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GAINING INSIGHT OF HOW ELEMENTARY SCHOOL STUDENTS IN THE REPUBLIC OF SRPSKA CONCEPTUALIZE GEOMETRIC SHAPE OF PARALLELOGRAM

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Abstract. In the Republic of Srpska elementary school system there no any official evaluation of mathematics education outcome exists. This work is a part of our broader research on students' geometrics knowledge in the primary schools. The goal is to investigate geometric competencies of children from six to ten years old at the elementary school level in the Republic of Srpska, Bosnia & Herzegovina. Towards this end, the authors offered geometric tasks about parallelograms to a small number (15) of elementary school students via interviews. In this paper, some selected results are presented with the focus on students' conceptualization of geometric shapes. The emphasis of this study is on classification, identifying differences, and defining geometric objects.

ZDM: B20**,** C30, G20
Key words: parallelogram, rectangle, square, cognitive processes, classification, geometric reasoning.

1. INTRODUCTION

International comparative studies of the early education, early mathematics education, and how education should look like have been widely discussed. Research suggests that early learning is important in order to offer a basic education for all children. The remaining question is how education for 6 to 10 years old children should be designed. In the Republic of Srpska, an entity of Bosnia & Herzegovina, there is only one approach – the traditional way. There is only one concept for elementary school mathematics education – tutorial according to the book approved by the Ministry of Education. We are unable to present real educational level of mathematics in the school system in the Republic of Srpska because there exists no any official research on elementary school mathematics education. For example, researchers of mathematics education in this region cannot say anything trustworthy about students' geometric competences. On the other hand, for the academic society of mathematicians, researchers of mathematics education and for society of elementary school teachers in the Republic of Srpska it is extremely important to know what the real mathematics competences after finishing the elementary school (the 1^{st} – 5^{th} grades of the primary school) are. So, the only way to acquire an insight about students' geometrical competences after finishing the first five grades of the elementary school is a partially collected data throughout interviews with randomly chosen elementary schools students.

In this article, the authors present established educational levels of learning geometry using rectangles and squares depending on interviews with 15 children. These included: three 2^{nd} grade students, three 3^{rd} grade students, three 4^{th} grade students, and three 5^{th} grade students from a local primary school in Bijeljina. The ages of these children range from 6 to 10 years old. This report is the first research on elementary school students' geometric competences in the Republic of Srpska. According to this study, the authors can conclude: elementary school students have problems in identifying the concept of square and rectangle. The data collected appears to be important not only to the academic society of elementary school teachers but also to the whole society of the Republic of Srpska.

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2. THEORETICAL BACKGROUND

When we talk about the concept of a parallelogram in the lower grades of the elementary school, we primarily think about the concept of rectangle and the concept of square. Indeed, rectangle and square are one of the very first geometric shapes presented to students. These shapes are introduced in the 2^{nd} grade at the level of visualization (according to van Hiele's classification [30]) within the theme "Objects in space" [X, pp. 4-28] that includes the lesson "Rectangle, square, triangle and circle" [X, pp. 16-17]. In fact, geometric figures, which are first presented to students, are the drawings and models of the cube, cuboid, pyramid, ball, etc. Students need to recognize rectangles and squares in the shapes. The images of the shapes, offered to the students in [X] without any explanation, are shown in Figure 1. The students conclude that the two shapes are not the same.

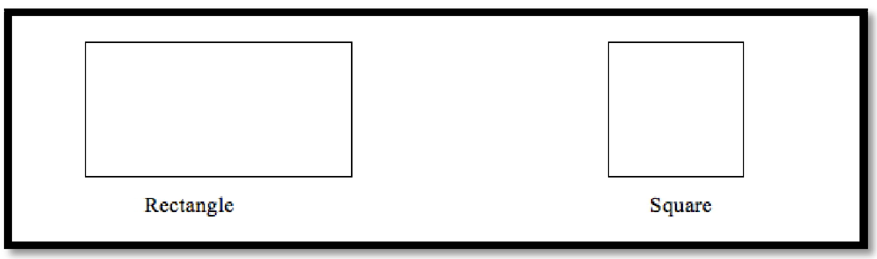

Figure 1. Images of rectangle and square according to [X].

In the 3^{rd} grade, students expand their knowledge about rectangle and square [Y, pp. 32-35]. They learn to mark vertices and sides and to draw them [Y, pp. 34-35]. All this is taught within the theme "Geometric figures" [Y, pp. 27-35]. The concepts that precede them are angle, polygon, open and closed lines [Y, pp. 30, 31]. Therefore, students first recognize an angle and a polygon and only after that they learn about square and rectangle. In the fourth grade, the concepts of rectangle and square are also mentioned [Z, pp. 82-88]. Students revisit their previous knowledge about these concepts and acquire new knowledge; namely, they learn to draw these concepts with the help of a setsquare and a ruler, or a compass and a setsquare. Note that this appears within the lesson "Right angle and quadrilaterals" [Z, p. 83]. In addition, students learn to define these terms so that the quadrilateral with four right angles is called a rectangle and the rectangle with all equal sides is called a square [Z, p. 83, Task 3].

Alternatively, one can define a rectangle based on a parallelogram: the rectangle is a parallelogram with at least one right angle; more precisely, it has four right angles. Students of the 2^{nd}, 3^{rd} and 4^{th} grades cannot comprehend this statement. Even though these descriptions are mathematically correct, with a deeper look at these definitions, it is clear that there are some methodological flaws. The students might perceive the first definition that says a rectangle is a quadrilateral with four right angles, as being incomplete. Indeed, the students could say: It is a square that is a quadrilateral with four right angles, isn't it? It seems to the authors that these students cannot understand that the class of squares (determined by two predicates) is a subclass of the rectangular class (determined by one predicate). Therefore, it is methodologically necessary to add two pairs of equal sides in the description of the rectangle in order for the students to accept that what they see is correctly described using the given description – the rectangle is a quadrilateral with four right angles and two pairs of equal sides.

Based on this, the second definition that the rectangle with all equal sides is called a square was brought into question. In the 5th grade, students learn about surface of rectangles and squares. From the above overview, we conclude that the concept of parallelogram is not represented in lower grades. The term parallelogram is not used in any description of the notions of rectangle and square. We believe that this is negligence. As we have mentioned before, one can define a rectangle based on a parallelogram: the rectangle is a parallelogram with four right angles. There is a deficiency when we translate the material or perceptual representations to speech (this is especially related to the textbooks).

So, further we will observe the quality of students' knowledge according to these two concepts, rectangle and square. It is necessary to mention that one of the main tasks of geometry in the primary school is for students to become familiar with the concept of parallelogram through geometry lessons. Students perceive these objects (rectangle and square) and their properties through visualization. In addition, students have to learn to construct these figures using geometric instruments. Therefore, the concept of parallelogram is also important for students.

3. THE CONCEPT OF SHAPE

The focus of this paper is to develop geometric concept of a parallelogram. We will illustrate what constitutes the notion of concept before we present two general theoretical models concerning concept development. Some empirical results concerning student's development of geometric concepts of rectangles and squares will be presented.

3.1. Formation of a concept

Franke [16, p. 72] defines the notion of concept as follows: "We speak of a concept, if it not only represents one single object – or incidence and so on – but a category or a class that is associated with it, in which the concrete object can be classified". According to Franke [16], a comprehensive conception of a geometric shape, as a concept for objects, includes the ability to:

- recognize the shape: know the name of the shape, know so-called 'almost definition' of the shape, know further examples of this category and recognize the characteristics of the category of these objects (level zero in van Hiele's classification [30]);
- know characteristics of relevant elements of the category of objects and at the end know characteristics of these elements and their interrelation (level one in van Hiele's classification [30]). Therefore, students' understanding of the concept of geometric shape is manifested by the following proficiencies:
- recognition of the shape,
- knowledge of the name of this shape,
- identification of essential elements of the shape,
- construction of 'almost description' of the shape,
- recognition of other objects in the category of some objects, and
- recognition and knowledge of the most of the category of objects' characteristics.

There are different suggestions on how such a comprehensive conception develops. One suggestion is illustrated in the following section.

3.2. Conceptualisation theories

Szagun [29] proposes two theoretical approaches that illustrate how a concept develops. In the "semantic feature hypothesis" general features are learned before specific features. For example, the child has learned the word "triangle", which is connected with one semantic feature that it is a "closed zigzag line". Accordingly, the child would first call every recognizable polygon (four line polygon, five line polygon, etc.) as a triangle. Over time, other semantic features are added, such as "triangle is a polygon with exactly three line segments", so that the word "triangle" could be distinguished from the names of other polygons. So, we accept into this concept that the notion "triangle" is pinpointed by two predicates: 'closed zigzag line' and 'number 3'. In contrast, in the "prototype theory", which is considered as the psychologically more real theory, some members of a category are categorized as more typical than others

[29, p. 134]. For example, a closed zigzag line yielding a convex polygon (triangle, convex four line polygon or convex five line polygon) is a more typical object of the polygon category. Since all the members in this category have many mutual characteristics, there are different subcategories determined by another subcategory characterized by more predicates. However, in order to give a complete picture of what we know about students' formation of the notion of parallelogram and how this notion develops research findings about geometric concepts will be presented.

4. EMPIRICAL BACKGROUND

In this section, we will be paraphrasing statements found in [18, p. 37]. The French psychologist Raymond Duval [10, 11] approaches geometry from a cognitive viewpoint and distinguishes four ways of "cognitive apprehensions" for a "geometric figure": perceptual, sequential, discursive and operational. The function of figures in the drawing can evoke perceptual apprehension and at least one of the other three. It is important that in any situation, a figure has its own organizational law and a process of visual stimulating. When we talk about perceptual understanding of a figure (represented in lower grades), we know that it is mostly related to the recognition of a shape in the plane or within the environment. As a matter of fact, students' perception of a figure presented to them, in this case, parallelogram, is determined by the laws of geometric organization and by pictorial symbols. Perceptual understanding indicates the possibility to name figures. It also indicates the ability to recognize the subfigures in the presented figure. For example, students can recognize a square in the cube. Sequential understanding refers to the construction of figures or describing their construction. The organization of elementary figural units does not depend on the perceptual law and the pictorial symbol law. This organization depends on technical limitations and their mathematical properties. (In the introduction, we noted that students learn to construct parallelogram, rectangle and square using a ruler and a set square – a tool shaped as a right-angled triangle, or a ruler and a compass). Discursive understanding refers to the language or speech and it is based on the fact that mathematical characteristic of a figure cannot be determined through perceptual apprehension.

In any geometric representation, perceptual understanding of a geometric figure's characteristics can remain on the level of intuitive description (for example, the definition of rectangle or square). However, it is important to emphasize the value of discursive understanding, which, through operational understanding, can give insight into solving the problem under consideration in the given figure. Operational understanding depends on different ways of modifying a given figure: the metrological, the optical, and the positional way. The metrological way relates to the division of whole figure into parts of different forms, or a combination of them in another figure or sub-figure (reconfiguration). The optical way is represented as part of the larger or smaller figure, or when we have a slant. The positional way refers to its position or orientation. Each of these modifications can be performed mentally and physically, through various operations. These operations construct specific figural processes that provide figures with heuristic function. In the geometric problems, one or more of these operations can provide an overview of figural modifications that gives insight into the solution of the problem. Duval [11] describes teaching and learning in relation to the cognitive approach, and three different cognitive processes, each of which has its own characteristics:

- The processes of visualization for representation are presented in the 2D or 3D space.
- The processes of construction are required for the representation of geometric structures by geometric instruments.
- The processes of reasoning require an organized description or an organized debate.

Each mathematical concept is bound to the function of representation. Students need to accept the notion that there exists a mathematical object constructed through abstraction and that in the nature there may be no real object to represent this abstraction. D'Amore [7] states that this statement can be useful when applied to a variety of different "registers of semiotic representation" to gain access to mathematical concepts and mathematical objects. The coordination of many registers of semiotic representation appears to be fundamental for the conceptual learning of objects: we must not confuse an object with its representation and that must be recognized on each of the possible representations [9]. So, the authors decided to conduct a survey with students about parallelogram in the lower grades of the elementary school regarding a picture, drawing, name, and definition.

In the last two decades, many researchers investigated how elementary school students understand the concepts of rectangle and square. In the course of data collection for the present research, the authors consulted with a large number of sources including [4-6, 8-15, 17, 18, 20-28]. It seems fair to say that the van Hiele's levels of geometric reasoning [30] is the most popular theoretical framework to acquire an insight on how elementary school students conceptualize the concepts of rectangle and square. We can distinguish the following three major groups of semiotic representation: material representation (in paper, wood, plastic, etc.), a drawing (made either by pencil and paper or on a computer screen using geometric software, etc.), and a discursive representation (a description with words using a mixture of natural and formal languages) [8, p. 696]. According to many contemporary researchers (e.g., [25]) a comprehensive conception of geometric shapes is shown through being able to: (0) recognize the shapes, (1) name the shapes, (2) give definition of the shapes, (3) show further examples of this category, and (4) name all the properties. However, this description was given for secondary school students only. For elementary school students we expect that they are able to do (0) and (1), and give some 'almost-definitions' – acceptable descriptions of geometric shapes.

5. EMPIRICAL STUDY

5.1. Research Questions

We have prepared four questions for the students about this topic. The first question concerned the drawing. The students were asked to recognize and classify a rectangle and a square in a drawing. Therefore, the first question was related to perception, that is, to the identification and naming of squares and rectangles. We concluded that this question is directly aimed at the level of visualization, i.e., level zero in van Hiele's classification [30]. The second question concerned students' perception of the difference between squares and rectangles. In the third and the fourth questions the students were given a task to express in their own words what rectangle is and what square is. In Figure 2 we show a connection of these questions with Duval's ways of apprehensions for geometric figures, in particular with perceptual and discursive approaches.

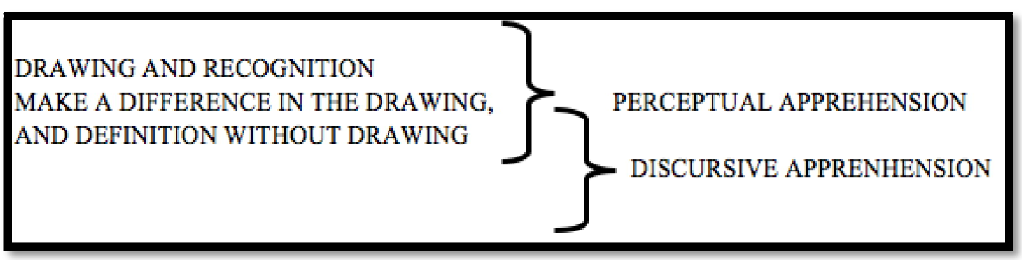

Figure 2. Two types of apprehension of geometric figures (borrowed from [31]).

Based on the illustration presented in Figure 2, it is clear that we intended to find out what perceptual apprehension for a parallelogram in lower grades is, i.e., approaching geometry discursively. Note that we have intentionally deleted sequential apprehension since we believe that students should not have any problems with drawing rectangles and squares.

5.2. Subjects

The research included 15 children, of which three (Stefan, Sergej, Luka) are 2nd grade students, three (Pavle, Andrea, Ognjen) are 3rd grade students, three (Petar, Tamara, Nikolina) are 4th grade students and three (Sabina, Adelisa, Ema) are 5th grade students of a local primary school in Bijeljina. The ages of the children ranged from 6 to 10 years old.

5.3. Method

The study was conducted in the form of interviews, the origins of which coincide with Piaget's early investigations into children's thinking [19]. The order of the tasks as well as the material was predetermined but, in accordance with the nature of the interviews, this order could be altered or complemented if some of the child's answers happened to be interesting or leading into another direction. There were altogether two points of investigation, without intervention, one at the beginning of the school year in 2012 and another one at the end of the school year in 2013.

5.4. Tasks

In order to investigate children's knowledge of shapes of rectangles and squares and to illustrate the concept formation of the children, different tasks were conducted in the interviews of which the following will be presented in the paper:

(1) naming, explaining and correlating rectangles and squares,
(2) drawing rectangles and squares, and
(3) identifying and discerning rectangles and squares.

(1) Naming, explaining and correlating shapes

At this task, the children were shown different geometric shapes (squares, rectangles, triangles and circles). First, they were asked to name these shapes and then correlate them to a hole in a scarf, which had the shape of one of the geometric figures. Afterwards, they were asked to explain the notions of rectangle and square "to somebody who never saw either figure".

(2) Drawing shapes

In order to examine the children's transfer from knowledge about a shape into its representation, they were asked to draw a rectangle and a square on a piece of paper [3, p. 34]. Then they were asked to draw another rectangle and square that are slightly different from the first pair. Afterwards, another rectangle and square had to be drawn, different from the first two pairs. This should continue until the child's way of drawing the figures is different from their previous drawings.

(3) Identifying and discerning shapes

Another task of giving hints on the conceptualization of the children was a shape-selection task according to procedure described in [3, 6, 25, 27]. The children were asked to "put a mark on each of the shapes that is a rectangle or a square" in the drawing (Figure 3) of separate geometric figures. After several shapes were marked, the interviewer asked questions such as: "Why did you choose this one?", "How did you know which one is a rectangular and which one is a square?", "Why didn't you choose that one?".

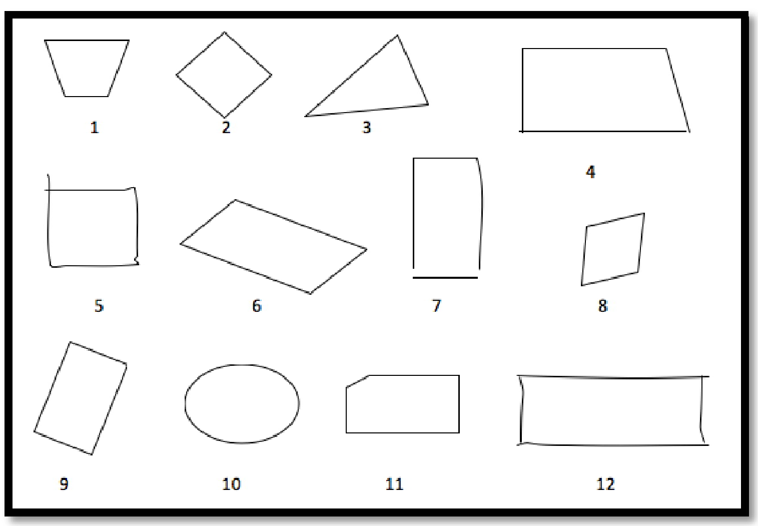

Figure 3. Identifying rectangles and squares using shapes from [27].

6. RESULTS

Some of our colleagues who read earlier drafts of this paper raised the following question: Why is the sample of students used in your research sufficient for drawing conclusions? When they saw the sample of 15 participants only, they expressed an opinion that we have to add a few sentences in order to explain our belief that the sample was adequate for our intention. In this paper, we formulate a hypothesis and give some valid reasons for it. However, for the confirmation of the hypothesis it is indeed necessary to make a more comprehensive research. Such an investigation has to be carried out under the supervision of the Ministry of Education of the Republic of Srpska as an external evaluator of mathematics education outcomes.

In the first question, the students were given the drawings shown in Figure 3 where, based on observations, they had to give an answer to the task to sort out rectangles and squares.

Question (1): *Identify rectangle/square*

2nd grade students, seven years old.	Shapes	**Stefan**	**Sergej**	**Luka**
	rectangles	3, 5, 6, 10	4, 5, 6, 7, 8, 9, 11, 12	6, 7, 9
	squares	6, 7	4, 6, 8, 11	2, 8.

3nd grade students, eight years old.	Shapes	**Pavle**	**Andrea**	**Ognjen**
	rectangles	9, 6	14, 4, 7, 9, 6	6, 7, 9
	squares	2, 8	5, 8, 2	8.

4th grade students, nine years old.	Shapes	**Petar**	**Tamara**	**Nikolina**
	rectangles	9, 6	14, 4, 7, 9, 6	6, 7, 9
	squares	2, 8	5, 8, 2	8.

5th grade students, ten years old.	Shapes	**Sabina**	**Adisa**	**Ema**
	rectangles	6, 9, 4	6, 7, 9, 12	3, 5, 8, 10
	squares	2, 8	8, 5, 2	6, 7

Question (2): *Explain what is the difference between rectangle and square?*

Stefan, Sergej, Luka, 2nd grade students, seven years old.
Stefan: The rectangle is this (*shows shape 12*), and the square is this (*shows shape 5*).
Sergej: A rectangle has two vertical sides, a square the same.
Luka: In the squares the four sides are equal, in the rectangles adjacent.

Pavle, Andrea, Ognjen, 3rd grade students, eight years old.
Pavle: Square has four equal sides, a rectangle just two.
Andrea: Square has four equal sides, rectangle two longer and two shorter sides.
Ognjen: All the sides in square are equal, and not in a rectangle.

Petar, Tamara, Nikolina, 4th grade students, nine years old.
Pavle: Square has four equal sides, a rectangle just two.
Andrea: Square has four equal sides, rectangle two longer and two shorter sides.
Ognjen: All the sides in a square are equal, and not in a rectangle.

Sabina, Adelisa, Ema, 5th grade students, ten years old.
Sabina: Square has all four sides equal.
Adelisa: Square has the same sides, a rectangle and has two and two the same (she shows).
Ema: The squares have all equal sides, neither side of rectangles are equal.

Question (3): *Explain, in words, what is a rectangle.*
Question (4): *Explain, in words, what is a square.*

Stefan, Sergej, Luka, 2nd grade students, seven years old.
Stefan: This rectangle is 12, 4, 6 (points at the shapes). The square is 5, 8 (points at the shapes).
Sergej: A rectangle has two vertical sides. Square has two vertical lines.
Luka: Rectangle. Square. It is a geometric shape that is the same.

Pavle, Andrea, Ognjen, 3rd grade students, eight years old.
Pavle: A rectangle is a square that has the same the upper and lower side. The square is a part of the cube.
Andrea: A rectangle is a geometric figure with two long and two short sides. Square is a geometric figure with four sides the same.
Ognjen: A rectangle is a geometric figure with four sides that are not all the same. Square is a geometric figure with four sides that are all the same.

Petar, Tamara, Nikolina, 4th grade students, nine years old.
Petar: A rectangle is a geometric figure or shape (he is thinking), the shape, shape. The square is the geometric shape.
Tamara: Rectangle is a shape, with two short and two long lines. Square is also a figure has four equal sides.
Nikolina: A rectangle has four straight lines the upper and lower line is longer than from the sides (points at a shape). Square has four straight lines and they are the same length.

Sabina, Adelisa, Ema, 5th grade students, ten years old.
Sabina: A rectangle is a figure that is drawn right and diagonally. The square is a figure with four equal sides.
Adelisa: A rectangle is a geometric figure that has two and two the same equal sides . The square is a geometric figure all four sides of which are equal.
Ema: A rectangle is a geometric figure which does not have all sides the same. The square is a geometric figure all sides of which are equal.

7. RESEARCH RESULTS AND THEIR INTERPRETATION

Based on the students' responses to the above four questions, we can see that there are problems that occur among students during the adoption of the concept of parallelogram, in this case, rectangle and square. Although at the first glance it seems to us that these geometrical concepts are easy to adopt for students without much difficulty and that they should not have a problem with these concepts, this study shows that this is not so. We will first consider Question 1. Obviously, students can distinguish between square and circle, or between rectangle and triangle, but there is a number of students who are not able to distinguish rectangle from square. In addition, there are some problems in distinguishing between rectangle and "non-rectangles" similar to distinguishing between squares and "non-squares". There has been another problem: obviously, students have difficulty recognizing rotations of geometric figures, although to a lesser extent than what van de Walle underlined [31].

Based on all of the above, we can conclude that students have some problems with the visualization of geometric figures, i.e., with the recognition of rectangles and squares. Note that the studies of Vighi [31] and Acuña [1] have also shown that students have problems with perceptual representations at the level of visualization; for example, a triangle is usually seen as an isosceles triangle.

The second question dealt with how students see the differences between the rectangle and the square. The analysis of students' responses indicates that their thinking is based on perception, visualization, and observing a noticeable difference. Also it can be noted that their notion of square is closer to them then the notion of rectangle. It is noticeable that students in their judgments take one aspect only. Also, based on how students perceive differences between these two geometrical concepts, it is easy to guess the answer to the third and fourth questions.

The third question was referred to the understanding of discursive concept of squares and rectangles. Analysing students' responses, it is evident that they favour their own definitions that are mainly based on pictures. This is also confirmed in [3]. It is also notable that students give their definitions based on one attribute of a geometric object, equal side of square, while another attribute, right angle, is ignored. This is supported by the fact that students much easier define square than rectangle, probably because of the fact that rectangle is more complicated than square, since it has two pairs of equal sides.

8. CONCLUDING REMARKS

From this study we can make the following conclusion: students in the elementary school have some problems in identifying the concept of square and rectangle, not so much in relation to other geometric shapes like circle or triangle, or rotated geometric shapes, as much as when the rectangle is compared to "non-rectangle". Why is it so? There are, probably, two reasons. The first reason is that in lower grades the concepts of square and rectangle are introduced. Particularly, in the 3rd grade, after the concepts of angle and polygon are introduced, where the difference between quadrangle and quadrilateral is not discussed. Likewise, after the lessons "Right angle and quadrilaterals" in the 4th grade the same error is repeated. The second reason is that teachers only opt for geometric shapes aspects from a collection of geometric shapes using words: this is a rectangle and this is a square, without comparing the shapes to different types of quadrilaterals. In addition, teachers almost never include curves or "non-straight" sides in shapes so that many students could have difficulty understanding the concept of side. From the analysis of the students' responses, we conclude that they see sides as sides even when they are "non-straight" or "curvy".

Most of the student participants were able to detect differences between rectangles and squares, but their thinking was deeply based on visualization. So, it is logical to have problems defining these geometric shapes. The question then arises as to why is it so? A possible answer is: schoolteachers, in general, pay little attention to the definition of geometric objects. As a proof of this observation one can refer to definitions given in the textbooks. As was already mentioned in the introduction, these definitions are incomplete. Teachers do not see a reason why students need to define geometric objects. Therefore, although at the first glance rectangle and square are one of the easiest geometric objects, it is obvious that students have some problems with understanding them. In particular, students have problems with perceiving the elements of those geometric figures and with recognizing relationships among these elements.

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