Parametric sensitivity of three-phase induction motor

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Abstract. We proposed a joint algorithm of calculating of transient and steady-state parametric sensitivity of the three-phase induction motor. Differential equations of device are presented in normal Cauchy's form. Transient process is obtained for given initial conditions, steady-state - for those that exclude the transient reaction. The results of simulations are given.

Keywords: transient and steady-state parametric sensitivities, three phase induction motor.

Introduction.

Calculation of the parametric sensitivity of electrical devices is the final stage of problem analysis, which is building a bridge to the task of synthesis. Stages of calculation of transient and steady-state processes and determine the static stability derived steady-state processes precede this stage. We propose a common algorithm that is based on the general theory of nonlinear differential equations to solve the complete problem of analysis. Two-point boundary value problem is solved for ordinary differential equations of the electromechanical state.

To solve this problem it was necessary to first: construct a mathematical model of the device, as well as auxiliary model of parametric sensitivity [1]. This was the basis construction of monodromy matrix of, and on its basis simulation transient and steady-state process, but at the same time steady-state parametric sensitivity.

Mathematical model.

We write the equations of the electromagnetic state of motor as [1]

$$\frac{di}{dt} = A(u - \Omega'\Psi - Ri), \qquad (1)$$

where

$$\begin{array}{|c|}\hline \lambda_S \\ \hline \lambda_R \\ \hline \end{array}, \ \lambda = u, \Psi, i; \quad \mathrm{A} = \begin{array}{|c|c|}\hline A_S & A_{SR} \\ \hline A_{RS} & A_R \\ \hline \end{array};$$
$$\Omega' = \begin{array}{|c|c|}\hline & \\ \hline & \Omega \\ \hline \end{array}; \quad R = \begin{array}{|c|c|}\hline R_S & \\ \hline & R_R \\ \hline \end{array}. \qquad (2)$$

Here $i_k=(i_{kA},i_{kB})_t$, $k=S, R$ are columns of phase currents of the stator winding and transformed currents of rotor winding; $u_k=(u_{kA},u_{kB})_t$, $k=S, R$ are columns of phase voltages of the stator winding; A_S, A_{SR}, A_{RS}, A_R are matrices

$$A_S = \alpha_S(1-\alpha_S G);\ A_{SR} = A_{RS} = -\alpha_S\alpha_R G;$$
$$A_R = \alpha_R(1-\alpha_R G), \qquad (3)$$

where G, Ω are matrices

$$G = \begin{array}{|c|c|}\hline T + b_A i_A & b_B i_A \\ \hline b_A i_B & T + b_B i_B \\ \hline \end{array}, \quad \Omega = \frac{\omega}{\sqrt{3}} \begin{array}{|c|c|}\hline -1 & -2 \\ \hline 2 & 1 \\ \hline \end{array}. \qquad (4)$$

Moreover

$$b_A = b(2i_A + i_B);\quad b_B = b(i_A + 2i_B);\quad b = \frac{2}{3}\frac{R-T}{i_m^2}; \qquad (5)$$

$$R = \frac{1}{\alpha_S + \alpha_R + \rho};\qquad T = \frac{1}{\alpha_S + \alpha_R + \tau}.$$

Here τ, ρ are inverse static and differential inductance, we find them by characteristic of magnetization of machine (no-operation state) as:

$$\tau = \left[\frac{\psi_m(i_m)}{i_m}\right]^{-1};\quad \rho = \left[\frac{d\psi_m(i_m)}{di_m}\right]^{-1}, \qquad (6)$$

where i_m is the module of spatial vector currents magnetization

$$i_m = 2\sqrt{(i_A^2 + i_A i_B + i_B^2)/3};\ \ i_A = i_{SA} + i_{RA};\ i_B = i_{SB} + i_{RB}. \qquad (7)$$

In the absence of saturation characteristics of magnetization degenerates into a straight line $i_m = \alpha_m\psi_m$, where α_m is inverse main inductance, and the matrix (4) according to (6) – in diagonal

$$G = \frac{1}{\alpha_S + \alpha_R + \alpha_m} \begin{array}{|c|c|}\hline 1 & \\ \hline & 1 \\ \hline \end{array}, \qquad (8)$$

which greatly simplifies equation (1). In this case, we get the simplest of all known mathematical model of asynchronous motor; R_S, R_R – matrix of resistances

$$R_s = \begin{array}{|c|c|}\hline r_S & \\ \hline & r_S \\ \hline \end{array};\ R_R = \begin{array}{|c|c|}\hline r_R & \\ \hline & r_R \\ \hline \end{array}, \qquad (9)$$

Moreover α_S, α_R are inverse inductance of dissipation of stator winding and rotor; r_S is resistance of stator phases; r_R is given resistance of rotor winding; Ω is matrix of angular velocity ω.

Components column of complete linkages of stator windings and rotor we found so:

$$\Psi_{kj} = \frac{1}{\tau}(i_{Sj} + i_{Rj}) + \frac{1}{\alpha_k} i_{kj},\quad j = A, B;\ k = S, R. \qquad (10)$$

Elements of columns voltage stator and rotor

$$u_S = (U_m \sin(\omega_0 t),\ U_m \sin(\omega_0 t - 120^0))_t;\quad u_R = 0, \qquad (11)$$

where U_m, ω_0 are amplitude and circular frequency of voltage.

The equation of mechanical condition has the form

$$\frac{d\omega}{dt} = \frac{p_0}{J}(M_E - M(\omega)),\ M_E = \sqrt{3}p_0(\Psi_{SA}i_{SB} - \Psi_{SB}i_{SA}), \qquad (12)$$

where $M(\omega)$ is mechanical moment; p_0 is number of pairs of magnetic poles; J is moment of inertia of the rotor; M_E is electromagnetic moment.

The system of differential equations (1), (12) is a mathematical model of the A-asynchronous motor. It is intended for the analysis of transient and steady-state processes. For practical use it is necessary to know the following input data: resistance and inverse inductance of dissipation of stator windings and rotor; characterization in no-operation state, while neglecting the saturation of the main magnetic circuit – the inverse of the primary inductance of the machine, the number of pairs of magnetic poles and the moment of inertia of the rotor. Input signals are: phase of supply voltage and mechanical moment on the shaft.

The solution of Cauchy's problem.

The system of ordinary differential equations (1), (12) we write in the general form

$$\frac{dx}{dt} = f(x,t), \qquad x = (i,\omega)_t. \tag{13}$$

Integration of differential equations (13) for given initial conditions $x(t)_{|t=+0} = x(0)$ constitutes the Cauchy's problem for a given system of differential equations, which presents a problem of calculating the transient electromechanical processes of motor. We must first find the matrix of monodromy to solve two-point boundary value problem.

The matrix of monodromy.

We will use column of unknown x (13). But for building auxiliary model of sensitivity, we will form column of unknown y

$$y = (\Psi,\omega)_t. \tag{14}$$

Appropriate (14) the differential equation (1) has the form

$$\frac{d\Psi}{dt} = u - \Omega'\Psi - Ri, \tag{15}$$

The matrix of monodromy we write in the form [1]

$$\Phi = (\mathrm{A}z, w)_t, \tag{16}$$

where

$$z = \frac{\partial\Psi}{\partial x(0)}; \quad w = \frac{\partial\omega}{\partial x(0)}. \tag{17}$$

Variation equations for calculating submatrix (17) we obtain by differentiation by $x(0)$ equations of electromechanical state (12), (15).

Differentiating (15), we obtain

$$\frac{dz}{dt} = -(\Omega' + R\mathrm{A})z - \frac{\partial\Omega'}{\partial\omega} w\Psi. \tag{18}$$

Differentiating by $x(0)$ (12), we obtain

$$\frac{dw}{dt} = \frac{p_0}{J}\left(\sqrt{3}p_0\Big(\frac{\partial\Psi_{SA}}{\partial x(0)} i_{SB} + \Psi_{SA}\frac{\partial i_{SB}}{\partial x(0)} - \frac{\partial\Psi_{SB}}{\partial x(0)} i_{SA} - \Psi_{SB}\frac{\partial i_{SA}}{\partial x(0)}\Big) - \frac{\partial M(\omega)}{\partial\omega} w\right). \tag{19}$$

Derivatives $\partial\Psi_{SA}/\partial x(0)$, $\partial\Psi_{SB}/\partial x(0)$, $\partial i_{SA}/\partial x(0)$, $\partial i_{SB}/\partial x(0)$ are elements of matrices z, Az, so they are known.

Therefore, the construction of matrix of monodromy asynchronous motor requires integrating the equations of first variation (18), (19).

The solution of two-point boundary value problem.

There are some initial conditions $x(0)$, which in the integration (13) on the interval from 0 to T enable enter directly into the periodic solution, bypassing the transient response. These initial conditions we consider as an argument equation of periodicity

$$f\big(x(0)\big) = x(0) - x\big(x(0),T\big) = 0, \tag{20}$$

where T – period.

The solution of nonlinear transcendental equation (20) we will implement by the Newton's iteration method

$$x(0)^{(s+1)} = x(0)^{(s)} - f'\Big(x(0)^{(s)}\Big)^{-1} f\Big(x(0)^{(s)}\Big). \tag{21}$$

Jacobi's matrix we get by differentiation by $x(0)$ the objective function (20)

$$f'\big(x(0)\big) = E - \Phi(T), \tag{22}$$

where

$$\Phi(T) = \left.\frac{\partial x\big(x(0),t\big)}{\partial x(0)}\right|_{t=T}. \tag{23}$$

Matrix (23) is the desired monodromy matrix of (16) at time $t=T$. Her multipliers give a complete answer about static stability found periodic state.

At the s-th iteration Newton's formulas (21) linear variational equation (18), (19) are subject compatible integration with the non-linear (1), (12) in the time interval $[0,T]$. As a result, we find the objective function (20) and the desired Jacobi's matrix (22), (23), which fully defines right part of iterative formulas (21) and then - and its desired left side $x(0)^{(s+1)}$. The process of Iteration ends when it reaches a given accuracy entering in periodic solution.

The matrix of monodromy Φ (23) is, in fact, a matrix of sensitivity to initial conditions. Each of the line can be considered as a gradient of certain variable in the space of initial conditions, and each column describes the sensitivity of the whole set of variables to the same initial conditions. Therefore, the differential equation (18), (19) can be regarded as the model sensitivity to initial conditions.

The model of parametric sensitivity.

The problem of calculating the of parametric sensitivity the easiest to solve variational methods as a simple addition to the algorithm accelerated search of periodic solutions of nonlinear differential equations based on Newton's iteration (21).

We denote vector of constant parameters as

$$\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_n). \quad (24)$$

Elements of column λ are any constants parameters which in turn may be functions of other structural constant parameters. Calculation of parametric sensitivity on these parameters is performed similarly, but derivatives by λ should be taken according to the rules of differentiation of complex functions.

Matrix of parametric sensitivities determined as the derivative

$$S = \frac{\partial x}{\partial \lambda}. \quad (25)$$

Argument x, we find from equation (13) we write the more general form:

$$dx/dt = f_1(x, \lambda, t), \quad (26)$$

Differentiating (26) by λ we obtain linear parametric equation

$$\frac{dS}{dt} = \frac{\partial f_1(x,\lambda,t)}{\partial x} S + \frac{\partial f_1(x,\lambda,t)}{\partial \lambda}. \quad (27)$$

In steady-state condition $x(0)=x(T)$, so equation (27) is also $S(t)$ a periodic solution.

Taking partial derivatives by x and t in the right part (1), (12) is quite a difficult task, but it is also not feasible. Therefore, we introduce the matrix of auxiliary parametric sensitivities χ elative to some other vector y (14):

$$\chi = \frac{dy}{d\lambda}. \quad (28)$$

The equation of state of the investigated object in relation to the vector y, we write also in the general form:

$$dy/dt = f_2(y, \lambda, t), \quad (29)$$

f_2 is T– periodic by t.

Differentiating (29) by λ and taking into account (25), (28), we obtain

$$\frac{d\chi}{dt} = \frac{\partial f_2(y,\lambda,t)}{\partial y}\chi + \frac{\partial f_2(y,\lambda,t)}{\partial \lambda}. \quad (30)$$

Equation (14) also has a periodic solution $\chi(t)$. Function $\chi(t)$, besides performed supporting roles, constitutes often independent interest.

Matrix of parametric sensitivities S in our case repeats (16)

$$S = (\mathrm{A}\chi, \eta)_t, \quad (31)$$

where

$$S = \frac{\partial (i,\omega)_t}{\partial \lambda}; \quad \chi = \frac{\partial \Psi}{\partial \lambda}; \quad \eta = \frac{\partial \omega}{\partial \lambda}. \quad (32)$$

We provide the equation (15) form (29)

$$\frac{d\Psi}{dt} = u - \Omega'\Psi - RL^{-1}\Psi, \quad (33)$$

where L^{-1} is inverse matrix of static inductances [1]

$$L^{-1} = T\begin{vmatrix} \alpha_S(\alpha_R+\tau) & & -\alpha_S\alpha_R & \\ & \alpha_S(\alpha_R+\tau) & & -\alpha_S\alpha_R \\ -\alpha_S\alpha_R & & \alpha_R(\alpha_S+\tau) & \\ & -\alpha_S\alpha_R & & \alpha_R(\alpha_S+\tau) \end{vmatrix}. \quad (34)$$

Recall, that the T we obtain from (5).

To obtain equation (30) is sufficient according to (28) to differentiate by λ (33)

$$\frac{d\chi}{dt} = -(\Omega + R\mathrm{A})\chi + F. \quad (35)$$

Here

$$F = \frac{\partial U}{\partial \lambda} + RL^{-1}\frac{\partial L}{\partial \lambda} I - \frac{\partial \Omega}{\partial \lambda}\Psi - \frac{\partial R}{\partial \lambda} I. \quad (36)$$

where L is matrix of static inductances [1]

$$L = \begin{vmatrix} l_S + l_\tau & & l_\tau & \\ & l_S + l_\tau & & l_\tau \\ l_\tau & & l_R + l_\tau & \\ & l_\tau & & l_R + l_\tau \end{vmatrix}, \quad (37)$$

moreover $l_S=1/\alpha_S$, $l_R=1/\alpha_R$ is inductance of dissipation of stator windings and rotor; $l_\tau=1/\tau$ is basic static inductance (see (6)).

Differentiating by λ (12), we obtain

$$\frac{d\eta}{dt} = \frac{p_0}{J}\left(\frac{\partial M_E}{\partial \lambda} - \frac{\partial M(\omega)}{\partial \omega}\eta\right) + \\ + p_0\left(M_E - M(\omega)\right)\frac{\partial (1/J)}{\partial \lambda}, \quad (38)$$

where

$$\frac{\partial M_E}{\partial \lambda} = \sqrt{3} p_0 (\chi_{SA} i_{SB} + \Psi_{SA} S_{SB} - \chi_{SB} i_{SA} - \Psi_{SB} S_{SA}). \quad (39)$$

Matrix of parametric sensitivity (31) we will be divided into columns and write as a string

$$S = (S_1, S_2, \ldots, S_m), \quad (40)$$

where m is the number of elements of the vector of constant parameters $\lambda=(\lambda_1,\lambda_2,\ldots,\lambda_m)$, λ=const. Moreover

$$S_i = d(i,\omega)_t / \partial \lambda_i, \ i = 1, 2, \ldots, m \quad (41)$$

are vectors of parametric sensitivities elements of the vector x to some constant parameters.

The condition of periodicity S we write similarly to (20)

$$F(S_i(0)) = S_i(0) - S_i(S_i(0), T) = 0, \ i = 1, 2, \ldots, n. \quad (42)$$

Equation (42), we also solve the iteration Newton's method, but since (35) and (38) are linear equations then solution we get per one iteration. In the zero approximation (21) will

$$S_i\left(0\right)^{(1)} = F'\left(S_i\left(0\right)^{(0)}\right)^{-1} S_i\left(T\right)^{(0)}, \quad i = 1,\ldots,n. \qquad (43)$$

Jacobi's matrix is expressed through a known matrix (16), obtained from the calculation of periodic solution $x(t) = x(t+T)$:

$$F'\left(S_i\left(0\right)^{(0)}\right) = E - \Phi(T). \qquad (44)$$

In a single iteration of formula (43) compatible integration on the interval [0,*T*] are differential equations (1), (12) with the initial conditions, excluding transient reaction, and (35), (38) with zero initial conditions. Moving in advance according to (31) from $S_i(0)$ to $\chi_i(0)$ we get the periodic solution: $\chi(t)+\chi(t+T)=0,\ \eta(t)+\eta(t+T)=0$. Sensitivity S we find from (31).

Periodic solutions are not always easy to use. Therefore, we can move to their root-mean-square values

$$\mathbf{S} = \sqrt{\frac{1}{T}\int_0^T S(t)^2\,dt}. \qquad (45)$$

Recall that integral of matrix taken as matrix of integrals of its individual elements.

Time discretization given differential equations and differential equations of sensitivity (to initial conditions and parametric) performed by explicit or implicit methods. Especially harmoniously combined them compatible solution in the case of implicit methods, because Jacobi's matrices the main equation and equation of objectives match.

The results of simulations.

On Fig. 1 – Fig. 4 are shown the results of the simulation.

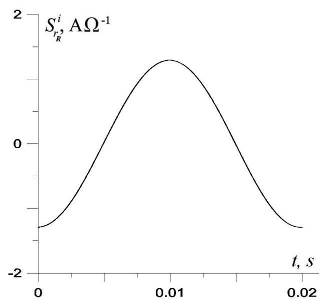

Fig. 1. Steady-state parametric sensitivity of the stator current to the resistance of rotor winding

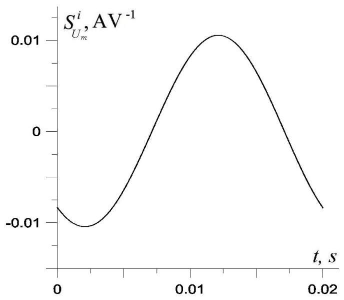

Fig. 2. Steady-state parametric sensitivity of the stator current to the amplitude of voltage

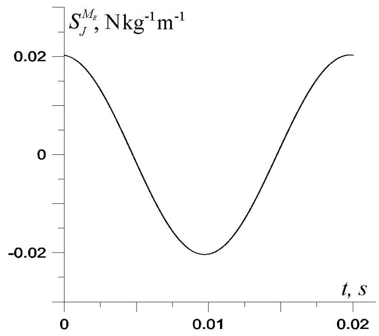

Fig. 3. Steady-state parametric sensitivity of the electromagnetic moment to moment of inertia of the rotor

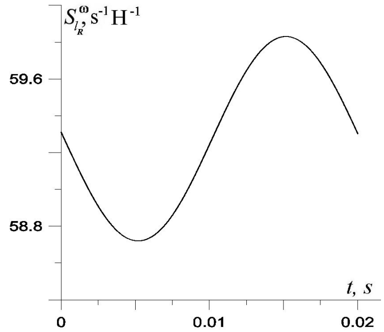

Fig. 4. Steady-state parametric sensitivity of the angular velocity to the inductance dissipation of rotor winding

Below there is a table of full root-mean-square sensitivities (45) of stator current, angular velocity and electromagnetic moment to the corresponding constant parameters

	r_S	r_R	U_m	J	l_R	l_S
i_{SA}	4.9	0.9	7.4E-03	1.4E-04	19.1	378.6
ω	1.2	2.2	1.2E-03	3.2E-04	59.2	46.9
M_E	37.3	1.9	5.1E-02	1.4E-02	2085.6	2117.5

Conclusion.

If the calculation of steady-state processes of electrical machines reduced to two-point boundary value problem for ordinary differential equations of the electromechanical state, then built on this basis algorithms allow to count transient and steady-state processes, static stability and parametric sensitivity on a common mathematical basis of the general theory of non-linear ordinary differential equations. The proposed algorithm is effective method of analysis of transient and steady-state processes in problems of electromechanics. It makes it possible to obtain desired results with predetermined accuracy. That will not provide the methods of timeless space.

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