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TRILHA PRINCIPAL

Moving Mesh for the Numerical Solution of Partial Differential Equations

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Abstract. In this paper, we present moving meshes for the numeric resolution of partial differential equations. We describe some important concepts on this topic and point to existing body of work for the solution of partial differential equations using the methods of finite volumes and finite elements, both with moving meshes.

***Keywords* — mesh generation; adaptive mesh refining; moving meshes.**

I. INTRODUCTION

Generally, systems development in engineering require using CAD tools (*computer-aided design*). In those tools, computational simulation techniques are frequently used to model and investigate physical phenomena in different areas of science. According to Budd, Huang e Russell (2009), examples of those phenomena occur in several application, as in fluid and gas dynamics, conservative laws, non linear optics, combustion, detonation, meteorological forecasting models, pollution studies in rivers, oceans and atmosphere, thermodynamic, electromagnetism and aerodynamic models and also in the prospecting and extraction of oil. Usually, these phenomena are modeled by partial differential equations (PDEs); for instance, Liu (2003) studied several problems related to the area of solid mechanics, structures fluid flux and their mathematical models.

Computational simulation techniques are often used to investigate those phenomena. Usually, the numerical simulation of physical phenomena involves two main parts: the mathematical modeling that describes the phenomenon in which we are interested and the implementation of numerical techniques for the mathematical model to be computationally solved.

One way to find an approximate solution of partial differential equations is by discretize the PDE using a numerical method. Among the many existing methods, we mention the finite elements and the finite volume methods. In those methods, we solve the differential equations by replacing the terms by algebraic expressions that involve the unknown function. When performing a numerical approximation, the solution is found for a discrete number of points with a certain error deriving from the approximation. If the method converges, the numerical solution will be closer to the exact solution as the number of points increases. We can consider this *mesh* of points and its connections as the domain discretized geometrically.

Phenomena models can be created using partial differential equations which in many cases have great variation in small regions of the domain under study. In order to decrease the error in those regions of great variation, we refine the mesh. Using a fine and uniform mesh throughout the whole domain will greatly increase the computational cost, increasing excessively the number of points also in regions where this refinement is not necessary. An alternative is to position a large amount of point in the regions of the mesh of great variation and a smaller set of points in the regions of smaller variation. With this *adaptive refinement*, the total number of points is much smaller than in the situation of a uniform mesh. Nevertheless, as Huang e Russell (2011) explain, adaptive refinement cannot be used as a panacea. For problems with smoother variations in the solution, we should rather use a uniform mesh instead of a non-uniform one because using a uniform mesh is possible to find a solution as efficient as using a non-uniform mesh.

According to Olivier e Alauzet (2011), in the last decades numerical simulation had an important role in science and in engineering due to the development of high speed computers and to the advancement of storage capacity in computer systems. These advancements allowed for the simulations to achieve good results both in terms of precision and performance. Achieving those results means to solve complex differential equations that model complex physical problems whereas trying to minimize the errors that are inherent to a numerical approximation. Refining the mesh is a way to minimize those inherent errors in the numerical approximation but even with adaptive refinement, the inclusion of new point increases the computational effort for solving the problem when compared to the previous iteration that had less points. Hence, for systems with relevant dimensions, the techniques used must be sufficiently efficient in order to allow precise results with low computational cost.

Several mesh adaptation techniques were developed for adaptive refinement, which include new points inclusion and mobile meshes, which is based on the movement of points from the initial mesh. As we already described, in the adaptation through refinement, we insert new points in the mesh in the regions where the error is great or the solution gradient is very steep. In the adaptation with moving meshes, the original amount of points is kept the same, but we move the points to the regions of higher variation in the solution. Keeping the amount of points in the mesh is beneficial since the inclusion of points increases the computational cost to solve the problem, as stated. In this text we describe mesh adaptation to the case where its application is needed and we focus on moving meshes.

In section II, we present important concepts on mesh generation. In section III, we describe some works on moving meshes and in section IV, we address some final considerations.

II. Some Concepts on Mesh Generation

In this context, the approach of adaptation for the numerical solution of partial differential equations can be divided into three categories: the *h*-refinement, the *p*-refinement and the *r*-refinement.

In the *h*-refinement approach, we begin the simulation with an initial mesh and this mesh is refined or simplified by including or removing points. Usually, the strategy to include or remove points is guided by a *a posteriori* estimation of the solution error. This is usually called as adaptive mesh refinement in the finite volume method community.

In the *p*-refinement approach, Budd, Huang e Russell (2009) explain that we discretize the PDEs using finite elements with polynomials of a specific order, which is increased or decreased according to a *a posteriori* estimation of the solution error. With this combination, we find the *hp*-refinement subcategory, whose goal is to find the solution within a specific error limited by the refinement procedures (BUDD; HUANG; RUSSELL, 2009).

In the *r*-refinement, the number of points in the mesh is fixed and those points are moved in order to be concentrated in the regions of greater variation of the solution as a function of time. In the community of finite volumes, they are usually called moving meshes. According to Eleftheriou (2011), this refinement approach can be in general used to transient problems because the mesh mobility makes it easier to deal with time integrators. Nevertheless, its limitation is in the difficulty to define an adequate time interval, due to the fact that the nodes vary their position with time, which may cause mesh tangling. Besides, the applicability of the *r*-refinement is limited to to the fixed number of degrees of freedom and to a constant connectivity of the mesh polygons. Because of that, *r* adaptation is typically used to speed the computational process instead of being used to find a specific precision. Please refer to Askes (2000) for details on this subject.

According to Huang and Russell (2011), the methods that use moving meshe are still on an early development phase. Many of them are in the experimental stage and almost all of them require additional mathematical justification. As those authors also point out, a rigorous analysis of the moving meshes methods to solve PDEs that are time dependent was performed only on some very simple models and hence there will probably be many ways to be developed in order to improve their efficiency and robustness of those models. For instance, we still need more systematic numerical studies on how to decrease the cost of the solution of a whole mesh and PDEs, as well as studies on how to balance the spatial and time adaptation of a mesh.

Huang and Russell (2011) also explain that an important factor in the methods of moving meshes is the adequate choice of a mesh density function, which controls the concentration of points in the mesh according to the principle of equidistribution and typically measures the difficulty of the numerical approximation of the problem being solved. According to those authors, the selection of the mesh density function can be based on the estimation of the interpolation error *a posteriori* with the optimal limit for the interpolation error or on the solution error, which is obtained through the corresponding equidistributed mesh.

The principle of equidistribution was originally presented by Boor (1973). According to this principle, we seek to rearrange the nodes of a one-dimensional mesh so that a certain metric is equally distributed along each sub-interval of the mesh. This metric can be, for instance, a measure of the error which will be compared to the measure of a desirable element, hypothetically optimal. The difference between each element in the mesh and the desirable element will be, approximately the same for all existing elements.

According to Askes (2000), some topological restrictions such

as non convex corners in multidimensional problems may prevent a ideal point movement. Hence, we cannot guarantee that equidistribution will be satisfied for all the points in the mesh. Admit that the position of node x_i, for $i=1..N$, in which N is the number of nodes, be defined in order to guarantee that a certain metric, called weight function or monitor function $M(x)$, be distributed equitably along the domain. This distribution is made according to the formula $\int_{x_{i-1}}^{x_i} M(x)dx = \int_{x_i}^{x_{i+1}} M(x)dx, 2 \le i \le N-1$, whose discrete approximation is given by

$$M_{i-1}\,\delta x_{i-1} = M_i\,\delta x_i, 2 \le i \le N-1, \quad (1)$$

In which $\delta x_{i-1} = x_i - x_{i-1}$ is the local size of the mesh and M_{i-1} represents a discrete estimation of $M(x)$ in the interval $[x_{i-1}, x_i]$.

Among the first applications of the equidistribution principle, we mention the works of Dwyer, Sanders and Kee (1979), Dwyer, Raiszadeh and Otey (1981), Gnoffo (1980) and White (1982), who applied it to solve problems in fluids mechanics and heat transfer in one dimension. White (1982) used arc length as monitor function,

$$M(u) = \sqrt{1+|u'|^2}. \quad (2)$$

Consider an example that will help to understand the main idea behind the equidistribution principle. Let us use the function $f(x) = \tanh\left(\frac{1-x}{0,1}\right)$. Consider a subset in the interval $[0,1]$ and suppose that $x_0 < x_1 < \ldots < x_n$, in which $x_0 = 0$ and $x_n = 1$. In this context, in the graphic on the left of figure (1), the mesh is divided uniformly and in the graphic on the right of the same figure, we generate the mesh according to the principle of equidistribution. In this case, the monitor function was based on arc length, expressed by (2) and the points are distribute equally in the curve, satisfying (1). Similar examples can be found in Zegeling (1996), Li, Tang and Zhang (2000) and Tan (2005).

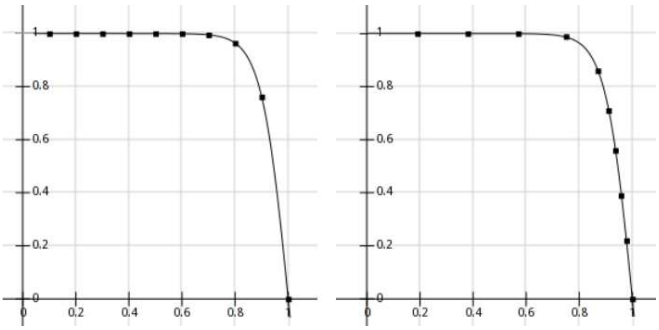

Figure 1: Comparison between the principle of equidistribution using arc length (on the right) and a mesh uniformly divided (on the left) with 10 points. Example adapted from Tan (2005).

In several works (see, for instance, Huang and Russell (2011) and its references), the equidistribution principle (BOOR, 1973) plays a fundamental part in the project of the mesh movement strategies for the one-dimensional case. Clearly, the adaptation in multidimensions is much more complicated that the problem in a single dimension. Huang and Russell (2011) explain that the equidistribution principle, by specifying only the volume of the mesh elements is not complete enough to determine a multidimensional mesh. An additional alignment condition is necessary to specify the geometry and the orientation of the mesh elements. Huang and Russell (2011) present the basic principles of multidimensional mesh adaptivity, including the necessary alignment condition.

In multidimensional meshes, adaptivity is guided by a monitor function that is solution dependent, namely, it is a symmetric positive-definite matrix function naturally related to a mesh density function in one dimension. See also Huang and Rusell (2011) for details on this subject. The monitor function defines a metric on the physical domain. According to Marlow (201), the idea of a monitor function is to convert the vague concept of "moving the mesh points to where activity is greater" into a mathematical solid procedure or, at least, into something that can be numerically quantified. According to Huang and Russell (2011), the monitor function is always a non-negative function.

Multidimensional mesh adaptation, according to Huang and Russell (2011) can be understood as a technique to create a uniform mesh in a certain metric space. This technique specifies the size, the geometry and the orientation of the mesh elements. It also provides a natural control of the equidistribution and of the alignment, also having the role of ensuring that the mesh is duly aligned with the behavior of the physical solution. Huang and Russell (2011) analyze and relate the quality of the mesh (in the mathematical precision sense) to this equidistribution and to the alignment condition. These authors present some interpretations in the discrete perspective using meshes and in the continuous one, using coordinate transformations.

According to Huang and Russell (2011), the problem of computing solutions of PDEs using moving meshes can be split into the following three problems.

- A mesh density function, also called monitor function, is necessary to guide the redistribution of the mesh points during the evolution of the PDE. This monitor function usually is restricted both to equidistribute this point movement and to find a mesh relaxation in the search of the equidistributed state. The choice of a monitor function may depend on the arc length of the solution in one-dimensional problems, in the solution curvature and in errors calculated *a posteriori*.
- Once determined the monitor function, we must verify a mesh that is equidistributed in some way. The equidistribution problem by itself is an algebraic non linear problem.
- The PDE is then discretized both in the computational

domain of the mesh and in the original physical domain and usually finite elements or finite volumes are used.

Practically, whatever the choice of monitor function, we must use some smoothing or spatial and time relaxation in order to improve the quality of the mesh, decreasing the distortion of the elements. See Huang and Russell (2011) for explanations on how to find an optimal monitor function for a certain limit of error interpolation or an *a posteriori* estimation or error and a monitor function based on other physical or geometrical considerations. A monitor function based on interpolation error can be found using the Taylor polynomial approximation for one dimension and for the Sobolev space approximation for multidimensions.

Many monitor functions include solution derivatives and in order to calculate those derivatives we perform approximations. Through this way, we define a monitor function *a posteriori.*

According to Huang and Russell (2011), monitor functions based on physical and geometrical considerations must take into account the distance or the area between the interfaces and can use a mesh as parameter, adapting the new mesh in order to come as close as possible to the reference mesh. The interpolation error treatment in the generalized Sobolev spaces, both in isotropic and anisotropic meshes, offers a result that shows how the choice of the optimal monitor function takes to an error that is delimited by an optimal solution. This error depends on a factor whose order is $1/n$, in which n is the number of elements in the mesh (HUANG; RUSSELL, 2011).

Interpolation error limits with non optimal and optimal monitor functions for a *a posteriori* error limit are also described by Huang and Russell (2011), which also presents several practical aspects of the computation of monitor functions.

Huang and Russell (2011) also present several PDEs for moving meshes (*moving mesh partial differential equations*, MMPDEs) for time dependent problems. They also develop several mesh equation for stationary state problems by using the equidistribution principle. MMPDEs are continuous versions of the mesh moving strategies formulated in terms of coordinate transformations. They also approaches practical questions of implementation, including the discretization of mesh equations, physical PDEs and the proceedings of a global solution.

Huang and Russell (2011) also approach the questions related to mesh adaptation in the multidimensional context, which is a quite challenging topic. For high dimensional spaces, we need advanced calculus tools to transform PDEs from the physical space into the computational space.

III. WORKS ON MOVING MESHES

Several moving meshes methods were developed and applied for one and two dimensions in the resolution of several problems, as we can see, for example in Tang (2005) reviewed the techniques for moving meshes and their applications in computational fluid dynamics. We will now comment some works with moving meshes.

Cao, Huang and Russell (2001) studied several error indicators for the finite elements method. These authors analyzed the error indicator based on the solution gradient, on the interpolation errors and in *a posteriori* error estimation in order to define the monitor function. Huang (2001) introduced the concepts of spatial balance and scale invariance and studied how to build PDE with moving meshes with some desirable properties.

Huang and Sun (2003) used finite elements methods interpolation theory to estimate errors. Liu and Shen (2003) proposed a Fourier spectral method to treat a phase field problem for the mixture of two incompressible fluids.

Zegeling (2004) discussed an adaptive mesh method based on a tensorial product approach. Zegeling (2005) described an adaptive moving mesh technique and its application in magnetic and hydrodynamics convection-diffusion models.

Liu, Qin and Xia (2006) proposed a simple and efficient technique of mesh dynamic deformation to calculate unstable flux problems with geometric deformation, relative body movement or shape variation due to aerodynamics optimization and the interaction between the fluid and the structure.

Tan (2007) applied an adaptive moving mesh technique with quadrangular meshes to the resolution of magnetic-hydrodynamic problems. Tan, Lim and Khoo (2007) developed a phase field model for the mixture of two incompressible fluids using quadrangular meshes and the equations of Navier-Stokes and Allen-Cahn.

Marlow (2010, 2011) described an adaptive method to solve parabolic non-linear PDEs with moving frontiers using moving meshes with continuous finite elements. McNally, Lyra and Passy (2012) compared the results from different codes to the solution of the Kelvin-Helmholtz instability problem.

Several studies, such as Mackenzie (1996), Dam and Zegeling (2006), Tan et al. (2004), Tan, Tang and Zhang (2006), Springel (2005, 2009, 2011), involving discretization by finite volumes, used moving meshes to solve PDEs. A large part of those researches was applied to the solution of phenomenal with relatively low computational effort and high approximation precision, in order to build adaptable meshes to the solution of the phenomenon under study.

Besides, several other methods, specially multidimensional ones, were developed and used successfully. For example, see Beni, Mostafavi and Pouliot (2008), Greif et al. (2011), Heß and Springel (2010, 2012), Pakmor, Bauer and Springel (2011) and Muñoz et al. (2012).

IV. Final Considerations

In this text, we introduced moving meshes and approached different types of mesh adaptation, with emphasis on the moving meshes methods. We also described the equidistribution principle with the presentation of a one-dimensional example. In the end, we mentioned several works on moving meshes that were recently developed.

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Refernces

1. ASKES, H. Advanced Spatial Discretisation Strategies for Localised Failure. Mesh Adaptivity and Meshless Methods. Tese (PhD thesis) — Technische Universiteit Delft, Delft – Nederlands, may 2000. Available at: <http://repository.tudelft.nl/view/ir/uuid%3A1f6cd5e6-08ad-4f0f-ba40-ea51ee0f4c16/>. Accessed: 23/04/2013.
2. BENI, L. H.; MOSTAFAVI, M. A.; POULIOT, J. Dynamic field process simulation within gis: the voronoi approche. The International Archives of the Photogrammetry, Remote Sensing and Spatial Information Sciences, Department of Geomatics, Laval University, Quebec, Canada, v. 37, p. 891–898, 2008. Available at: <http://www.isprs.org/proceedings/XXXVII/congress/2_pdf/8_ICWG-II-IV/01.pdf>. Accessed: 23/04/2013.
3. BOOR, C. de. Good approximation by splines with variables knots II. Springer Lecture Notes Series, Springer-Verlag, Berlin, v. 363, 1973. Available at: <ftp://ftp.cs.wisc.edu/Approx/goodappr.pdf>. Accessed: 30/04/2013.
4. BUDD, C. J.; HUANG, W.; RUSSELL, R. D. Adaptivity with moving grids. Acta Numerica, v. 18, p. 111–241, 2009.
5. CAO, W.; HUANG, W.; RUSSELL, R. D. Comparison of two-dimensional r-adaptive finite element methods using various error indicators. Mathematics and Computers in Simulation, v. 56, n. 2, p. 127–143, 2001. ISSN 0378-4754. <ce:title>Method of lines</ce:title>. Available at: <http://www.sciencedirect.com/science/article/pii/S0378475401002853>. Accessed: 23/04/2013.
6. DAM, A. van; ZEGELING, P. A. A robust moving mesh finite volume method applied to 1D hyperbolic conservation laws from magnetohydrodynamics. Journal of Computational Physics, Academic Press Professional, Inc., San Diego, CA, USA, v. 216, n. 2, p. 526–546, August 2006. ISSN 0021-9991. Available at: <http://dx.doi.org/10.1016/j.jcp.2005.12.014>. Accessed: 23/04/2013.
7. DWYER, H.; RAISZADEH, F.; OTEY, G. A study of reactive diffusion problems with stiff integrators and adaptive grids. In: REYNOLDS, W.; MACCORMACK, R. (Ed.). Seventh International Conference on Numerical Methods in Fluid Dynamics. Springer Berlin Heidelberg, 1981, (Lecture Notes in Physics, v. 141). p. 170–175. ISBN 978-3-540-10694-4. Available at: <http://dx.doi.org/10.1007/3-540-10694-4_24 >.
8. DWYER, H.; SANDERS, B.; KEE, R. An adaptive grid method for problems in fluid mechanics and heat transfer. In: . Williamsburg, Virginia: American Institute of Aeronautics and Astronautics, 1979. p. 195–203. Available at: <http://opac.inria.fr/record=b1070874>.
9. ELEFTHERIOU, T. Moving Mesh Methods using monitor functions for the Porous Medium Equation. Tese (PhD em Matemática) — University of Reading, 2011.
10. GNOFFO, P. Complete supersonic flowfields over blunt bodies in a generalized orthogonal coordinate system. American Institute of Aeronautics and Astronautics, v. 18, n. 6, p. 611–612, 1980.
11. GREIF, T. H.; SPRINGEL, V.; WHITE, S. D. M.; GLOVER, S. C. O.; CLARK, P. C.; SMITH, R. J.; KLESSEN, R. S.; BROMM, V. Simulations on a moving mesh: The clustered formation of population III protostars. The Astrophysical Journal, v. 737, n. 2, p. 75, 2011. Available at: <http://stacks.iop.org/0004-637X/737/i=2/a=75>. Accessed: 23/04/2013.
12. Heß, S.; Springel, V. Particle hydrodynamics with tessellation techniques. Monthly Notices of the Royal Astronomical Society, v. 406, p. 2289–2311, August 2010. Available at: <http://adsabs.harvard.edu/abs/2010MNRAS.406.2289H>. Accessed: 23/04/2013.
13. Heß, S.; SPRINGEL, V. Gas stripping and mixing in galaxy clusters: A numerical comparison study. Monthly Notices of the Royal Astronomical Society, v. 426, p. 3112–3134, November 2012. Available at: <http://arxiv.org/abs/1208.0351>. Accessed: 23/04/2013.
14. HUANG, W. Practical aspects of formulation and solution of moving mesh partial differential equations. Journal of Computational Physics, v. 171, n. 2, p. 753–775, 2001. ISSN 0021-9991. Available at: <http://www.sciencedirect.com/science/article/pii/S0021999101968093>. Accessed: 23/04/2013.
15. HUANG, W.; RUSSELL, R. D. Adaptive moving mesh methods. New York: Springer, 2011. (Applied mathematical sciences). ISBN 978-1-4419-7915-5.
16. HUANG, W.; SUN, W. Variational mesh adaptation II: error estimates and monitor functions. Journal of Computational Physics, v. 184, n. 2, p. 619–648, 2003. ISSN 0021-9991. Available at: <http://www.sciencedirect.com/science/article/pii/S0021999102000402>. Accessed: 23/04/2013.
17. LI, R.; TANG, T.; ZHANG, P. Moving mesh methods in multiple dimensions based on harmonic maps. Journal of Computational Physics, v. 170, p. 562–588, 2000. Available at: <http://www.math.hkbu.edu.hk/~ttang/moving.html>. Accessed: 23/04/2013.
18. LIU, C.; SHEN, J. A phase field model for the mixture of two incompressible fluids and its approximation by a fourier-spectral method. Physica D: Nonlinear Phenomena, v. 179, n. 3-4, p. 211–228, 2003. ISSN 0167-2789. Available at: <http://www.sciencedirect.com/science/article/pii/S0167278903000307> .
19. LIU, G.-R. Mesh free methods: moving beyond the finite element method. First edition. New York: CRC Press, 2003. ISBN 0-8493-1238-8.
20. LIU, X.; QIN, N.; XIA, H. Fast dynamic grid deformation based on delaunay graph mapping. Journal of Computational Physics, Academic Press Professional, Inc., San Diego, CA, USA, v. 211, n. 2, p. 405–423, jan. 2006. ISSN 0021-9991. Available at: <http://dx.doi.org/10.1016/j.jcp.2005.05.025>. Accessed: 23/04/2013.
21. MACKENZIE, J. Moving Mesh Finite Volume Methods for One-Dimensional Evolutionary Partial Differential Equations. 1996.
22. MCNALLY, C. P.; LYRA, W.; PASSY, J.-C. A well-posed Kelvin-Helmholtz instability test and comparison. The Astrophysical Journal Supplement Series, v. 201, n. 2, p. 18, ago. 2012. Available at: <http://adsabs.harvard.edu/abs/2012ApJS..201...18M>. Accessed: 23/04/2013.
23. MARLOW, R. Moving mesh methods for solving parabolic partial differential equations. Tese (PhD School of Computing) — Universit of Leeds, 2010.
24. MARLOW, R. Moving mesh methods for solving parabolic partial

differential equations. 10th Conference Series on Numerical Methods for Fluid Dynamics, School of Computing, Leeds, v. 46, n. 1, p. 353–361, July 2011.

25. MUÑOZ, D.; SPRINGEL, V.; MARCUS, R.; VOGELSBERGER, M.; HERNQUIST, L. Multi-Dimensional, Compressible Viscous Flow on a Moving Voronoi Mesh. 2012. Available at: <http://arxiv.org/abs/1203.1037>. Accessed: 17/4/2013.
26. OLIVIER, G.; ALAUZET, F. A new changing-topology ale scheme for moving mesh unsteady simulations. In: Proceedings of 49th AIAA Aerospace Sciences Meeting and Exhibit. Gamma Project, INRIA Rocquencourt, 78 153 Le Chesnay, France: AIAA, 2011. (AIAA-2011-0474), p. 1–25. Available at: <https://www.rocq.inria.fr/gamma/gamma/Membres/CIPD/Frederic.Alauzet>. Accessed: 23/04/2013.
27. PAKMOR, R.; BAUER, A.; SPRINGEL, V. Magnetohydrodynamics on an unstructured moving grid. Institut für Theoretische Studien - Heidelberg, Germany, v. 418, n. ArXiv:1108.1792, p. 1392–1401, December 2011. Available at: <http://arxiv.org/abs/1108.1792>. Accessed: 23/04/2013.
28. SPRINGEL, V. The cosmological simulation code GADGET-2. Monthly Notices of the Royal Astronomical Society, v. 364, p. 1105–1134, dez. 2005. Available at: <http://adsabs.harvard.edu/abs/2005MNRAS.364.1105S>. Accessed: 23/04/2013.
29. SPRINGEL, V. E pur si muove: Galiliean-invariant cosmological hydrodynamical simulations on a moving mesh. Monthly Notices of the Royal Astronomical Society, Max-Planck-Institute for Astrophysics, v. 401, p. 791–851, October 2009. Available at: <http://arxiv.org/abs/0901.4107 >. Accessed: 18/04/2013.
30. SPRINGEL, V. Hydrodynamic simulations on a moving Voronoi mesh. Garching, September 2011. v. 1109.2218, n. arXiv:1109.2218, 35 p. Comments: 35 pages, 10 figures; invited review for the volume Tessellations in the Sciences: Virtues, Techniques and Applications of Geometric Tilings, Springer (accepted). Available at: <http://arxiv.org/abs/1109.2218>. Accessed: 23/04/2013.
31. TAN, Z. Moving Mesh Finite Volume Method and its Applications. Tese (Doutorado) — Hong Kong Baptist University, 2005.
32. TAN, Z. Adaptive moving mesh methods for two-dimensional resistive magneto-hydrodynamic PDE models. Computers & Fluids, v. 36, n. 4, p. 758 – 771, 2007. ISSN 0045-7930. Available at: <http://www.sciencedirect.com/science/article/pii/S0045793006000831 >. Accessed: 23/04/2013.
33. TAN, Z.; LIM, K. M.; KHOO, B. C. An adaptive mesh redistribution method for the incompressible mixture flows using phase-field model. Journal of Computational Physics, Academic Press Professional, Inc., San Diego, CA, USA, v. 225, n. 1, p. 1137–1158, July 2007. ISSN 0021-9991. Available at: <http://dx.doi.org/10.1016/j.jcp.2007.01.019>. Accessed: 23/04/2013.
34. TAN, Z.; TANG, T.; ZHANG, Z. A simple moving mesh method for one-and two-dimensional phase-field equations. Journal of Computational and Applied Mathematics, Elsevier Science Publishers B. V., Amsterdam, The Netherlands, v. 190, n. 1, p. 252–269, jun. 2006. ISSN 0377-0427. Available at: <http://dx.doi.org/10.1016/j.cam.2005.01.042>. Accessed: 23/04/2013.
35. TAN, Z.; ZHANG, Z.; HUANG, Y.; TANG, T. Moving mesh methods with locally varying time steps. Journal of Computational Physics, v. 200, n. 1, p. 347–367, 2004. ISSN 0021-9991. Available at: <http://www.sciencedirect.com/science/article/pii/S0021999104001706>. Accessed: 23/04/2013.
36. TANG, T. Moving Mesh Methods for Computational Fluid Dynamics. Hong Kong: Department of Mathematics, Hong Kong Baptist University, 2005. (Technical report).
37. WHITE, A. B. J. On the numerical solution of initial/boundary-value problems in one space dimension. SIAM Journal on Numerical Analysis, Society for Industrial and Applied Mathematics, v. 19, n. 4, p. 683–697, 1982. ISSN 00361429. Available at: <http://www.jstor.org/stable/2157026>.
38. ZEGELING, P. MFE revisited, Part 1: Adaptive Grid-Generation using the Heat Equation. Department of Mathematics, Utrecht University, 1996. Available at: <http://igitur-archive.library.uu.nl/math/2001-0703-151147/946.pdf>. Accessed: 12/06/2013.
39. ZEGELING, P. A. Tensor-product adaptive grids based on coordinate transformations. Journal of Computational and Applied Mathematics, Elsevier Science Publishers B. V., Amsterdam, The Netherlands, v. 166, n. 1, p. 343–360, abr. 2004. ISSN 0377-0427. Available at: <http://dx.doi.org/10.1016/j.cam.2003.09.018>. Accessed: 23/04/2013.
40. ZEGELING, P. A. On resistive MHD models with adaptive moving meshes. Journal of Scientific Computing, Plenum Press, New York, NY, USA, v. 24, n. 2, p. 263–284, ago. 2005. ISSN 0885-7474. Available at: <http://dx.doi.org/10.1007/s10915-004-4618-6>. Accessed: 23/04/2013.