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THE INCREASE IN EFFICIENCY OF INTERACTIVE LEARNING OF MATHEMATICS THROUGH THE IMPLEMENTATION OF MINI EXEMPLARY TEACHING

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Abstract: This paper is concerned with the topic of interactive teaching and learning in the primary mathematical education. The objective is to determine and empirically evaluate the method of increasing the efficiency of such teaching and learning. The theoretical basis for the development of the method consists of a suitable array of research. The quality of the method was statistically confirmed through empirical research which involved working with uniformed groups of second to fourth grade primary school pupils. The method used in this article was tested on a local pupils' population.

Key words: mini exemplary teaching, interactive learning, mathematics teaching method, empiric evaluation.

1. Introduction

Active learning in the primary mathematical education implies acquiring knowledge independently of the teacher's guidance through the thought processes of observation and comparison, abstraction and generalization, analogy, analysis and synthesis. Therefore, pupils are supposed to complete their activities by demonstrating flexibility of thinking and clearly articulated creativity.

Interactive teaching and learning, the most common strategy of the modern education, is a sub-category of interaction because it is predominantly related to interpersonal relationship. For preparing and carrying out interactive teaching, all contemporary educational systems, methods, forms, and means are used. Therefore, the choice of these systems, methods, forms and means depends on the age group of pupils, the objectives and tasks of teaching, and the content of every topic and teaching unit.

In the interactive teaching and learning of mathematics the pupils' activity takes the center stage and the role of the teacher is to guide, encourage and teach them how to learn mathematics. Behind every pupil's activity is a necessary feedback about the accuracy of the work done and the results developed, because this is a proven psychological need of any individual. Interactive learning of mathematics affects the development of cognitive and connotative abilities of a pupil in a more efficient way, improves critical thinking, creativity and the matter learnt is more permanently remembered [4, 9, 11, 16].

2. Theoretical background

The idea of *exemplary teaching/learning* was developed using the term *exemplar* (model set of examples) and its possible use. The core of this teaching method lies in choosing the characteristic teaching content from the syllabus, and methodically processing them using the exemplary method. Exemplar needs to contain the procedure by which the teaching content would be methodically treated. With the choice of a typical teaching content and its adequate treatment pupils are given "samples" for further work and studies inside and outside of the school.

Exemplary teaching has certain limitations, such as: difficulties in choosing typical exemplary contents and the problem of setting guideposts for analogous content. The biggest problem is defining the volume of exemplary content and appropriate analogous contents for pupils to process independently.

In teaching primary school mathematics, the exemplary method is used successfully for one teaching unit or a part of it, usually covered over two to four class periods. This form of teaching is termed mini exemplary teaching. In the primary school, exemplary teaching classes must be prepared with a special care and the pupils have to be gradually taught how to participate in it. Exemplary teaching significantly contributes to the interactivity of teaching and through that, to the learning of mathematics.

When it comes to an interactive, efficient process of forming a mathematical concept, or rather, its mental image, the choice of the exemplary content plays a significant role. During the first phase of the process of forming a mathematical concept, set of the examples must accurately represent the abstract mathematical concept. Only in this case the initial mental image will be accurate enough and the process of forming it will be simple and concise. Lesh [7] created a modified version of Bruner's [3] linear model for representing a concept. He particularly emphasized the interactive nature of interpreting the model for representing a concept.

Learning rules is interactively and efficiently conducted through an exemplar which contains one or two characteristic examples. After the interactive processing and treatment of the exemplars, with pupils' thinking activities the rule is instantly formulated verbally or by using symbols. Other examples which pupils tackle by analogy have only a confirmative role.

Understanding or adopting the rules in the mentioned way would be more complete and more permanent and in the course of learning or teaching, the cognitive abilities of a pupil would develop significantly faster.

Contemporary methodology of teaching mathematics puts the *principle of individualization* in the category of priority principles together with the principle of conscious activity [8, 13, 14]. In the primary school, especially in the lower grades, we justify this principle by the fact that every child, the moment they start school, is an individual with certain abilities and characteristics. In the learning of mathematics the most significant are the ability and potential for the development of the cognitive part of a person. The objectives of contemporary primary education demand, together with obligatory adoption of mathematical knowledge and skills, an optimal development of cognitive abilities of every pupil individually.

The stated is practically impossible to achieve through teaching and learning but it is possible through preparation and realization of an individual method for every pupil. That is why the principle of individualization and differentiation in teaching and learning are closely tied, that is, they together form *differentiated teaching.* Teaching contents are most commonly differentiated on the three levels: for the pupils with above average, average, and below average abilities. The obligatory contents cannot be differentiated in terms of the levels. Rather, the contents can be differentiated in terms of help provided to the pupils of different abilities.

Working in small groups implies dividing a class into groups to enable communication, that is, interaction among pupils in each group. Having in mind the working conditions in the classroom, the most rational thing would be to form groups of pupils who sit together at the school desks, which we refer to as shoulder partners. For interactive teaching in numerous groups of pupils, the most rational division is into groups which consist of four pupils from two neighboring school desks.

For interactive teaching of mathematics the structure of pupils within a group is of the crucial significance besides the number of pupils in the group. If groups are formed homogeneously according to the pupils' abilities and learning achievements, in that case every group needs to have tasks and assignments different in terms of complexity. For work in heterogeneous groups, provided that the above characteristics of the pupils are approximately equal, all groups can tackle the same tasks; that is, they can do undifferentiated group work.

An important condition for the structure of a group is to ensure cooperation among the pupils which means acknowledging the pupils' discretion when forming groups. "The main prerequisite for pupil-oriented teaching is the existence of cooperative atmosphere among the pupils" [15, p. 40]. It is necessary to motivate every pupil towards group work and therefore they should be allowed to use help of other members of the group and to perform only a part of the task individually. The activity of individuals within a group can be significantly increased in that way.

When forming heterogeneous groups, we need to have in mind the degree of heterogeneity. If the levels of pupils' knowledge and skills are significantly different, the tasks and the challenges are elusive for one part of the group, therefore the best pupils are often bored and do not know how to provide assistance to others in the group. As it is practically impossible to precisely determine the difference in pupils' knowledge and skills in a group that provides the optimal working conditions, we will accept the suggestions of Bennett & Cass [2, p. 83] "If a hierarchy structure of pupils can be established in one area, then it is best to group together within the hierarchy structure those who differ from each other in one or two teaching steps at the most". When this suggestion cannot be accomplished we modify or change the groups until a structure which allows interactive and efficient learning has been established.

We can conclude that a small group of pupils represents a natural environment in order for mathematical inference to be optimal. The small group environment encourages and stimulates verbal expression and mathematical inference of a pupil. Pupils ask each other questions, they negotiate, they want to prove their point and disprove the points of others. The fact is that not all pupils within a group contribute equally to the solution of mathematical problem tasks. However, it is enough that all the pupils within a group understand, confirm and support the course of the resolution as well as know how to explain the solution and the following mathematical inference.

3. Teaching method

The term *flexible differentiation* implies that teaching is conducted in an environment where the homogenous and heterogeneous groups of pupils overlap, and frontal work with the entire class is present.

When establishing our method, by the term "flexible differentiation" we refer to the way of teaching where help provided to pupils is differentiated by the principle of the minimal help. "In the conditions of equal program requirements, the problem of flexible differentiation of teaching mathematics comes down to the optimal usage of intuition and concretization, motivation and the level of difficulty of the tasks and levels of help provided for the pupils" [12, p. 400].

That type of teaching can be conducted through frontal form of work, through oral administration of differentiated help to all pupils along with feedback. It thereby helps all the pupils and still has the characteristics of flexibility: at the time of providing help and feedback, help is received (listened to or read) by only those pupils who are in need of help, while other pupils work independently. If a pupil is cognitively active in the period between being instructed and given feedback, regardless of the result of the activity, he or she is interacting.

Optimal differentiation is a necessary prerequisite for quality interactive teaching and learning and we opted to fulfill it by using flexible differentiation; that is, the differentiated help of a teacher to the highest achieving pupils in small groups. Pupils who provide assistance during group work, must be instructed and trained by the teacher to be a qualified and suitable substitute. To confirm the described standpoint we state the following [10, 13].

Pupils learn more efficiently when they help others but also make bigger progress towards independent learning when they receive a controlled amount of help [1, 6, 9]. When exchanging their ideas they verbally communicate and change roles during the observation of the problem, they sketch and gesture. The exchange of learning experiences between the pupils, regardless of the form in which it takes place, is extremely useful for those who receive help in that way, as well as for those who provide help. "While helping others, pupils are often placed in a situation to cognitively reconstruct information in a way which makes them understandable and useful to others." [5, p. 137]. It often occurs during that process that even those who provide help open new horizons and personal cognition in dialogue with others who seek further explanation.

By introducing a realistically possible individualization into differentiated teaching and learning, we make the most important step towards the quality of its realization and desired outcomes. According to the described theoretical background, the second step would be an optimal choice of teaching methods and didactic systems, as well as an appropriate linkage and integration of teaching. During the interactive learning of mathematical terms and rules, we mostly rely on mini exemplary teaching because we believe that its implementation can significantly contribute to the efficiency of an interactive learning of mathematics.

Based on the theoretical background used in this paper and our own research, we have determined a structure which contains the following description of common stages in the realization of the classes intended for processing of the teaching units.

Preparative stage:

Interactively reminding pupils about their prior knowledge and experiences which are in an immediate connection with the processing of contents anticipated for the operative stage.

Operative stage:

1. The teacher *determines an exemplar* (one or a few examples) primarily from the textbook.
2. Pupils, by engaging in thinking activities (observation, comparison of the elements of analysis and synthesis), *notice* the crucial elements for forming a term or adopting (understanding and determining the accuracy) the rules.
3. Pupils *analyze* texts which set *the term* or *the rule,* written in a notebook or student's book. They *simplify* it and engage in a thinking process of abstraction and generalization at the same time.
4. With the processing of new examples, pupils *confirm, expand and consolidate* the acquired knowledge.

In the previously described structure, pupils use most of the examples to confirm, expand and consolidate the setting of the term or the rule. They, thereby additionally, by using analogy, engage in thinking activities. At the same time, incomplete induction is applied through a reduced number of examples but it retains its role of a significant form of inference.

Verification stage

Interactive summary is pointed towards the processing of exemplars and a part of an example is used (one or two examples) for confirmation, expenditure and consolidation. For homework they finish the assignments used in the operative stage and are possibly given some new ones.

Illustration 1: Model of teaching addition with two or three addends

Each pair of pupils will prepare a 52-card deck and three cards with the numbers 1, 2 and 3 on them.

Preparative stage:

a)

1. The deck of cards is divided into two decks of 25 and 27 cards.

2. The card numbered 1 is placed on top the 25-card deck and the card numbered 2 on the 27-card deck. Then the sum of the cards is written down (25+27). The decks swap places and the new sum is written down (27+25).

3. Without doing any calculations, the following is written down (25+27=27+25).

After these activities the teacher asks pupils to name this property of addition and give reasons for it. (They have proven the commutative property of addition which they express in words: the sum of two addends does not change if they swap places. The reason for proving the property is described: the sum does not change because in both cases, it is the overall sum of the playing cards, which stays the same.)

b)

1. Three cards are taken off the 52-card deck and form a third deck. This way, three decks are formed, comprising 3, 22 and 27 cards.
2. The decks are arranged from the lowest to the highest number of cards and on them are three cards. A number of cards in each deck is written down on each of the three cards on the top.
3. Using pencils as brackets, two ways of writing down the sum of the cards are formed: (3+22)+27 and 3+(22+27).
4. Without doing any calculations, the following is written down: (3+22)+27=3+(22+27).

After these activities the teacher asks pupils to name this property of addition and give reasons for it. (They have proven the associative property of addition which they express in the following words: the sum of three addends does not change if the order of adding them is changed using brackets. The reason for proving the property is described: the sum does not change because in both cases, it is the overall sum of the playing cards, which stays the same.)

Operative stage:

When forming and conducting the exemplars, pupils take the three cards of the decks and rearrange them to get all possible strings of numbers of cards in the decks (permutations). While doing that, they do the following activities:

1. Write down two possible strings, with card numbered 1 at the beginning of the string (1, 2, 3 and 1, 3, 2); write down 2 possible string with card numbered 2 at the beginning of the string (2, 1, 3 and 2, 3, 1); write down 2 possible strings with card numbered 3 at the beginning of the string (3, 1, 2 and 3, 2, 1).
2. Return the cards to the decks so that the number of the cards in the decks is visible.
3. By using their pencils as brackets, they form the first two ways of writing down the addition process, predetermined in the preparative phase ((3+22)+27 and 3+(22+27)).

To avoid rearranging the decks, the pupils take off the cards with numbers and arrange them according to the strings they wrote down earlier. After arranging them, they turn to reveal the sides with the numbers of the cards in the decks. They place their pencils as brackets, between the cards, and write down two different addition orders each.

Here are all ten possible permutations:

1. For the string 1, 3, 2 they write down (3+27)+22 and 3+(27+22).
2. For the string 2, 1, 3 they write down (22+3)+27 and 22+(3+27).
3. For the string 2, 3, 1 they write down (22+27)+3 and 22+(27+3).
4. For the string 3, 1, 2 they write down (27+3)+22 and 27+(3+22).
5. For the string 3, 2, 1 they write down (27+22)+3 and 27+(22+3).

After these activities, the teacher gives a task to the students: Choose four ways to finish this addition and finish it without adding them up in stages.

$$(3+27)+22=30+22=22+(3+27)=22+30=22+(27+3)=22+30=(27+3)+22=30+22=52$$

After making sure the results of all different possibilities are the same, the pupils conclude that the brackets can be left out.

When forming the rule, the teacher encourages pupils to abstract and generalize cognitive activities. This is why the teacher asks them questions which they write down in their notebooks. They give their answers and after feedback from the teacher they may correct them.

1. If we had used some other number of cards or three decks with different numbers of cards, would the same rule apply? Explain why. (Yes, because in whatever order we add up three decks of cards, we always have the same sum.)
2. Which properties of addition did we use to prove this rule? (We used the commutative and associative properties of addition.)
3. How do we express this rule verbally? (Sum of any three numbers: a+b+c does not change when changing the order of addition so brackets are unnecessary)

When confirming, expanding and summarizing, pupils first write down: (18+2)+19=(2+18)+19; (18+19)+2=18+(19+2) and (19+2)+18=19+(18+2).

Verification stage

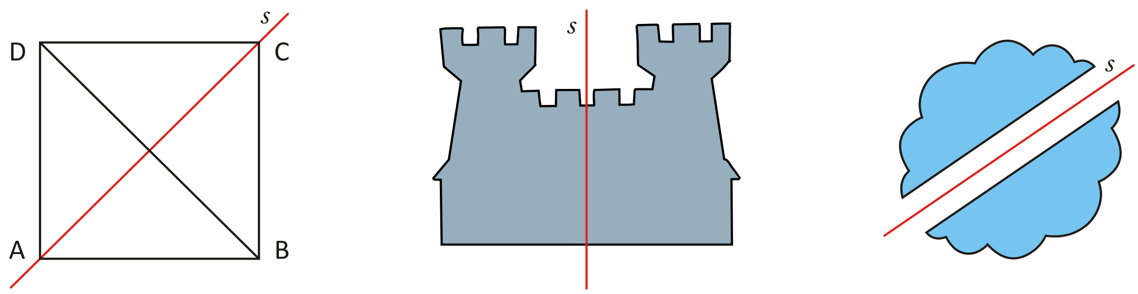
An interactive summary is directed towards briefly describing the exemplar. For homework, the pupils have to write down all ways of adding up numbers 2, 18 and 19, and take four different ways of finishing the addition.

Illustration 2: *Model of preparation of the teaching unit about plane axisymmetry*

Preparative stage:

The teacher, using heuristic guidance, introduces the term symmetry in a propaedeutic way, especially plane axisymmetry, relying on the pupils' previous experiences.

In nature, since the prehistoric period of humanity, men have noticed, and very much appreciated symmetry as a virtue of spatial and flat shapes and forms. The word *symmetry* comes from ancient Greek and its approximate meaning lies in the word *harmoniousness***.** In time, people learn to apply and understand symmetry more and more accurately in the geometrical sense. We can say with certainty that in their works they have long surpassed nature. The reasons for that are multiple and they lie primarily in practical and esthetic needs of the man. We will only tackle the axisymmetry of flat objects.



a) b) c)

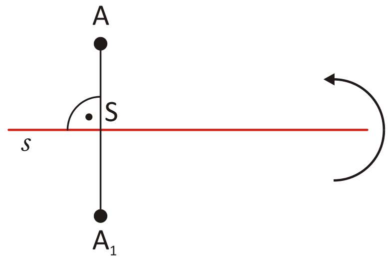
Figure 1. Sketches of symmetrical figures

What pupils notice first is that the spatial object (the fort) is shown as flat. By observing the pictures they notice (discern) the feature which makes them symmetrical in a geometrical sense.

Operative stage

In this stage the pupils perform an experiment in pairs which makes the exemplar. For a model of a plain they use a sheet of transparent paper and they draw a line s (it is known that the line divides the plain into two half-spaces). On the sheet, away from the line *s*, they mark a point A in such a position that after folding the paper it becomes visible on both sides of the sheet. Then they mark the point and unfold the sheet. These activities are illustrated on pictures 1 and 2, which they copy into their notebooks. By folding the sheet in the opposite direction they notice that point A_1 coincides with point A, that is that A_1 is mapped into A.

Picture 1



Picture 2

As point A can represent any point outside the line *s*, with the help of heuristic guidance of the teacher, pupils can conclude that by performing these activities they *mapped* (bijection) of the points from one half-space to another and vice versa. They write the following text into their notebooks "The mapping of the points we previously performed is called *plain axisymmetry* and the line *s* against which the mapping was done is called the *axis*. Points A and A_1 are *axisymmetrical* against the line *s*."

With the help of teacher's heuristic guidance they conclude that axial symmetry maps the point S (picture 2) against the axis s, and so will any other point belonging to the line *s*. After that they conclude that points A and A_1 are assigned with exactly one perpendicular line to the line *s*, as well as that $AS = A_1S$. The previously mentioned conclusions are derived based on the performed experiment, that is, after mapping the points of the plain.

If the following, or a similar, definition is in the student's book, the pupils read it and analyze it and if not, they copy it into their notebooks. *The pair of axial symmetrical points which belong to the line perpendicular to the axis of symmetry are on different ends of the axis of symmetry and are equally remote from the axis. For an object which maps itself when divided with an axis of symmetry we say that it is symmetrical, and for the axis of symmetry we say that it is the centerline. For two objects we say that they are axial symmetrical against the axis s if axial symmetry maps them to each other.*

By observing the pictures shown in the preparative stage, with heuristic guidance, they recap two conclusions.

1) All pictures represent symmetrical objects, only one picture marked with *c)* represents two objects axial symmetrical against the axis s.
2) On the picture marked with *a)* there is a square with an axis of symmetry defined by one diagonal. With heuristic guidance of the teacher the pupils determine the other three axis of symmetry of the square. They also conclude that the square, with all four axes of symmetry, maps itself (in four ways in total).

With the stated concluding the pupils confirm, expand and consolidate their knowledge.

Verification stage

After an interactive abstract, the pupils begin work on their tasks on the instructional leaflets and finish them for homework.

4. Description of empiric research

The study sample was selected from the population of fourth grade pupils of primary schools with approximately the same structure of pupils, working conditions and other characteristics. Bearing in mind the application of the working method with uniformed parallel groups, the first one is completely uniformed with 120 pupils according to their achievements in mathematics and the number of points recorded in the initial test. That was done by extracting pupils whose work would be measured because in each of the schools there was a significantly larger number than 240 pupils of the same grade. All pupils from the beginning of the second up to the end of fourth grade in all experimental groups worked according to the program. However, the experimental as well as the control group consisted of 30 pupils with a good average in mathematics, 40 with a very good average and 50 with excellent performance in mathematics. Forming of the mentioned uniformed groups was conducted after the initial test.

The teachers who conducted the work with the experimental group used *the theoretical basis of research, which is* generally marginal for *the application of our method and special instructions for preparation of all teaching units and lessons and their processing.* Hereby, the authors used the content and the structure of the book in common for both groups. For preparation of every class there are written basic guidelines or a complete preparation. Besides the written instructions all teachers were engaged in oral communication during the experiment, especially for the thematic revision of the part of the curriculum which was not put in writing within the preparation.

The *basic hypothesis* means that the application of special methodic of interactive teaching achieves statistically significantly better outcomes of teaching and learning within the experimental group compared to the control group. The *sub-hypotheses* are:

a) Each subgroup of the experimental group (pupils with good, very good and excellent average in mathematics in the second grade) will show statistically significantly better outcomes of teaching/learning compared to a corresponding subgroup in the control group.
b) The biggest difference in outcomes of teaching and learning in sub-groups of the experimental group and the corresponding subgroups in the control group is achieved by the subgroup of pupils with a very good average in mathematics.

Description of the testing and graphics

Initial research was conducted in the second grade of primary school. The first final testing was done at the end of the fourth grade and consisted of program contents of the second and the third grade. The second final testing was done immediately after the first one and it consisted of the program contents of the fourth grade. In that way the durability of the acquired knowledge and skills was also incorporated in the measuring.

To illustrate the results of all three tests we use a graphic display (see Appendix Figures 2 and 3).

Based on the charts and graphs and the testing of significance of the differences between the arithmetic averages we conclude that the achievements of the pupils from group E, on both final tests, significantly exceed the achievements of the group K, on the level of statistic reliability of 0.95. The same conclusion applies to all subgroups of pupils. We also notice that the biggest improvement was achieved by subgroup E with a very good average in mathematics in the second grade. In that way we empirically confirm the ground hypothesis and both sub-hypothesis.

From the graphic displays of sample (Figure 4, 5 and 6) dispersions it can be noticed that the medium square aberration of the arithmetic means on both final tests is lesser in the case of group E compared to the corresponding aberration in the case of group K. It can be concluded that the pupils in group E, after performing the work according to the described method, achieved greater balance in progress throughout their achievements.

5. Conclusion

With the application of exemplars, which is the most appealing to pupils, interactive processing of the planned teaching content is relieved of excess examples, and the deductive concluding is enriched with thinking activities. A suitably chosen exemplar acquires the features of a problem situation, and it dominantly affects the level and quality of interactivity in learning or forming notions as well as in learning mathematical rules. The outcomes of interactive work increase effects such as optimal development of cognitive and connotative ability, criticism, creativity, etc.

These statements primarily refer to the overcoming of the program contents in teaching mathematics, because so far they have not been respected as much. Through interactive learning, the matter learnt is better used in new situations in the process of learning mathematics. The transfer of learning is greater and the substance learnt lasts longer.

The most important advantage of interactive learning of mathematics, according to the described structure, is the greater engagement of pupils into the thinking activities on which the conclusion of the study is based. Therefore, the study was rooted in the assumption that there is a need to involve students is a variety of thinking activities including observation, comparison, abstraction, generalization, analysis, and synthesis, something that motivate creative thought and flexible reasoning.

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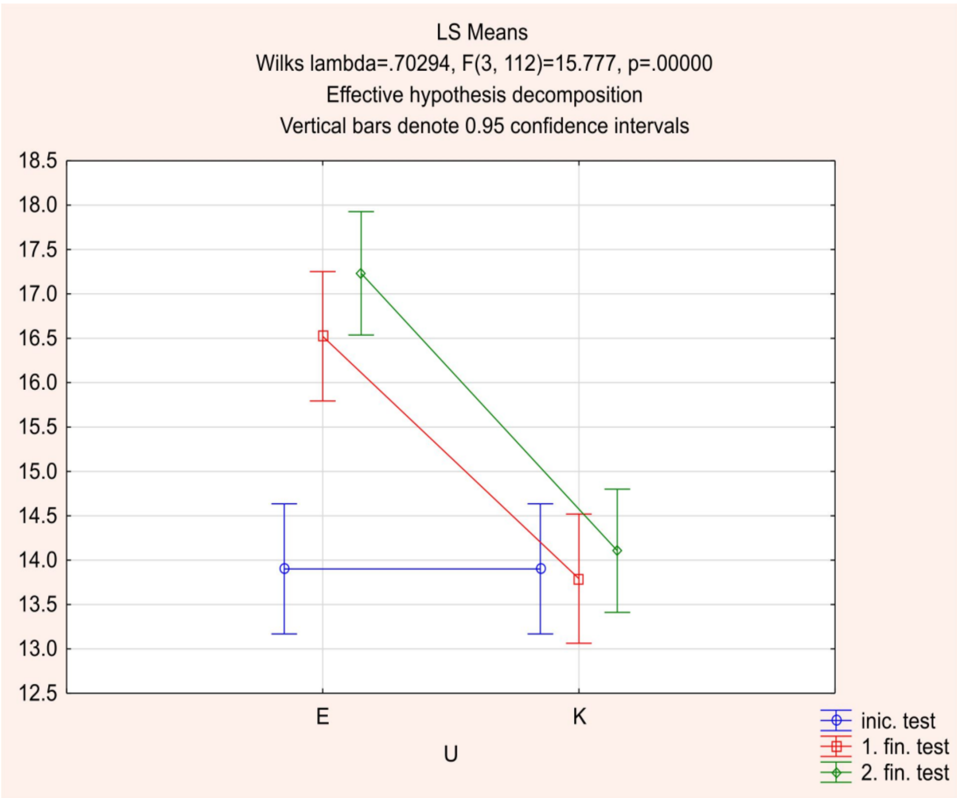
Appendix

Graphic display of the results of all three tests



Fuigure 2. Graph of the initial and final testing

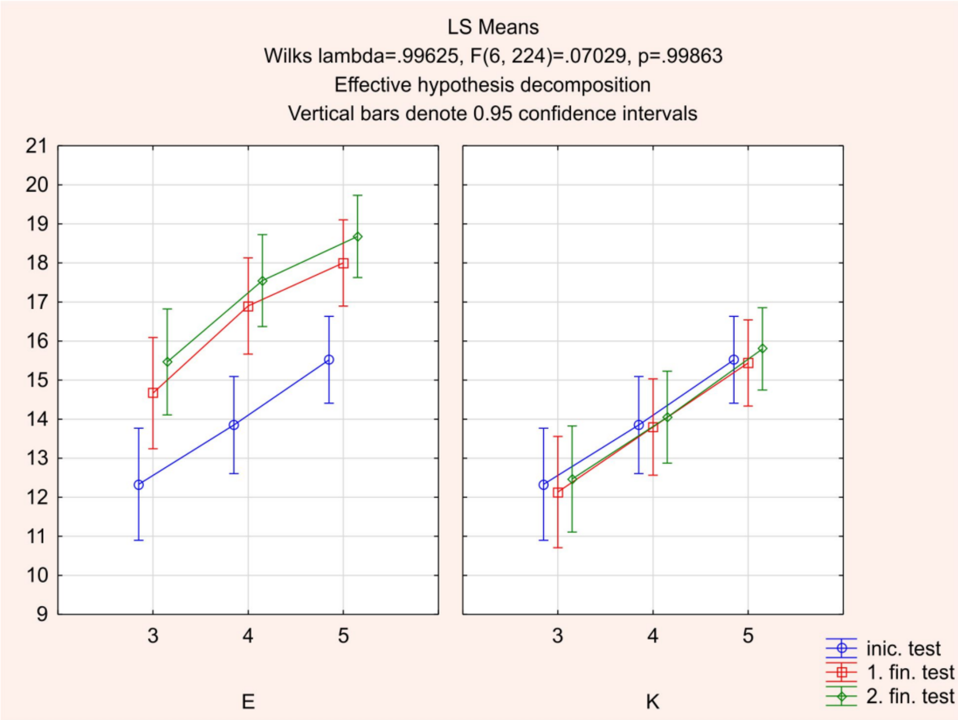


Figure 3. Graph of the initial and final testing by groups of pupils

Graphic display of sample dispersions

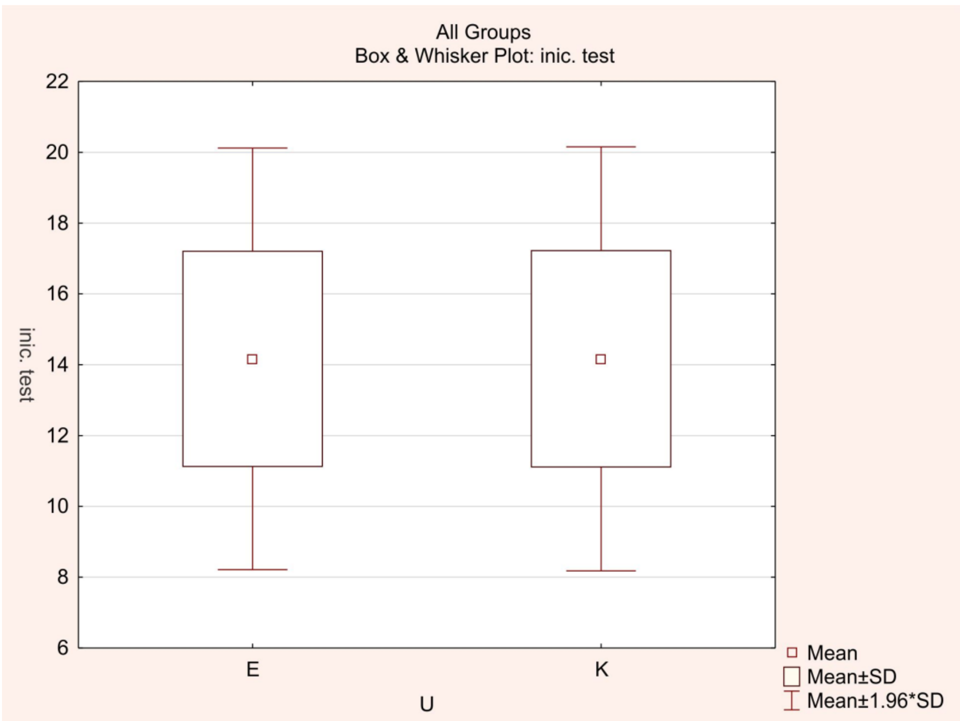


Figure 4. All groups - initial test

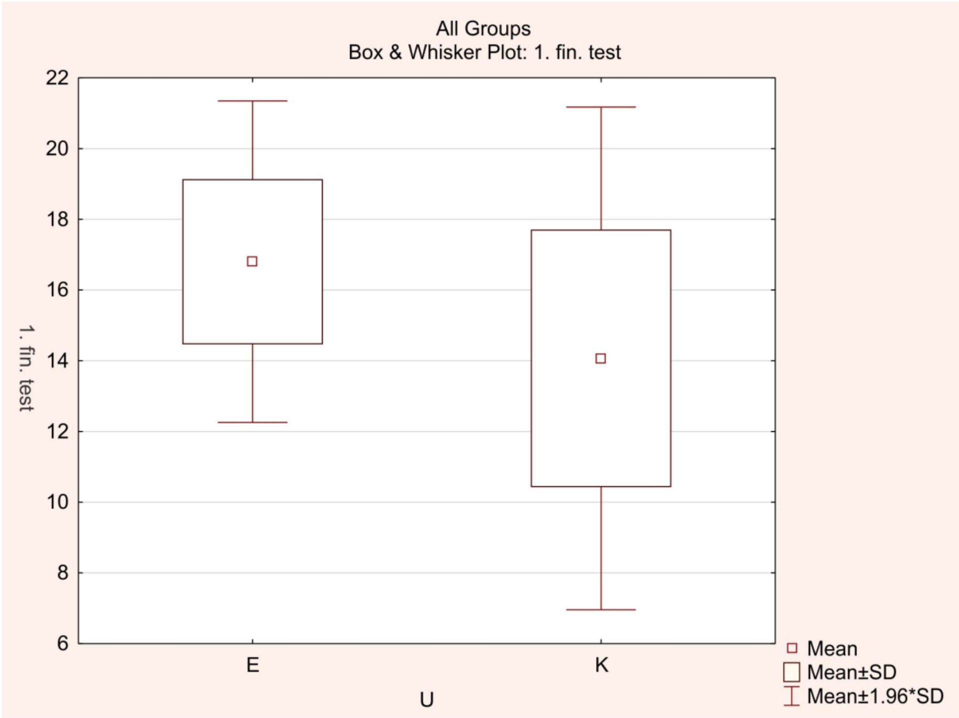


Figure 5. All groups - first final test

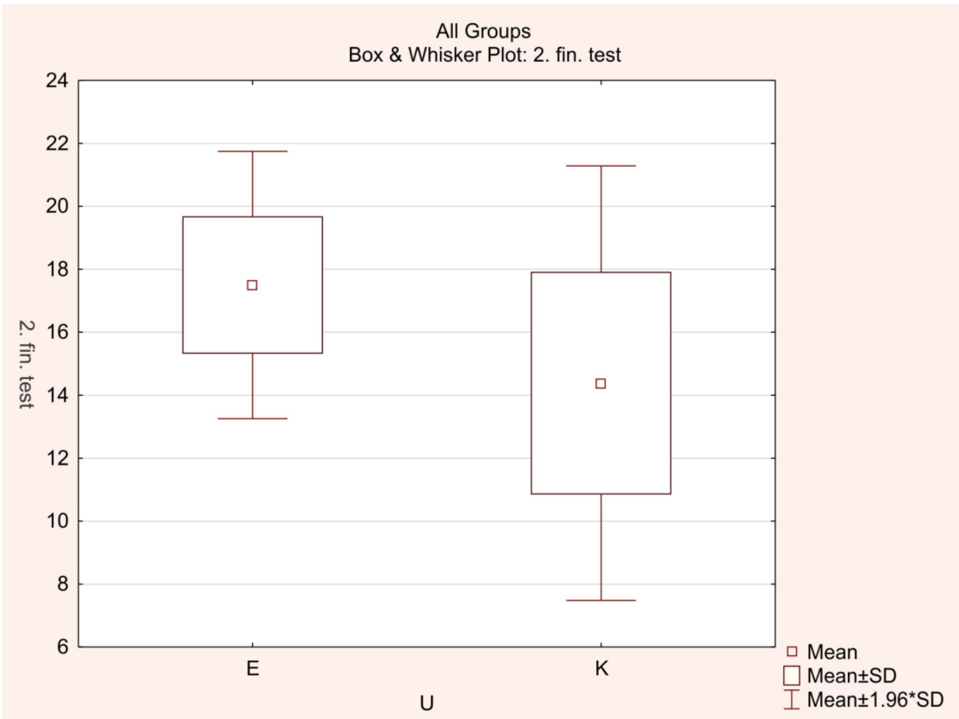


Figure 6. All groups - second final test

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