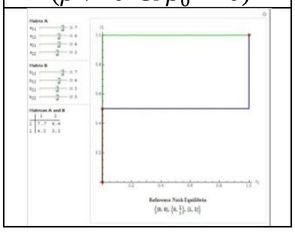
| 1. |
|---|
| $(\alpha < 0 \,\&\, \alpha_0 > 0 \,\&\, \alpha + \alpha_0 < 0) \cap (\beta > 0\ \,\&\, \beta + \beta_0 = 0)$ |
|  |

20. $NE = \left\{segmentul\ \left(1, -\frac{\alpha_0}{\alpha}\right);\ (1,1)\right\}$:

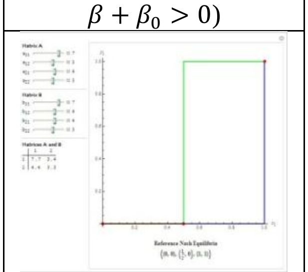
| 1. |
|---|
| $(\alpha > 0 \,\&\, \alpha_0 < 0 \,\&\, \alpha + \alpha_0 > 0) \cap (\beta < 0\ \,\&\, \beta + \beta_0 = 0)$ |
|  |

## Un segment și un punct distinct

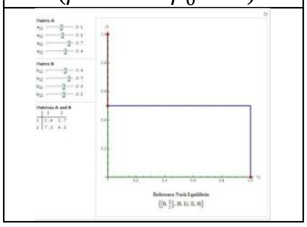
21. $NE = \left\{segmentul\ (0,0); \left(0, -\frac{\alpha_0}{\alpha}\right), punctul\ (1,1)\right\}$:

| 1. |
|---|
| $(\alpha > 0 \,\&\, \alpha_0 < 0 \,\&\, \alpha + \alpha_0 > 0) \cap (\beta > 0\ \,\&\, \beta_0 = 0)$ |
|  |

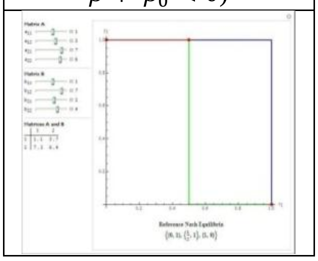
22. $NE = \left\{segmentul\ (0,0); \left(-\frac{\beta_0}{\beta}, 0\right), punctul\ (1,1)\right\}$:

| 1. |
|---|
| $(\alpha > 0 \,\&\, \alpha_0 = 0) \cap (\beta > 0\ \,\&\beta_0 < 0 \,\&\, \beta + \beta_0 > 0)$ |
|  |

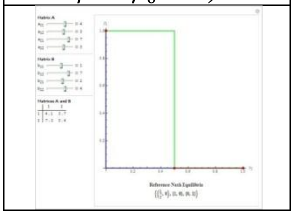
23. $NE = \left\{segmentul\ \left(0, -\frac{\alpha_0}{\alpha}\right); (0,1), punctul\ (1,0)\right\}$:

| 1. |
|---|
| $(\alpha < 0 \,\&\, \alpha_0 > 0 \,\&\, \alpha + \alpha_0 < 0) \cap (\beta < 0\ \,\&\, \beta_0 = 0)$ |
|  |

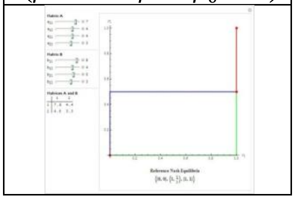
24. $NE = \left\{segmentul\ (0,1); \left(-\frac{\beta_0}{\beta}, 1\right), punctul\ (1,0)\right\}$:

| 1. |
|---|
| $(\alpha < 0 \,\&\, \alpha + \alpha_0 = 0) \cap (\beta < 0\ \,\&\, \beta_0 > 0 \,\&\, \beta +\ \beta_0 < 0)$ |
|  |

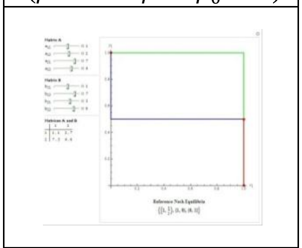
25. $NE = \left\{segmentul \left(-\frac{\beta_0}{\beta}, 0\right); (1{,}0), punctul\ (0{,}1)\right\}$:

| 1. |
|---|
| $(\alpha < 0\ \&\ \alpha_0 = 0) \cap$<br>$(\beta < 0\ \ \&\ \beta_0 > 0\ \&$<br>$\beta +\ \beta_0 < 0)$ |
|  |

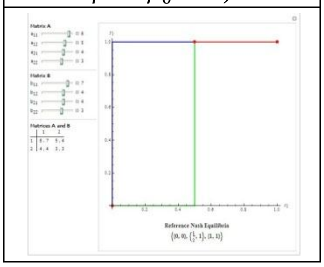
27. $NE = \left\{segmentul\ \left(-\frac{\beta_0}{\beta}, 1\right); (1{,}1), punctul\ (0{,}0)\right\}$:

| 1. |
|---|
| $(\alpha > 0\ \&\ \alpha_0 < 0\ \&$<br>$\alpha + \alpha_0 > 0) \cap$<br>$(\beta > 0\ \ \&\ \beta +\ \beta_0 = 0)$ |
|  |

26. $NE = \left\{segmentul\ (1{,}0); \left(1, -\frac{\alpha_0}{\alpha}\right), punctul\ (0{,}1)\right\}$:

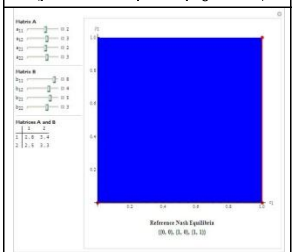
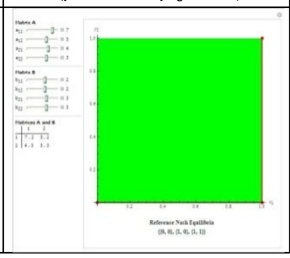
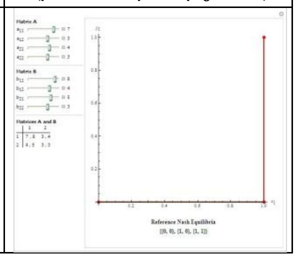
| 1. |
|---|
| $(\alpha < 0\ \&\ \alpha_0 > 0\ \&$<br>$\alpha + \alpha_0 < 0) \cap$<br>$(\beta < 0\ \ \&\ \beta +\ \beta_0 = 0)$ |
|  |

28. $NE = \left\{segmentul\ \left(1, -\frac{\alpha_0}{\alpha}\right);\ (1{,}1), punctul\ (0{,}0)\right\}$:

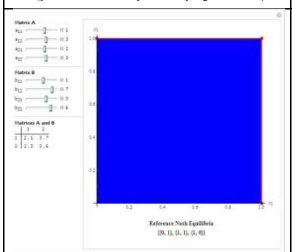
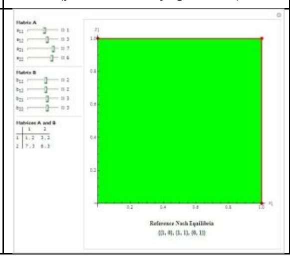
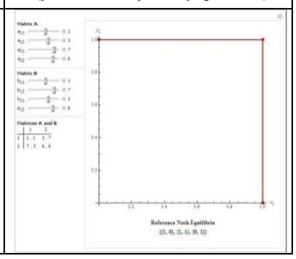
| 1. |
|---|
| $(\alpha > 0\ \ \&\ \alpha + \alpha_0 = 0) \cap$<br>$(\beta > 0\ \ \&\ \beta_0 < 0\ \&$<br>$\beta +\ \beta_0 > 0)$ |
|  |

**Linie frântă formată din două segmente**

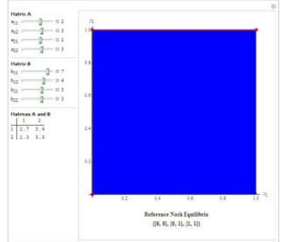
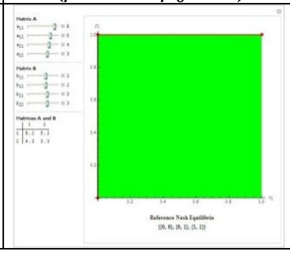
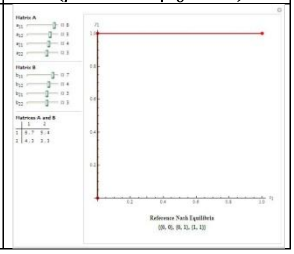
29. $NE = \{linia\ frântă\ (0{,}0); (1{,}0); (1{,}1)\}$:

| 1. | 2. | 3. |
|---|---|---|
| $(\alpha = 0\ \&\ \alpha_0 = 0) \cap$<br>$(\beta > 0\ \&\ \beta + \beta_0 = 0)$ | $(\alpha > 0\ \&\ \alpha_0 = 0) \cap$<br>$(\beta = 0\ \&\ \beta_0 = 0)$ | $(\alpha > 0\ \&\ \alpha_0 = 0) \cap$<br>$(\beta > 0\ \&\ \beta + \beta_0 = 0)$ |
|  |  |  |

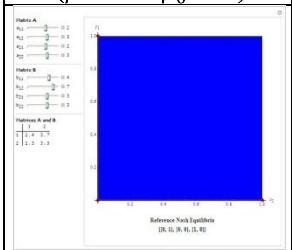
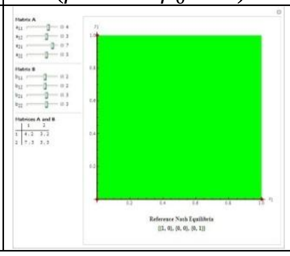
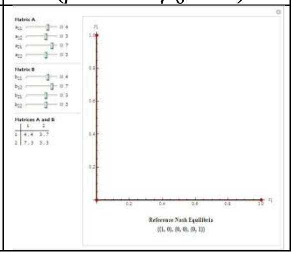
30. $NE = \{linia\ frântă\ (1{,}0); (1{,}1); (0{,}1)\}$:

| 1. | 2. | 3. |
|---|---|---|
| $(\alpha = 0\ \&\ \alpha_0 = 0) \cap$<br>$(\beta < 0\ \&\ \beta + \beta_0 = 0)$ | $(\alpha < 0\ \&\ \alpha + \alpha_0 = 0) \cap$<br>$(\beta = 0\ \&\ \beta_0 = 0)$ | $(\alpha < 0\ \&\ \alpha + \alpha_0 = 0) \cap$<br>$(\beta < 0\ \&\ \beta + \beta_0 = 0)$ |
|  |  |  |

31. $NE = \{linia\ frântă\ (0,0); (0,1); (1,1)\}$:

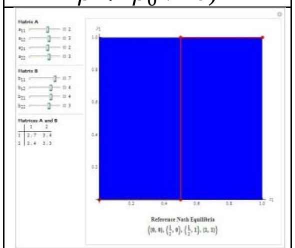
| 1. | 2. | 3. |
|---|---|---|
| $(\alpha = 0 \,\&\, \alpha_0 = 0) \cap$ $(\beta > 0 \,\&\, \beta_0 = 0)$ | $(\alpha > 0 \,\&\, \alpha + \alpha_0 = 0) \cap$ $(\beta = 0 \,\&\, \beta_0 = 0)$ | $(\alpha > 0 \,\&\, \alpha + \alpha_0 = 0) \cap$ $(\beta > 0 \,\&\, \beta_0 = 0)$ |
|  |  |  |

32. $NE = \{linia\ frântă\ (0,1); (0,0); (1,0)\}$:

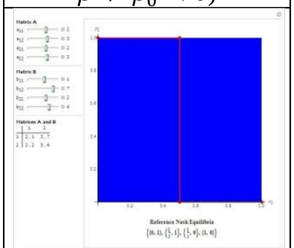
| 1. | 2. | 3. |
|---|---|---|
| $(\alpha = 0 \,\&\, \alpha_0 = 0) \cap$ $(\beta < 0 \,\&\, \beta_0 = 0)$ | $(\alpha < 0 \,\&\, \alpha_0 = 0) \cap$ $(\beta = 0 \,\&\, \beta_0 = 0)$ | $(\alpha < 0 \,\&\, \alpha_0 = 0) \cap$ $(\beta < 0 \,\&\, \beta_0 = 0)$ |
|  |  |  |

## Linie frântă formată din trei segmente

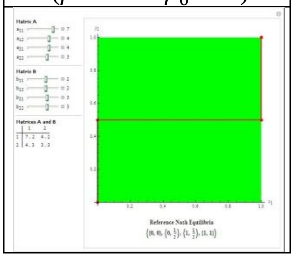
33. $NE = \left\{linia\ frântă\ (0,0); \left(-\frac{\beta_0}{\beta}, 0\right); \left(-\frac{\beta_0}{\beta}, 1\right); (1,1)\right\}$:

| 1. |
|---|
| $(\alpha = 0 \ \,\&\, \alpha_0 = 0) \cap$ $(\beta > 0 \ \,\&\, \beta_0 < 0 \,\&$ $\beta + \beta_0 > 0)$ |
|  |

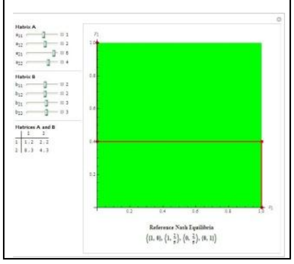
34. $NE = \left\{linia\ frântă\ (0,1); \left(-\frac{\beta_0}{\beta}, 1\right); \left(-\frac{\beta_0}{\beta}, 0\right); (1,0)\right\}$

| 1. |
|---|
| $(\alpha = 0 \ \,\&\, \alpha_0 = 0) \cap$ $(\beta < 0 \ \,\&\, \beta_0 > 0 \,\&$ $\beta + \beta_0 < 0)$ |
|  |

35. $NE = \left\{linia\ frântă\ (0,0); \left(0, -\frac{\alpha_0}{\alpha}\right); \left(1, -\frac{\alpha_0}{\alpha}\right); (1,1)\right\}$

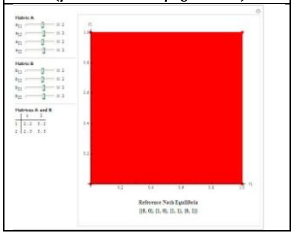
| 1. |
|---|
| $(\alpha > 0 \ \,\&\, \alpha_0 < 0 \,\&$ $\alpha + \alpha_0 > 0) \cap$ $(\beta = 0 \ \,\&\, \beta_0 = 0)$ |
|  |

36. $NE = \left\{linia\ frântă\ (0,1); \left(0, -\frac{\alpha_0}{\alpha}\right); \left(1, -\frac{\alpha_0}{\alpha}\right); (1,0)\right\}$

| 1. |
|---|
| $(\alpha < 0 \ \,\&\, \alpha_0 > 0 \,\&$ $\alpha + \alpha_0 < 0) \cap$ $(\beta = 0 \ \,\&\, \beta_0 = 0)$ |
| **  |

**Pătratul unitar**

37. $NE = \{dreptunghiul\ (0,0); (1,1)\}$:

| 1. |
|---|
| $(\alpha = 0\ \&\ \alpha_0 = 0) \cap$<br>$(\beta = 0\ \&\ \beta_0 = 0)$ |
|  |

**Concluzii**

În lucrare se propune un algoritm simbolic de determinare a mulţimii Nash, a cărui esenţă se rezumă la enumerarea tuturor cazurilor (formelor) posibile în funcţie de datele iniţiale: elementele matricelor de câştig. Ca bază teoretică se foloseşte metoda intersecţiei graficelor aplicaţiilor de răspuns optim [2-4]. Toate cazurile examinate sunt ilustrate prin imagini obţinute cu ajutorul programului elaborat în Wolfram Mathematica 9. Aceste ilustraţii servesc, totodată, şi ca demonstraţie a corectitudinii algoritmului propus.

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