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Hermite-Hadamard and Simpson Type Inequalities for Differentiable Quasi-Geometrically Convex Functions

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Abstract In this paper, the authors define a new identity for differentiable functions. By using of this identity, authors obtain new estimates on generalization of Hadamard and Simpson type inequalities for quasi-geometrically convex functions.

Keywords: *quasi-geometrically convex functions, hermite–hadamard type inequalities, simpson type inequality*

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1. Introduction

Let real function f be defined on some nonempty interval I of real line R. The function f is said to be convex on I if inequality

$$f(tx+(1-t)y)\le tf(x)+(1-t)f(y)$$

holds for all $x,y\in I$ and $t\in[0,1]$.

Following inequalities are well known in the literature as Hermite-Hadamard inequality and Simpson inequality respectively:

Theorem 1. Let $f:I\subseteq\mathbb{R}\to\mathbb{R}$ be a convex function defined on the interval I of real numbers and $a,b\in I$ with $a<b$. The following double inequality holds

$$f\left(\frac{a+b}{2}\right)\le\frac{1}{b-a}\int_a^b f(x)dx\le\frac{f(a)+f(b)}{2}.$$

Theorem 2. Let $f:[a,b]\to\mathbb{R}$ be a four times continuously differentiable mapping on (a,b) and $\left\|f^{(4)}\right\|_\infty=\sup_{x\in(a,b)}\left|f^{(4)}(x)\right|<\infty.$ Then the following inequality holds:

$$\left|\frac{1}{3}\left[\frac{f(a)+f(b)}{2}+2f\left(\frac{a+b}{2}\right)\right]-\frac{1}{b-a}\int_a^b f(x)dx\right|$$
$$\le\frac{1}{2880}\left\|f^{(4)}\right\|_\infty(b-a)^4.$$

In recent years, many athors have studied errors estimations for Hermite-Hadamard, Ostrowski and Simpson inequalities; for refinements, counterparts, generalization see [2,9,10].

The following definitions are well known in the literature.

Definition 1 ([7,8]). A function $f:I\subseteq(0,\infty)\to\mathbb{R}$ is said to be GA-convex (geometric-arithmatically convex) if

$$f(x^t y^{1-t})\le tf(x)+(1-t)f(y)$$

for all $x,y\in I$ and $t\in[0,1]$.

Definition 2 ([7,8]). A function $f:I\subseteq(0,\infty)\to(0,\infty)$ is said to be GG-convex (called in [13] geometrically convex function) if

$$f(x^t y^{1-t})\le f(x)^t f(y)^{(1-t)}$$

for all $x,y\in I$ and $t\in[0,1]$.

In [3], İşcan gave definition of quasi-geometrically convexity as follows:

Definition 3. A function $f:I\subseteq(0,\infty)\to\mathbb{R}$ is said to be quasi-geometrically convex on I if

$$f\left(x^t y^{1-t}\right)\le\sup\{f(x),f(y)\},$$

for any $x,y\in I$ and $t\in[0,1]$.

Clearly, any GA-convex and geometrically convex functions are quasi-geometrically convex functions. Furthermore, there exist quasi-geometrically convex functions which are neither GA-convex nor GG-convex [3].

For some recent results concerning Hermite-Hadamard type inequalities for GA-convex, GG-convex, quasi-geometrically convex functions we refer interestes reader to [1,3,4,5,6,11,12,14].

The goal of this article is to establish some new general integral inequalities of Hermite-Hadamard and Simpson type for quasi-geometrically convex functions by using a new integral identity.

2. Main Results

Let $f:I\subseteq(0,\infty)\to\mathbb{R}$ be a differentiable function on I°, the interior of I, throughout this section we will take

$$I_f(\lambda,\mu,a,b)=(\lambda-\mu)f\left(\sqrt{ab}\right)+\mu f(a)+(1-\lambda)f(b)-\frac{1}{\ln(b/a)}\int_a^b\frac{f(u)}{u}du$$

where $a,b\in I$ with $a<b$ and $\lambda,\mu\in\mathbb{R}$.

In order to prove our main results we need the following identity.

Lemma 1. Let $f:I\subseteq(0,\infty)\to\mathbb{R}$ be a differentiable function on I° such that $f'\in L[a,b]$, where $a,b\in I$ with $a<b$. Then for all $\lambda,\mu\in\mathbb{R}$ we have:

$$I_f(\lambda,\mu,a,b)=\ln(b/a)\left\{\int_0^{1/2}(t-\mu)a^{1-t}b^t f'\left(a^{1-t}b^t\right)dt+\int_{1/2}^{1}(t-\lambda)a^{1-t}b^t f'\left(a^{1-t}b^t\right)dt\right\}. \quad (1)$$

Proof. By integration by parts and changing the variable, we can state

$$\ln(b/a)\int_0^{1/2}(t-\mu)a^{1-t}b^t f'\left(a^{1-t}b^t\right)dt$$
$$=\int_0^{1/2}(t-\mu)df\left(a^{1-t}b^t\right)$$
$$=(t-\mu)f\left(a^{1-t}b^t\right)\Big|_0^{1/2}-\int_0^{1/2}f\left(a^{1-t}b^t\right)dt$$
$$=\left(\frac{1}{2}-\mu\right)f\left(\sqrt{ab}\right)+\mu f(a)-\frac{1}{\ln(b/a)}\int_a^{\sqrt{ab}}\frac{f(u)}{u}du$$

and similarly we get

$$\ln(b/a)\int_{1/2}^{1}(t-\lambda)a^{1-t}b^t f'\left(a^{1-t}b^t\right)dt$$
$$=\int_{1/2}^{1}(t-\lambda)df\left(a^{1-t}b^t\right)$$
$$=(t-\lambda)f\left(a^{1-t}b^t\right)\Big|_{1/2}^{1}-\int_{1/2}^{1}f\left(a^{1-t}b^t\right)dt$$
$$=(1-\lambda)f(b)-\left(\frac{1}{2}-\lambda\right)f\left(\sqrt{ab}\right)-\frac{1}{\ln(b/a)}\int_{\sqrt{ab}}^{b}\frac{f(u)}{u}du.$$

Adding the resulting identities we obtain the desired result.

Theorem 3 Let $f:I\subset(0,\infty)\to\mathbb{R}$ be a differentiable function on I° such that $f'\in L[a,b]$, where $a,b\in I^{\circ}$ with $a<b$. If $|f'|^q$ is quasi-geometrically convex on $[a,b]$ for some fixed $q\geq 1$ and $0\leq\mu\leq 1/2\leq\lambda\leq 1$, then the following inequality holds

$$I_f(\lambda,\mu,a,b)\leq\ln(b/a)\left(\sup\left\{\left|f'(a)\right|^q,\left|f'(b)\right|^q\right\}\right)^{\frac{1}{q}}\left\{C_1^{1-1/q}(\mu)C_3^{1/q}(\mu,q,a,b)+C_2^{1-1/q}(\lambda)C_4^{1/q}(\lambda,q,a,b)\right\} \quad (2)$$

where

$$C_1(\mu)=\mu^2-\frac{\mu}{2}+\frac{1}{8}, \quad (3)$$

$$C_2(\lambda)=\lambda^2-\frac{3\lambda}{2}+\frac{5}{8},$$

$$C_3(\mu,q,a,b)=\begin{cases}\dfrac{1}{2q\ln(b/a)}\Big[(1-2\mu)(ab)^{q/2}+4\mu a^{(1-\mu)q}L\left(a^{q\mu},b^{q\mu}\right)\\ \quad -a^{q/2}L\left(a^{q/2},b^{q/2}\right)-2\mu a^q\Big],\ 0<\mu\leq 1/2,\\ \dfrac{a^{q/2}}{2q\ln(b/a)}\left[b^{q/2}-L\left(a^{q/2},b^{q/2}\right)\right],\ \mu=0\end{cases}$$

$$C_4(\lambda,q,a,b)=\frac{1}{2q\ln(b/a)}\Big[2(1-\lambda)b^q-(2\lambda-1)(ab)^{q/2}-2L\left(a^q,b^q\right)+4\lambda a^{(1-\lambda)q}L\left(a^{q\lambda},b^{q\lambda}\right)-a^{q/2}L\left(a^{q/2},b^{q/2}\right)\Big],$$

and $L(a,b)$ is logarithmic mean defined by $L(a,b)=(b-a)/(\ln b-\ln a)$.

Proof. Since $|f'|^q$ is quasi-geometrically convex on $[a,b]$, for all $t\in[0,1]$

$$\left|f'\left(a^{1-t}b^t\right)\right|^q\leq\sup\left\{\left|f'(a)\right|^q,\left|f'(b)\right|^q\right\}.$$

Hence, using Lemma 1 and power mean inequality we get

$$I_f(\lambda,\mu,a,b)\leq\ln(b/a)\times\left\{\left(\int_0^{1/2}|t-\mu|dt\right)^{1-\frac{1}{q}}\left(\int_0^{1/2}|t-\mu|\left(a^{1-t}b^t\right)^q\sup\left\{\left|f'(a)\right|^q,\left|f'(b)\right|^q\right\}dt\right)^{\frac{1}{q}}\right.$$
$$\left.+\left(\int_{1/2}^{1}|t-\lambda|dt\right)^{1-\frac{1}{q}}\left(\int_{1/2}^{1}|t-\lambda|\left(a^{1-t}b^t\right)^q\sup\left\{\left|f'(a)\right|^q,\left|f'(b)\right|^q\right\}dt\right)^{\frac{1}{q}}\right\}$$
$$\leq\ln(b/a)\left(\sup\left\{\left|f'(a)\right|^q,\left|f'(b)\right|^q\right\}\right)^{\frac{1}{q}}$$
$$\times\left\{\left(\int_0^{1/2}|t-\mu|dt\right)^{1-\frac{1}{q}}\left(\int_0^{1/2}|t-\mu|\left(a^{1-t}b^t\right)^q dt\right)^{\frac{1}{q}}\right.$$
$$\left.+\left(\int_{1/2}^{1}|t-\lambda|dt\right)^{1-\frac{1}{q}}\left(\int_{1/2}^{1}|t-\lambda|\left(a^{1-t}b^t\right)^q dt\right)^{\frac{1}{q}}\right\},$$

where

$$\int_0^{1/2} |t-\mu|\,dt = C_1(\mu) = \mu^2 - \frac{\mu}{2} + \frac{1}{8},$$
$$\int_{1/2}^{1} |t-\lambda|\,dt = C_2(\lambda) = \lambda^2 - \frac{3\lambda}{2} + \frac{5}{8},$$
$$\int_0^{1/2} |t-\mu|\left(a^{1-t}b^t\right)^q dt = C_3(\mu,q,a,b),$$
$$\int_{1/2}^{1} |t-\lambda|\left(a^{1-t}b^t\right)^q dt = C_4(\lambda,q,a,b),$$

which completes the proof.

Corollary 1 Under the assumptions of Theorem 3 with $\lambda = \mu = 1/2$, the inequality (2) reduced to the following inequality

$$\left|\frac{f(a)+f(b)}{2} - \frac{1}{\ln(b/a)}\int_a^b \frac{f(u)}{u}du\right|$$
$$\le \left(\frac{1}{8}\right)^{1-1/q} \ln(b/a)\left(\sup\left\{\begin{array}{l}|f'(a)|^q,\\ |f'(b)|^q\end{array}\right\}\right)^{\frac{1}{q}} \{C_3^{1/q}(1/2,q,a,b)$$
$$\le \left(\frac{1}{8}\right)^{1-1/q} \ln(b/a)\left(\sup\left\{\begin{array}{l}|f'(a)|^q,\\ |f'(b)|^q\end{array}\right\}\right)^{\frac{1}{q}}$$
$$\times\left\{C_3^{1/q}(0,q,a,b) + C_4^{1/q}(1,q,a,b)\right\} + C_4^{1/q}(1/2,q,a,b)\Big\}.$$

Corollary 2 Under the assumptions of Theorem 3 with $\mu = 0$ and $\lambda = 1$, the inequality (2) reduced to the following inequality

$$\left|f\left(\sqrt{ab}\right) - \frac{1}{\ln(b/a)}\int_a^b \frac{f(u)}{u}du\right|$$
$$\le \left(\frac{1}{8}\right)^{1-1/q} \ln(b/a)\left(\sup\left\{|f'(a)|^q, |f'(b)|^q\right\}\right)^{\frac{1}{q}}$$
$$\times\left\{C_3^{1/q}(0,q,a,b) + C_4^{1/q}(1,q,a,b)\right\}.$$

Corollary 3 Under the assumptions of Theorem 3 with $\mu = 1/6$ and $\lambda = 5/6$, the inequality (2) reduced to the following inequality

$$\left|\frac{1}{3}\left[\frac{f(a)+f(b)}{2} + 2f\left(\sqrt{ab}\right)\right] - \frac{1}{\ln(b/a)}\int_a^b \frac{f(u)}{u}du\right|$$
$$\le \left(\frac{5}{72}\right)^{1-1/q} \ln(b/a)\left(\sup\left\{\begin{array}{l}|f'(a)|^q,\\ |f'(b)|^q\end{array}\right\}\right)^{\frac{1}{q}}$$
$$\times\left\{\begin{array}{l}C_3^{1/q}(1/6,q,a,b)\\ +C_4^{1/q}(5/6,q,a,b)\end{array}\right\}$$

Theorem 4 Let $f: I \subset (0,\infty) \to \mathbb{R}$ be a differentiable function on I° such that $f' \in L[a,b]$, where $a,b \in I^\circ$ with $a < b$. If $|f'|^q$ is quasi-geometrically convex on $[a,b]$ for some fixed $q > 1$ and $0 \le \mu \le 1/2 \le \lambda \le 1$, then the following inequality holds.

$$I_f(\lambda,\mu,a,b) \le \ln(b/a)\left(\sup\left\{|f'(a)|^q, |f'(b)|^q\right\}\right)^{\frac{1}{q}} \quad (4)$$
$$\times\left\{C_5^{1/p}(p,\mu)C_7^{1/q}(q,a,b) + C_6^{1/p}(p,\lambda)C_8^{1/q}(q,a,b)\right\}$$

where

$$C_5(p,\mu) = \frac{1}{p+1}\left[\mu^{p+1} + (\frac{1}{2}-\mu)^{p+1}\right],$$
$$C_6(p,\lambda) = \frac{1}{p+1}\left[(\lambda-\frac{1}{2})^{p+1} + (1-\lambda)^{p+1}\right],$$
$$C_7(q,a,b) = \frac{1}{2}a^{q/2}L\left(a^{q/2},b^{q/2}\right),$$
$$C_8(q,a,b) = L\left(a^q,b^q\right) - C_7(q,a,b)$$

and $\frac{1}{p} + \frac{1}{q} = 1$.

Proof. Since $|f'|^q$ is quasi-geometrically convex on $[a,b]$ and using Lemma 1 and Hölder inequality, we get

$$I_f(\lambda,\mu,a,b) \le \ln(b/a)$$
$$\times\left\{\left(\int_0^{1/2}|t-\mu|^p dt\right)^{\frac{1}{p}}\left(\int_0^{1/2}\left(a^{1-t}b^t\right)^q \sup\left\{\begin{array}{l}|f'(a)|^q,\\ |f'(b)|^q\end{array}\right\}dt\right)^{\frac{1}{q}}\right.$$
$$\left.+\left(\int_{1/2}^{1}|t-\lambda|^p dt\right)^{\frac{1}{p}}\left(\int_{1/2}^{1}\left(a^{1-t}b^t\right)^q \sup\left\{\begin{array}{l}|f'(a)|^q,\\ |f'(b)|^q\end{array}\right\}dt\right)^{\frac{1}{q}}\right\}$$
$$\le \ln(b/a)\left(\sup\left\{|f'(a)|^q, |f'(b)|^q\right\}\right)^{\frac{1}{q}}$$
$$\times\left\{\left(\int_0^{1/2}|t-\mu|^p dt\right)^{\frac{1}{p}}\left(\int_0^{1/2}\left(a^{1-t}b^t\right)^q dt\right)^{\frac{1}{q}}\right.$$
$$\left.+\left(\int_{1/2}^{1}|t-\lambda|^p dt\right)^{\frac{1}{p}}\left(\int_{1/2}^{1}\left(a^{1-t}b^t\right)^q dt\right)^{\frac{1}{q}}\right\},$$

here it is seen by simple computation that

$$\int_0^{1/2}|t-\mu|^p dt = \frac{1}{p+1}\left[\mu^{p+1} + (\frac{1}{2}-\mu)^{p+1}\right],$$
$$\int_{1/2}^{1}|t-\lambda|^p dt = \frac{1}{p+1}\left[(\lambda-\frac{1}{2})^{p+1} + (1-\lambda)^{p+1}\right],$$
$$\int_0^{1/2}\left(a^{1-t}b^t\right)^q dt = \frac{a^{q/2}}{2}L\left(a^{q/2},b^{q/2}\right)$$
$$\textit{and} \int_{1/2}^{1}\left(a^{1-t}b^t\right)^q dt = L\left(a^q,b^q\right) - \frac{a^{q/2}}{2}L\left(a^{q/2},b^{q/2}\right).$$

Hence, the proof is completed.

Corollary 4 *Under the assumptions of Theorem 4 with* $\lambda = \mu = 1/2$, *the inequality (4) reduced to the following inequality*

$$\left| \frac{f(a)+f(b)}{2} - \frac{1}{\ln(b/a)} \int_a^b \frac{f(u)}{u} du \right|$$
$$\leq \ln(b/a) \left(\sup\left\{ \left| f'(a) \right|^q, \left| f'(b) \right|^q \right\} \right)^{\frac{1}{q}}$$
$$\times \left(\frac{1}{2^{p+1}(p+1)} \right)^{1/p} \left\{ C_7^{1/q}(q,a,b) + C_8^{1/q}(q,a,b) \right\}.$$

Corollary 5 *Under the assumptions of Theorem 4 with* $\mu = 0$ *and* $\lambda = 1$, *the inequality (4) reduced to the following inequality*.

$$\left| f\left(\sqrt{ab}\right) - \frac{1}{\ln(b/a)} \int_a^b \frac{f(u)}{u} du \right|$$
$$\leq \ln(b/a) \left(\sup\left\{ \left| f'(a) \right|^q, \left| f'(b) \right|^q \right\} \right)^{\frac{1}{q}}$$
$$\times \left(\frac{1}{2^{p+1}(p+1)} \right)^{1/p} \left\{ C_7^{1/q}(q,a,b) + C_8^{1/q}(q,a,b) \right\}.$$

Corollary 6 *Under the assumptions of Theorem 4 with* $\mu = 1/6$ *and* $\lambda = 5/6$, *the inequality (4) reduced to the following inequality*

$$\left| \frac{1}{3}\left[\frac{f(a)+f(b)}{2} + 2f\left(\sqrt{ab}\right) \right] - \frac{1}{\ln(b/a)} \int_a^b \frac{f(u)}{u} du \right|$$
$$\leq \frac{\ln(b/a)}{2} \left(\sup\left\{ \left| f'(a) \right|^q, \left| f'(b) \right|^q \right\} \right)^{\frac{1}{q}}$$
$$\times \left(\frac{1+2^{p+1}}{6^{p+1}(p+1)} \right)^{1/p} \left\{ C_7^{1/q}(q,a,b) + C_8^{1/q}(q,a,b) \right\}.$$

Theorem 5 *Let* $f: I \subset (0,\infty) \to \mathbb{R}$ *be a differentiable function on* I° *such that* $f' \in L[a,b]$, *where* $a,b \in I^\circ$ *with* $a < b$. *If* $|f'|^q$ *is quasi-geometrically convex on* $[a,b]$ *for some fixed* $q > 1$ *and* $0 \leq \mu \leq 1/2 \leq \lambda \leq 1$, *then the following inequality holds*

$$I_f(\lambda,\mu,a,b)$$
$$\leq \ln(b/a) \left(\sup\left\{ \begin{matrix} \left| f'(a) \right|^q, \\ \left| f'(b) \right|^q \end{matrix} \right\} \right)^{\frac{1}{q}} \tag{5}$$
$$\times \left\{ C_7^{1/p}(p,a,b) C_5^{1/q}(q,\mu) + C_8^{1/p}(p,a,b) C_6^{1/q}(q,\lambda) \right\}$$

where C_5, C_6, C_7, C_8 are defined as in Theorem 4 and $\frac{1}{p} + \frac{1}{q} = 1$.

Proof. Since $|f'|^q$ is quasi-geometrically convex on $[a,b]$ and using Lemma 1 and Hölder inequality, we get

$$I_f(\lambda,\mu,a,b) \leq \ln(b/a)$$
$$\times \left\{ \left(\int_0^{1/2} \left(a^{1-t} b^t \right)^p dt \right)^{\frac{1}{p}} \left(\int_0^{1/2} |t-\mu|^q \sup\left\{ \begin{matrix} \left| f'(a) \right|^q, \\ \left| f'(b) \right|^q \end{matrix} \right\} dt \right)^{\frac{1}{q}} \right.$$
$$\left. + \left(\int_{1/2}^{1} \left(a^{1-t} b^t \right)^p dt \right)^{\frac{1}{p}} \left(\int_{1/2}^{1} |t-\lambda|^q \sup\left\{ \begin{matrix} \left| f'(a) \right|^q, \\ \left| f'(b) \right|^q \end{matrix} \right\} dt \right)^{\frac{1}{q}} \right\}$$
$$\leq \ln(b/a) \left(\sup\left\{ \left| f'(a) \right|^q, \left| f'(b) \right|^q \right\} \right)^{\frac{1}{q}}$$
$$\left\{ \left(\int_0^{1/2} \left(a^{1-t} b^t \right)^p dt \right)^{\frac{1}{p}} \left(\int_0^{1/2} |t-\mu|^q dt \right)^{\frac{1}{q}} \right.$$
$$\left. + \left(\int_{1/2}^{1} \left(a^{1-t} b^t \right)^p dt \right)^{\frac{1}{p}} \left(\int_{1/2}^{1} |t-\lambda|^q dt \right)^{\frac{1}{q}} \right\},$$
$$\leq \ln(b/a) \left(\sup\left\{ \left| f'(a) \right|^q, \left| f'(b) \right|^q \right\} \right)^{\frac{1}{q}}$$
$$\left\{ C_7^{1/p}(p,a,b) C_5^{1/q}(q,\mu) + C_8^{1/p}(p,a,b) C_6^{1/q}(q,\lambda) \right\}.$$

Hence, the proof is completed.

Corollary 7 *Under the assumptions of Theorem 5 with* $\lambda = \mu = 1/2$, *the inequality (5) reduced to the following inequality*

$$\left| \frac{f(a)+f(b)}{2} - \frac{1}{\ln(b/a)} \int_a^b \frac{f(u)}{u} du \right|$$
$$\leq \ln(b/a) \left(\sup\left\{ \left| f'(a) \right|^q, \left| f'(b) \right|^q \right\} \right)^{\frac{1}{q}}$$
$$\times \left(\frac{1}{2^{q+1}(q+1)} \right)^{1/q} \left\{ C_7^{1/p}(p,a,b) + C_8^{1/p}(p,a,b) \right\}.$$

Corollary 8 *Under the assumptions of Theorem 5 with* $\mu = 0$ *and* $\lambda = 1$, *the inequality (5) reduced to the following inequality*

$$\left| f\left(\sqrt{ab}\right) - \frac{1}{\ln(b/a)} \int_a^b \frac{f(u)}{u} du \right|$$
$$\leq \ln(b/a) \left(\sup\left\{ \left| f'(a) \right|^q, \left| f'(b) \right|^q \right\} \right)^{\frac{1}{q}}$$
$$\times \left(\frac{1}{2^{q+1}(q+1)} \right)^{1/q} \left\{ C_7^{1/p}(p,a,b) + C_8^{1/p}(p,a,b) \right\}.$$

Corollary 9 *Under the assumptions of Theorem 5 with* $\mu = 1/6$ *and* $\lambda = 5/6$*, the inequality (5) reduced to the following inequality*

$$\left|\frac{1}{3}\left[\frac{f(a)+f(b)}{2}+2f\left(\sqrt{ab}\right)\right]-\frac{1}{\ln(b/a)}\int_a^b \frac{f(u)}{u}du\right|$$

$$\leq \frac{\ln(b/a)}{2}\left(\sup\left\{\left|f'(a)\right|^q,\left|f'(b)\right|^q\right\}\right)^{\frac{1}{q}}$$

$$\times\left(\frac{1+2^{q+1}}{6^{q+1}(q+1)}\right)^{1/q}\left\{C_7^{1/p}(p,a,b)+C_8^{1/p}(p,a,b)\right\}.$$

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