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TRILHA PRINCIPAL

Two Examples of Application of TOPSIS to Decision Making Problems

Renato A. Krohling, & Talles T.M. de Souza

***Abstract* — In this work, we apply the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) to two case studies. The first example is concerned with the evaluation of the best response alternatives in case of accident with oil spill in the sea. The second example considers the rental evaluation of residential properties. Simulations results are promising and show the feasibility of the technique.**

***Keywords* — Decision making, multi-criteria analysis, TOPSIS.**

I. Introduction

Multicriteria decision making problems are usually characterized by a finite number of alternatives and by several criteria (attributes) that are usually conflicting and a vector of weights that indicates the importance of each criterion. Lots of effort was applied to the development of several methodologies to help solve the multicriteria decision making problem, with several important advances being made along the way.

A technique for decision making that is widely used is known as TOPSIS (Technique for Order Preference by similarity to Ideal Solution), which is a used to evaluate de performance of alternatives through their similarity to the ideal solution [7]. According to this technique, the best alternative is the one closest to the positive ideal solution and most distant from the negative ideal solution [2, 6].

The ideal positive solution is the one that maximizes benefit criteria and minimizes cost criteria, while the ideal negative solution is the one that maximizes cost criteria and minimizes benefit criteria. That means that the positive ideal solution is made of the highest possible values at all benefit criteria, while the negative ideal solution is made up of the highest possible values in the cost criteria.

For the last years, TOPSIS has been applied to decision making problems in Brazil. A methodology to evaluate the effect of urbanization on water beds being aggregated through TOPSIS and ELECTRE TRI was proposed in [1]. In [5] TOPSIS was used to evaluate the sanitation services in the Velhas River basin in MG. In [10] a method of prioritizing surveillance actions in the presence of pesticides in superficial waters was applied in some regional basins. In this case, TOPSIS was used to select the hierarchy of basins with higher contamination potential. In [11] TOPSIS was used in decision support in the choice of transport route for grain production. In [14, 15] a methodology to guide strategic decisions on the choice of highest priority projects in Brazil's transport infrastructure using TOPSIS was proposed. In [8, 9] fuzzy TOPSIS was applied to the problem of oil spill in the southern shore of ES and in [16] it was used for the selection of hedge fund managers.

In this article our goal is to apply the TOPSIS technique in two case studies: i) to evaluate the best alternatives for contention in case of oil spills in Espírito Santo's southern shores. ii) for the selection of best alternatives in rental evaluation of residential properties. The rest of this article is organized as follows: in section 2 we briefly describe TOPSIS. Results for both studies are presented in section 3, while conclusions and future directions finalize the article in section 4.

II. Multicriteria Decision Making

The decision matrix A is made of alternative and criteria is described by:

$$A = \begin{matrix} & \begin{matrix} C_1 & \dots & C_n \end{matrix} \\ \begin{matrix} A_1 \\ \dots \\ A_m \end{matrix} & \begin{pmatrix} x_{11} & \cdots & x_{1n} \\ \vdots & \ddots & \vdots \\ x_{m1} & \cdots & x_{mn} \end{pmatrix} \end{matrix} \quad (1)$$

R.A. Krohling is a professor at the Production Engineering Department and Adjunct Professor at the Graduate Program in Computer Science (PPGI) at the Federal University of Espírito Santo - UFES, Av. Fernando Ferrari, s/n. Prédio CT VII CEP 29060-270, Vitória, ES (e-mail: krohling.renato@gmail.com).

T.T.M. de Souza is an undergraduate student at the Computer Engineering course at the Computer Science Department of UFES, Av. Fernando Ferrari, s/n. Prédio CT VII CEP 29060-270, Vitória, ES (e-mail: talles@gmail.com).

where $A_1, A_2, ..., A_m$ are feasible alternatives, , $C_1, C_2, ..., C_n$ are criteria and x_{ij} indicates the performance of alternative A_i according to criterion C_j. The weight vector $W = (w_1, w_1, ..., w_n)$ is composed by the individual weights for every criterion C_j is held to $\sum_{i=1}^{n} w_j = 1$. Data from matrix A come from different sources and therefore they must be normalized so that they can be transformed into a non dimensional matrix and the comparison among criteria is possible. In this work, the matrix A is normalized for each criterion C_j according to the following formula:

$$p_{ij} = \frac{x_{ij}}{\sum_{i=1}^{m} x_{ij}}, \text{com } i = 1, ..., m, \ j = 1, ..., n \tag{1}$$

This way, the normalized decision matrix A_n represents the relative performance of the alternatives and can be described by $A_n = (p_{ij})_{mxn}$, com $i = 1, ..., m$, e $j = 1, ..., n$.

Now, we describe the multicriteria decision making technique TOPSIS.

A. TOPSIS (Technique for Order Preference by Similarity to Ideal Solution)

Generally, the evaluation criteria can be classified into two groups: *benefit* and *cost*. Benefit means that a higher value is best, while for cost, the opposite is held true. The algorithm to calculate the best alternative according to TOPSIS [6] is described by the following steps:

Step 1: Calculate the positive ideal solutions A^+ (benefits) and he negative ideal solutions A^- (costs) according to the following formula:

$$A^+ = (p_1^+, p_2^+, ..., p_m^+) \tag{2}$$

$$A^- = (p_1^-, p_2^-, ..., p_m^-) \tag{3}$$

where

$$p_j^+ = \left(\max_i p_{ij}, j \in J_1; \min_i p_{ij}, j \in J_2\right) \tag{4}$$

$$p_j^- = \left(\min_i p_{ij}, j \in J_1; \max_i p_{ij}, j \in J_2\right) \tag{5}$$

where J_1 and J_2 represent the criterion benefit and cost, respectively.

Step 2: Calculate the Euclidean distances between A_i and A^+ (benefits) and between A_i and A^- (cost) using the following formula:

$$d^+ = \sqrt{\sum_{j=1}^{n} w_j (p_j^+ - p_{ij})^2} \quad \text{com } i = 1, ..., m. \tag{6}$$

$$d^- = \sqrt{\sum_{j=1}^{n} w_j (p_j^- - p_{ij})^2} \quad \text{com } i = 1, ..., m. \tag{7}$$

Step 3: Calculate the relative closeness ξ_i for each alternative A_i in relation to the positive ideal solution A^+ using:

$$\xi_i = \frac{d_i^-}{d_i^+ + d_i^-}. \tag{8}$$

In the next section we apply this technique to two case studies.

III. Experimental Results

A. Case Study 1: Oil spill in the southern shore of Espírito Santo

The first case study considers a problem involving a possible accident causing an oil spill in the sea at the southern shore of the state of Espírito Santo [3]. The decision matrix adopted from [8, 9] shown in Table 1 is composed by 10 alternatives and 2 criteria, C_1 being a cost criterion and C_2 a benefit criterion, according to the terminology used in TOPSIS. We normalized the decision matrix as described in the previous section. The weight vector is $W = (w_1, w_2) = (0.5107, 0.4893)$ [8]. The result of the application of TOPSIS is shown in Table 2. As we can see, alternative 8 represented the best strategy according to TOPSIS.

B. Case Study 2: Rental evaluation of residential properties

The second case study considers a problem involving the evaluation of rental prices in the city of Volta Redonda, RJ. The information on the decision matrix presented here are available in [4, 13]. The decision matrix A in Table 3 is made of 15 alternatives and 8 criteria. The normalized decision is shown in Table 4. The weight vector is listed in Table 5. The results of the application of TOPSIS are shown in Table 6. As we can see, the alternative 5 is the best one according to TOPSIS.

A recent study on real state evaluation using a hybrid approach combining TODIM and neural networks was proposed in [12]. Similarly, TOPSIS could also be used, for it is also based in distance metrics, i.e., the best alternative is the one closer to the positive ideal solution and farther away from the negative ideal solution.

IV Conclusions

In this paper we applied the decision support technique known as TOPSIS (*Technique for Order Preference by Similarity to Ideal Solution*) to two case studies: i) to evaluate the best alternatives for contention in case of oil spills in Espírito Santo's southern shores. ii) for the selection of best alternatives in rental evaluation of residential properties.

In both case studies investigated, we could verify the feasibility of TOPSIS, which is very effective and very simple in terms of computational implementation. One of its main advantages is the fact that it does not require parameter tuning. If the decision matrices with uncertainties are described by fuzzy numbers, then *fuzzy TOPSIS* [9] might be used to solve those problems.

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REFERÊNCIAS

[1] Castro, L.M.A. de. Proposição de metodologia para avaliação dos efeitos da urbanização nos corpos de água. Tese de Doutorado em Saneamento, Meio Ambiente e Recursos Hídricos. Universidade Federal de Minas Gerais, Minas Gerais, 2007.

[2] Ertuğrul, I., & Karakaşoğlu, N. Performance evaluation of Turkish cement firms with fuzzy analytic hierarchy process and TOPSIS methods. *Expert Systems with Applications*, 36(1), pp. 702-715,2009.

[3] Ferreira, J.P. Análise de estratégias de resposta a derramamento de óleo pesado no litoral do Espírito Santo utilizando modelagem computacional, Dissertação de Mestrado, Programa de Pós-Graduação em Engenharia Ambiental, Universidade Federal do Espírito Santo, Espírito Santo, 2006.

[4] Gomes, L.F.A.M. & Rangel, L.A.D. An application of the TODIM method to the multicriteria rental evaluation of residential properties, *European Journal of Operational Research* ,193, 204-211, 2009.

[5] Heller, P.G.B. Avaliação dos serviços de saneamento de quatro municípios da bacia hidrográfica do Rio das Velhas/MG. Uma abordagem da dimensão tecnológica. Dissertação de Mestrado em Saneamento, Meio Ambiente e Recursos Hídricos da Escola de Engenharia. Universidade Federal de Minas Gerais. Minas Gerais, 2007.

[6] Huang, J. Combining entropy weight and TOPSIS method for information system selection. In *Proceedings of the IEEE Conference on Cybernetics and Intelligent Systems, CIS* 2008, pp. 1281-1284, 2008.

[7] Hwang, C. L. & Yoon, K. P. *Multiple attributes decision making methods and applications*. Berlin: Springer-Verlag, 1981.

[8] Krohling, R.A. & Campanharo, V.C. Fuzzy TOPSIS para tomada de decisão multicritério: Uma aplicação para o caso de acidentes com derramamento de óleo no mar, Anais do *XLI SBPO - Simpósio Brasileiro em Pesquisa Operacional, SOBRAPO*, Porto Seguro, BA, de 1-4 de setembro, 2009.

[9] Krohling, R.A. & Campanharo, V.C. Fuzzy TOPSIS for group decision making: A case study for accidents with oil spill in the sea. *Expert Systems with Applications*, 38(4), pp. 4190-4197, 2011.

[10] Menezes, C.T. Métodos para priorização de ações de vigilância da presença de agrotóxicos em águas superficiais: Um estudo em Minas Gerais.Dissertação de Mestrado em Saneamento, Meio Ambiente e Recursos Hídricos da Escola de Engenharia. Universidade Federal de Minas Gerais. Minas Gerais, 2007.

[11] Miranda, L.M. de. Contribuição a um modelo de análise multicritério para apoio à decisão da escolha do corredor de transporte para escoamento da produção de granéis agrícolas de Mato Grosso. Tese de Doutorado em Engenharia de Transportes. Universidade Federal do Rio de Janeiro, Rio de Janeiro, 2008.

[12] Moreira, D.S., Silva, R.S. & Fernandes, A.M.R. Engenharia de avaliações de imóveis apoiada em técnicas de análise multicritério e redes neurais artificiais, *Revista de Sistemas de Informação da FSMA* , n.6, pp. 49-58, 2010.

[13] Rangel, L.A.D & Gomes, L.F.A.M. Avaliação multicritério de imóveis residenciais: Uso combinado dos métodos UTA-CR e TODIM. *Investigación Operativa*, v. XVII, p. 113-127, 2009.

[14] Silva, R.B da. Análise estruturada das dimensões do desenvolvimento sustentável e a interdependência com a função transporte: Proposta de metodologia para orientar decisões estratégicas na implantação de projetos prioritários para a infraestrutura de transporte no Brasil. Tese de Doutorado em Engenharia Oceânica. Universidade Federal do Rio de Janeiro – COPPE/UFRJ, Rio de Janeiro, 2008.

[15] Silva, R.B. da & Cavalcanti Netto, M.A. Uma estrutura de apoio à decisão para orientar a escolha de projetos prioritários para a infraestrutura de transporte no Brasil, Anais do *XLII SBPO - Simpósio Brasileiro em Pesquisa Operacional, SOBRAPO*, Bento Gonçalves, RS, de 30 de Agosto – 03 de Setembro, 2010.

[16] Valladares, C.F.G. O processo para seleção de gestores de fundos de investimento utilizando o método multicritério TOPSIS. Dissertação de Mestrado Profissionalizante em Administração. Faculdades Ibmec RJ, 2011.

Table 1. Decision Matrix

Alternatives	C_1 (x10^3)	C_2 (x10^3)
A1	8.627	5.223
A2	9.838	4.023
A3	10.374	3.495
A4	8.200	5.659
A5	5.854	7.989
A6	8.108	5.790
A7	6.845	7.083
A8	5.738	8.238
A9	5.858	8.189
A10	6.269	7.808

Table 2. Alternative Ordering

	ξ_i	Classification
A1	0.3695	8
A2	0.1131	9
A3	0.0000	10
A4	0.4615	7
A5	0.9568	3
A6	0.4859	6
A7	0.7584	5
A8	**1.0000**	**1**
A9	0.9816	2
A10	0.8989	4

Table 3. Decision Matrix [4]

Alternatives	Criteria							
	C1	C2	C3	C4	C5	C6	C7	C8
A1	3	290	3	3	1	6	4	0
A2	4	180	2	2	1	4	2	0
A3	3	347	1	2	2	5	1	0
A4	3	124	2	3	2	5	4	0
A5	5	360	3	4	4	9	1	1
A6	2	89	2	3	1	5	1	0
A7	1	85	1	1	1	4	0	1
A8	5	80	2	3	1	6	0	1
A9	2	121	2	3	0	6	0	0
A10	2	120	1	3	1	5	1	0
A11	4	280	2	2	1	7	4	1
A12	1	90	1	1	1	5	2	0
A13	2	160	3	3	2	6	1	1
A14	3	320	3	3	2	8	2	1
A15	4	180	1	4	1	6	1	1

Table 4. Normalized decision matrix [4]

Alternatives	Criteria							
	C1	C2	C3	C4	C5	C6	C7	C8
A1	0.068	0.103	0.1	0.075	0.045	0.069	0.174	0
A2	0.091	0.064	0.067	0.05	0.045	0.046	0.087	0
A3	0.068	0.123	0.033	0.05	0.091	0.057	0.043	0
A4	0.068	0.044	0.067	0.075	0.091	0.057	0.174	0
A5	0.114	0.127	0.1	0.1	0.182	0.103	0.043	0.143
A6	0.045	0.031	0.067	0.075	0.045	0.057	0.043	0
A7	0.023	0.03	0.033	0.025	0.045	0.046	0	0.143
A8	0.114	0.028	0.067	0.075	0.045	0.069	0	0.143
A9	0.045	0.043	0.067	0.075	0	0.069	0	0
A10	0.045	0.042	0.033	0.075	0.045	0.057	0.043	0
A11	0.091	0.099	0.067	0.05	0.091	0.08	0.174	0.143
A12	0.023	0.032	0.33	0.025	0.045	0.057	0.087	0
A13	0.045	0.057	0.1	0.075	0.091	0.069	0.043	0.143
A14	0.068	0.113	0.1	0.075	0.091	0.092	0.087	0.143
A15	0.091	0.064	0.033	0.1	0.045	0.069	0.043	0.143

Table 5. Weight for each criterion [4]

Criteria	Weights	Normalized Weights
C1	5	0.25
C2	3	0.15
C3	2	0.10
C4	4	0.20
C5	1	0.05
C6	2	0.10
C7	1	0.05
C8	2	0.10

Table 6. Alternative Ordering

	ξ_i	Classification
A1	0.5414	7
A2	0.3432	10
A3	0.3851	9
A4	0.4547	8
A5	**1**	**1**
A6	0.1364	12
A7	0.2609	11
A8	0.5627	5
A9	0.0896	14
A10	0.1200	13
A11	0.8209	2
A12	0	15
A13	0.5565	6
A14	0.8139	3
A15	0.6582	4