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Symmetric Identities Involving *q*-Frobenius-Euler Polynomials under Sym (5)

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Abstract Following the definition of *q*-Frobenius-Euler polynomials introduced in [3], we derive some new symmetric identities under sym (5), also termed symmetric group of degree five, which are derived from the fermionic *p*-adic *q*-integral over the *p*-adic numbers field.

Keywords: *Symmetric identities, q-Frobenius-Euler polynomials, Fermionic p-adic q-integral on* $\mathbb{Z}_p$ *Invariant under* S_5

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1. Introduction

As it is known, the Frobenius-Euler polynomials $H_n(x)$ for $\lambda \in \mathbb{C}$ with $|\lambda| > 1$ are defined by means of the power series expansion at $t = 0$

$$\sum_{n=0}^{\infty} H_n(x)\frac{t^n}{n!} = \frac{1-\lambda}{e^t - \lambda}e^{xt}. \tag{1.1}$$

Taking x = 0 in the Eq. (1.1), we have $H_n(0) := H_n$ that is widely known as *n*-th Frobenius-Euler number cf. [3,4,5,8,17,18,21].

Let p be chosen as a fixed odd prime number. Throughout this paper, we make use of the following notations: $\mathbb{Z}_p$ denotes topological closure of $\mathbb{Z}$, $\mathbb{Q}$ denotes the field of rational numbers, $\mathbb{Q}_p$ denotes topological closure of $\mathbb{Q}$, and $\mathbb{C}_p$ indicates the field of *p*-adic completion of an algebraic closure of $\mathbb{Q}_p$. Let $\mathbb{N}$ be the set of natural numbers and $\mathbb{N}^* = \mathbb{N} \cup \{0\}$.

For *d* an odd positive number with *(p,d) = 1*, let

$$X := X_d = \lim_{\overleftarrow{n}} \mathbb{Z} / dp^N\mathbb{Z} \;\; and \;\; X_1 = \mathbb{Z}_p$$

and

$$a + dp^N\mathbb{Z}_p = \left\{x \in X \mid x \equiv a \left(\operatorname{mod} dp^N\right)\right\}$$

where $a \in \mathbb{Z}$ lies in $0 \le a \le dp^N$. See, for details, [1,2,3,4,6-17].

The normalized absolute value according to the theory of *p*-adic analysis is given by $|p|_p = p^{-1}$. q can be considered as an indeterminate a complex number $q \in \mathbb{C}$ with $|q| < 1$, or a *p*-adic number $q \in \mathbb{C}_p$ with $|q-1|_p < p^{-\frac{1}{p-1}}$ and $q^x = \exp(x \log q)$ for $|x|_p \le 1$. It is always clear in the content of the paper.

Throughout this paper, we use the following notation:

$$[x]_q = \frac{1-q^x}{1-q}. \tag{1.2}$$

which is called *q*-extension of *x*. It easily follows that $\lim_{q\to 1}[x]_q = x$ for any *x*.

Let f be uniformly differentiable function at a point $a \in \mathbb{Z}_p$, which is denoted by $f \in UD(\mathbb{Z}_p)$. Then the *p*-adic *q*-integral on $\mathbb{Z}_p$ (or sometimes called *q*-Volkenborn integral) of a function *f* is defined by Kim [10]

$$I_q(f) = \int_{\mathbb{Z}_p} f(x) d\mu_q(x) = \lim_{N\to\infty} \frac{1}{\left[p^N\right]_q} \sum_{x=0}^{p^N-1} f(x) q^x. \tag{1.3}$$

It follows from the Eq. (1.3) that

$$\begin{aligned} \lim_{q\to -1} I_q(f) = I_{-1}(f) &= \int_{\mathbb{Z}_p} f(x) d\mu_{-1}(x) \\ &= \lim_{N\to\infty} \sum_{x=0}^{p^N-1} f(x)(-1)^x. \end{aligned} \tag{1.4}$$

Thus, by the Eq. (1.4), we have

$$I_{-1}(f_n)+(-1)^{n-1}I_{-1}(f)=2\sum_{s=0}^{n-1}(-1)^{n-s-1}f(s)$$

where $f_n(x)=f(x+n),(n\in\mathbb{N}.)$. For the applications of fermionic p-adic integral over the p-adic numbers field, see the references, e. g., [1,2,3,4,6,7,9,11,12,16].

In [3], the q-Frobenius-Euler polynomials are defined by the following p-adic fermionic q-integral on $\mathbb{Z}_p$, with respect to μ_{-1}:

$$H_{n,q}\left(x\mid-\lambda^{-1}\right)=\frac{\lambda+1}{2}\int_{\mathbb{Z}_p}\lambda^y[x+y]_q^n d\mu_{-1}(y). \quad (1.5)$$

Upon setting x = 0 into the Eq. (1.5) gives $H_{n,q}(0):=H_{n,q}$ which are called n-th q-Frobenius-Euler number.

By letting $q\to1^-$ in the Eq. (1.5), it yields to

$$\lim_{q\to1^-}H_{n,q}\left(x\mid-\lambda^{-1}\right):=H_n\left(x\mid-\lambda^{-1}\right)=\frac{\lambda+1}{2}\int_{\mathbb{Z}_p}\lambda^y(x+y)^n d\mu_{-1}(y).$$

Recently, many mathematicians have studied the symmetric identities on some special polynomials, see, for details, [1,6,7,9,12]. Some of mathematicians also investigated some applications of Frobenius-Euler numbers and polynomials (or its q-analog) cf. [3,4,5,13,14,15,16]. Moreover, Frobenius-Euler numbers at the value λ = −1 in (1.1) are Euler numbers that is closely related to Bernoulli numbers, Genocchi numbers, etc. For more information about these polynomials, look at [1-21] and the references cited therein.

In the present paper, we obtain not only new but also some interesting identities which are derived from the fermionic p-adic q-integral over the p-adic numbers field. The results derived here is written under Sym (5).

2. Symmetric Identities Involving *q*-Frobenius-Euler Polynomials

For $w_i\in\mathbb{N}$ with $w_i\equiv1(\mathrm{mod}\,2)$ with $i\in\{1,2,3,4,5\}$, by the Eqs. (1.3) and (1.5), we obtain

$$\int_{\mathbb{Z}_p}\lambda^{w_1w_2w_3w_4y}e^{\left[\begin{array}{l}w_1w_2w_3w_4y\\+w_1w_2w_3w_4w_5x\\+w_5w_4w_2w_3i\\+w_5w_4w_1w_3j\\+w_5w_4w_1w_2k\\+w_5w_3w_1w_2h\end{array}\right]_q t}d\mu_{-1}(y)$$
$$=\frac{2}{\lambda+1}\lim_{N\to\infty}\sum_{y=0}^{p^N-1}(-1)^y\lambda^{w_1w_2w_3w_4y}\times e^{\left[\begin{array}{l}w_1w_2w_3w_4y\\+w_1w_2w_3w_4w_5x\\+w_5w_4w_2w_3i\\+w_5w_4w_1w_3j\\+w_5w_4w_1w_2k\\+w_5w_3w_1w_2h\end{array}\right]_q t} \quad (2.1)$$
$$=\frac{2}{\lambda+1}\lim_{N\to\infty}\sum_{l=0}^{w_5-1}\sum_{y=0}^{p^N-1}(-1)^{l+y}\lambda^{w_1w_2w_3w_4(l+w_5y)}\times e^{\left[\begin{array}{l}w_1w_2w_3w_4(l+w_5y)+w_1w_2w_3w_4w_5x\\+w_5w_4w_2w_3i+w_5w_4w_1w_3j\\+w_5w_4w_1w_2k+w_5w_3w_1w_2h\end{array}\right]_q t}.$$

Taking

$$\frac{\lambda+1}{2}\sum_{i=0}^{w_1-1}\sum_{j=0}^{w_2-1}\sum_{k=0}^{w_3-1}\sum_{h=0}^{w_4-1}\left[(-1)^{i+j+k+h}\times\lambda^{\left(\begin{array}{l}w_5w_4w_2w_3i+w_5w_4w_1w_3j\\+w_5w_4w_1w_2k+w_5w_3w_1w_2h\end{array}\right)}\right]$$

on the both sides of Eq. (2.1) gives

$$\frac{\lambda+1}{2}\sum_{i=0}^{w_1-1}\sum_{j=0}^{w_2-1}\sum_{k=0}^{w_3-1}\sum_{h=0}^{w_4-1}\left[(-1)^{i+j+k+h}\times\lambda^{\left(\begin{array}{l}w_5w_4w_2w_3i+w_5w_4w_1w_3j\\+w_5w_4w_1w_2k+w_5w_3w_1w_2h\end{array}\right)}\right]$$
$$\times\int_{\mathbb{Z}_p}\lambda^{w_1w_2w_3w_4y}e^{\left[\begin{array}{l}w_1w_2w_3w_4y\\+w_1w_2w_3w_4w_5x\\+w_5w_4w_2w_3i\\+w_5w_4w_1w_3j\\+w_5w_4w_1w_2k\\+w_5w_3w_1w_2h\end{array}\right]_q t}d\mu_{-1}(y)$$
$$=\lim_{N\to\infty}\sum_{i=0}^{w_1-1}\sum_{j=0}^{w_2-1}\sum_{k=0}^{w_3-1}\sum_{h=0}^{w_4-1}\sum_{l=0}^{w_5-1}\sum_{y=0}^{p^N-1}(-1)^{i+j+k+h+y+l} \quad (2.2)$$
$$\times\lambda^{\left[\begin{array}{l}w_1w_2w_3w_4(l+w_5y)+w_5w_4w_2w_3i\\+w_5w_4w_1w_3j+w_5w_4w_1w_2k+w_5w_3w_1w_2h\end{array}\right]}$$
$$\times e^{\left[\begin{array}{l}w_1w_2w_3w_4(l+w_5y)+w_1w_2w_3w_4w_5x\\+w_5w_4w_2w_3i+w_5w_4w_1w_3j\\+w_5w_4w_1w_2k+w_5w_3w_1w_2h\end{array}\right]_q t}.$$

Note that the equation (2.2) is invariant for any permutation $\sigma\in S_5$. Hence, we have the following theorem.

Theorem 1. *Let* $w_i\in\mathbb{N}$ *with* $w_i\equiv1(\mathrm{mod}\,2)$ *with* $i\in\{1,2,3,4,5\}$. *Then the following*

$$\frac{\lambda+1}{2}\sum_{i=0}^{w_{\sigma(1)}-1}\sum_{j=0}^{w_{\sigma(2)}-1}\sum_{k=0}^{w_{\sigma(3)}-1}\sum_{h=0}^{w_{\sigma(4)}-1}(-1)^{i+j+k+h}$$
$$\times\lambda^{\left(\begin{array}{l}w_{\sigma(5)}w_{\sigma(4)}w_{\sigma(2)}w_{\sigma(3)}i\\+w_{\sigma(5)}w_{\sigma(4)}w_{\sigma(1)}w_{\sigma(3)}j\\+w_{\sigma(5)}w_{\sigma(4)}w_{\sigma(1)}w_{\sigma(2)}k\\+w_{\sigma(5)}w_{\sigma(3)}w_{\sigma(1)}w_{\sigma(2)}h\end{array}\right)}$$
$$\times\int_{\mathbb{Z}_p}\lambda^{w_{\sigma(1)}w_{\sigma(2)}w_{\sigma(3)}w_{\sigma(4)}\left(l+w_{\sigma(5)}y\right)}$$
$$\exp([w_{\sigma(1)}w_{\sigma(2)}w_{\sigma(3)}w_{\sigma(4)}y$$
$$+w_{\sigma(1)}w_{\sigma(2)}w_{\sigma(3)}w_{\sigma(4)}w_{\sigma(5)}x$$
$$w_{\sigma(5)}w_{\sigma(4)}w_{\sigma(2)}w_{\sigma(3)}i$$
$$+w_{\sigma(5)}w_{\sigma(4)}w_{\sigma(1)}w_{\sigma(3)}j$$
$$+w_{\sigma(5)}w_{\sigma(4)}w_{\sigma(1)}w_{\sigma(2)}k$$
$$+w_{\sigma(5)}w_{\sigma(3)}w_{\sigma(1)}w_{\sigma(2)}h]_q t)d\mu_{-1}(y)$$

holds true for any $\sigma \in S_5$.

By Eq. (1.2), we easily derive that

$$\begin{aligned}&\left[\begin{array}{l}w_1w_2w_3w_4y+w_1w_2w_3w_4w_5x+w_5w_4w_2w_3i\\+w_5w_4w_1w_3j+w_5w_4w_1w_2k+w_5w_3w_1w_2h\end{array}\right]_q\\&=\left[w_1w_2w_3w_4\right]_q\left[\begin{array}{l}y+w_5x+\frac{w_5}{w_1}i\\+\frac{w_5}{w_2}j+\frac{w_5}{w_3}k+\frac{w_5}{w_4}h\end{array}\right]_{q^{w_1w_2w_3w_4}}.\end{aligned} \tag{2.3}$$

From Eq. (2.1) and (2.3), we obtain

$$\begin{aligned}&\int_{\mathbb{Z}_p}\lambda^{w_1w_2w_3w_4y}e^{\left[\begin{array}{l}w_1w_2w_3w_4y\\+w_1w_2w_3w_4w_5x\\+w_5w_4w_2w_3i\\+w_5w_4w_1w_3j\\+w_5w_4w_1w_2k\\+w_5w_3w_1w_2h\end{array}\right]_q t}d\mu_{-1}(y)\\&=\sum_{n=0}^{\infty}\left[w_1w_2w_3w_4\right]_p^n\\&\times\left(\int_{\mathbb{Z}_p}\left\{\begin{array}{l}\lambda^{w_1w_2w_3w_4y}\\\times\left[\begin{array}{l}y+w_5x+\frac{w_5}{w_1}i\\+\frac{w_5}{w_2}j+\frac{w_5}{w_3}k\\+\frac{w_5}{w_4}h\end{array}\right]^n_{q^{w_1w_2w_3w_4}}\end{array}\right\}d\mu_{-1}(y)\ \frac{t^n}{n!}\right),\end{aligned} \tag{2.4}$$

from which, we have

$$\begin{aligned}&\int_{\mathbb{Z}_p}\lambda^{w_1w_2w_3w_4y}\left[\begin{array}{l}w_1w_2w_3w_4y\\+w_1w_2w_3w_4w_5x\\+w_5w_4w_2w_3i\\+w_5w_4w_1w_3j\\+w_5w_4w_1w_2k\\+w_5w_3w_1w_2h\end{array}\right]^n_q d\mu_{-1}(y)\\&=\frac{2}{\lambda+1}\left[w_1w_2w_3w_4\right]_q^n H_{n,q^{w_1w_2w_3w_4}}\\&\left(w_5x+\frac{w_5}{w_1}i+\frac{w_5}{w_2}j+\frac{w_5}{w_3}k+\frac{w_5}{w_4}h\mid-\lambda^{-w_1w_2w_3w_4}\right).\\&(n\geq 0).\end{aligned} \tag{2.5}$$

Thus, by Theorem 1 and (2.5), we procure the following theorem.

Theorem 2. *For* $w_i\in\mathbb{N}$ *with* $w_i\equiv 1(\operatorname{mod}2)$ *with* $i\in\{1,2,3,4,5\}$, *the following*

$$\begin{aligned}&\left[w_{\sigma(1)}w_{\sigma(2)}w_{\sigma(3)}w_{\sigma(4)}\right]_q^n\\&\times\sum_{i=0}^{w_{\sigma(1)}-1}\sum_{j=0}^{w_{\sigma(2)}-1}\sum_{k=0}^{w_{\sigma(3)}-1}\sum_{h=0}^{w_{\sigma(4)}-1}(-1)^{i+j+k+h}\\&\times\lambda^{\left(\begin{array}{l}w_{\sigma(5)}w_{\sigma(4)}w_{\sigma(2)}w_{\sigma(3)}i+w_{\sigma(5)}w_{\sigma(4)}w_{\sigma(1)}w_{\sigma(3)}j\\+w_{\sigma(5)}w_{\sigma(4)}w_{\sigma(1)}w_{\sigma(2)}k+w_{\sigma(5)}w_{\sigma(3)}w_{\sigma(1)}w_{\sigma(2)}h\end{array}\right)}\\&\times H_{n,q^{w_{\sigma(1)}w_{\sigma(2)}w_{\sigma(3)}w_{\sigma(4)}}}\\&\left(\begin{array}{l}w_{\sigma(5)}x+\frac{w_{\sigma(5)}}{w_{\sigma(1)}}i+\frac{w_{\sigma(5)}}{w_{\sigma(2)}}j+\frac{w_{\sigma(5)}}{w_{\sigma(3)}}k\\+\frac{w_{\sigma(5)}}{w_{\sigma(4)}}h\mid-\lambda^{-w_{\sigma(1)}w_{\sigma(2)}w_{\sigma(3)}w_{\sigma(4)}}\end{array}\right)\end{aligned}$$

holds true for any $\sigma\in S_5$.

It is shown by using the definition of $[x]_q$ that

$$\begin{aligned}&\left[y+w_5x+\frac{w_5}{w_1}i+\frac{w_5}{w_2}j+\frac{w_5}{w_3}k+\frac{w_5}{w_4}h\right]^n_{q^{w_1w_2w_3w_4}}\\&\sum_{m=0}^{n}\binom{n}{m}\left(\frac{\left[w_5\right]_q}{\left[w_1w_2w_3w_4\right]_q}\right)^{n-m}\\&\times\left[\begin{array}{l}w_4w_2w_3i+w_4w_1w_3j\\+w_4w_1w_2k+w_3w_1w_2h\end{array}\right]^{n-m}_{q^{w_5}}\\&\times q^{m\left(\begin{array}{l}w_5w_4w_2w_3i+w_5w_4w_1w_3j\\+w_5w_4w_1w_2k+w_5w_3w_1w_2h\end{array}\right)}\left[y+w_5x\right]^m_{q^{w_1w_2w_3w_4}}.\end{aligned} \tag{2.6}$$

Taking $\int_{\mathbb{Z}_p}\lambda^{w_1w_2w_3w_4y}d\mu_{-1}(y)$ on the both sides of Eq.(2.6) gives

$$\begin{aligned}&\int_{\mathbb{Z}_p}\lambda^{w_1w_2w_3w_4y}\left[\begin{array}{l}y+w_5x+\frac{w_5}{w_1}i\\+\frac{w_5}{w_2}j\\+\frac{w_5}{w_3}k+\frac{w_5}{w_4}h\end{array}\right]^n_{q^{w_1w_2w_3w_4}}d\mu_{-1}(y)\\&=\sum_{m=0}^{n}\binom{n}{m}\left(\frac{\left[w_5\right]_q}{\left[w_1w_2w_3w_4\right]_q}\right)^{n-m}\\&\times\left[w_4w_2w_3i+w_4w_1w_3j+w_4w_1w_2k+w_3w_1w_2h\right]^{n-m}_{q^{w_5}}\\&\times q^{m\left(w_5w_4w_2w_3i+w_5w_4w_1w_3j+w_5w_4w_1w_2k+w_5w_3w_1w_2h\right)}\\&\times\int_{\mathbb{Z}_p}\lambda^{w_1w_2w_3w_4y}\left[y+w_5x\right]^m_{q^{w_1w_2w_3w_4}}d\mu_{-1}(y)\\&=\frac{2}{1+\lambda}\sum_{m=0}^{n}\binom{n}{m}\left(\frac{\left[w_5\right]_q}{\left[w_1w_2w_3w_4\right]_q}\right)^{n-m}\\&\times\left[w_4w_2w_3i+w_4w_1w_3j+w_4w_1w_2k+w_3w_1w_2h\right]^{n-m}_{q^{w_5}}\\&\times q^{m\left(w_5w_4w_2w_3i+w_5w_4w_1w_3j+w_5w_4w_1w_2k+w_5w_3w_1w_2h\right)}\\&\times H_{n,q^{w_1w_2w_3w_4}}\left(w_5x\mid-\lambda^{-w_1w_2w_3w_4}\right).\end{aligned} \tag{2.7}$$

By the Eq. (2.7), we have

$$\left[w_1w_2w_3w_4\right]_q^n\frac{\lambda+1}{2}\sum_{i=0}^{w_1-1}\sum_{j=0}^{w_2-1}\sum_{k=0}^{w_3-1}\sum_{h=0}^{w_4-1}(-1)^{i+j+k+h}$$

$$\times\lambda^{\left(w_5w_4w_2w_3i+w_5w_4w_1w_3j+w_5w_4w_1w_2k+w_5w_3w_1w_2h\right)}$$

$$\times\int_{\mathbb{Z}_p}\lambda^{w_1w_2w_3w_4y}\begin{bmatrix} y+w_5x+\frac{w_5}{w_1}i \\ +\frac{w_5}{w_2}j \\ +\frac{w_5}{w_3}k+\frac{w_5}{w_4}h \end{bmatrix}_{q^{w_1w_2w_3w_4}}^{n} d\mu_{-1}(y)$$

$$=\sum_{m=0}^{n}\binom{n}{m}\left[w_1w_2w_3w_4\right]_q^m\left[w_5\right]_q^{n-m}$$
$$\times H_{n,q^{w_1w_2w_3w_4}}\left(w_5x\mid-\lambda^{-w_1w_2w_3w_4}\right)$$
$$\times\sum_{i=0}^{w_1-1}\sum_{j=0}^{w_2-1}\sum_{k=0}^{w_3-1}\sum_{h=0}^{w_4-1}(-1)^{i+j+k+h}$$
$$\times\lambda^{\left(w_5w_4w_2w_3i+w_5w_4w_1w_3j+w_5w_4w_1w_2k+w_5w_3w_1w_2h\right)}$$
$$\times q^{m\left(w_4w_2w_3i+w_4w_1w_3j+w_4w_1w_2k+w_3w_1w_2h\right)}$$
$$\times\left[w_2w_4w_3i+w_1w_3w_4j+w_1w_2w_4k+w_1w_2w_3h\right]_{q^{w_5}}^{n-m}$$
$$=\sum_{m=0}^{n}\binom{n}{m}\left[w_1w_2w_3w_4\right]_q^m\left[w_5\right]_q^{n-m}$$
$$\times H_{n,q^{w_1w_2w_3w_4}}\left(w_5x\mid-\lambda^{-w_1w_2w_3w_4}\right)$$
$$\times C_{n,q^{w_5}}\left(w_1,w_2,w_3,w_4\mid m\right), \tag{2.8}$$

where

$$C_{n,q^{w_5}}\left(w_1,w_2,w_3,w_4\mid m\right)$$
$$=\sum_{i=0}^{w_1-1}\sum_{j=0}^{w_2-1}\sum_{k=0}^{w_3-1}\sum_{h=0}^{w_4-1}(-1)^{i+j+k+h}$$
$$\times\lambda^{\left(w_5w_4w_2w_3i+w_5w_4w_1w_3j+w_5w_4w_1w_2k+w_5w_3w_1w_2h\right)} \tag{2.9}$$
$$\times q^{m\left(w_4w_2w_3i+w_4w_1w_3j+w_4w_1w_2k+w_3w_1w_2h\right)}$$
$$\times\left[w_2w_4w_3i+w_1w_3w_4j+w_1w_2w_4k+w_1w_2w_3h\right]_{q^{w_5}}^{n-m}.$$

Consequently, by (2.9), we get the following theorem.

Theorem 3. *Let* $w_i\in\mathbb{N}$ *with* $w_i=1(\bmod 2)$ *with* $i\in\{1,2,3,4,5\}$. *Then the following expression*

$$\sum_{m=0}^{n}\binom{n}{m}\left[w_{\sigma(1)}w_{\sigma(2)}w_{\sigma(3)}w_{\sigma(4)}\right]_q^m\left[w_5\right]_q^{n-m}$$
$$\times H_{n,q^{w_{\sigma(1)}w_{\sigma(2)}w_{\sigma(3)}w_{\sigma(4)}}}\left(w_{\sigma(5)}x\mid-\lambda^{\left[\begin{smallmatrix}-w_{\sigma(1)}w_{\sigma(2)}\\ \times w_{\sigma(3)}w_{\sigma(4)}\end{smallmatrix}\right]}\right)$$
$$\times C_{n,q^{w_{\sigma(5)}}}\left(w_{\sigma(1)},w_{\sigma(2)},w_{\sigma(3)},w_{\sigma(4)}\mid m\right)$$

holds true for some $\sigma\in S_5$.

3. Conclusion

We have derived some new interesting identities of q-Frobenius-Euler polynomials. We also showed that these symmetric identities are written by symmetric group of degree five.

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