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Minimum weight design of prestressed concrete beams by a modified grid search technique

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ABSTRACT

In this study, a computer program, which employs modified grid search optimization technique, has been developed for the minimum weight of prestressed concrete beams under flexure. Optimum values of prestressing force, eccentricity and cross-sectional dimensions are determined subject to constraints on the design variables and stresses. The developed computer program offers a rapid practical and interactive method for realizing optimum design of beams of general I-shaped cross-section with eight geometrical design variables. The computer program can assist a designer in producing efficient designs rapidly and easily. Two examples, one of which is available in the literature and the other is modified form of it, have been solved for minimum cross-sectional area designs and the results were found to be in good agreement.

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1. Introduction

Optimization is generally defined as the best condition. For prestressed concrete, the best condition can be defined as to find the minimum cross-section, minimum prestressing or minimum cost of a beam. Several analytical studies of optimum design of prestressed beams have been reported in the literature (Morris, 1978; Taylor, 1987; Cohn and MacRae, 1983; Jones, 1984; Saouma and Murad, 1984; Birkeland, 1974; Fereig, 1994; Wang, 1970; Kirsh, 1972). In these studies, linear programming methods and non-linear programming procedures such as gradient methods have been used as optimization techniques. However, the grid search optimization method, which is simple and effective for programming, has not been used for the optimization of prestressed concrete beams before.

In the present study, a modified grid search optimization method (Çağatay, 1996) has been applied to the prestressed concrete beam. The advantages of the method can be defined as: with no restriction on the number and type of constraints on the design variables and stresses.

The proposed method is applied to an example problem available in the literature and the results are in good agreement.

A computer program has been developed employing the modified grid search optimization method, which can assist a designer in producing efficient designs rapidly and easily.

2. Problem Formulation

In general, most papers on prestressed concrete beam design and optimization deal with idealized I beam section with six dimensions (Morris, 1978; Taylor, 1987; Cohn and MacRae, 1983; Jones, 1984) due to simplicity. But, here, the method has been formulated for a general I shaped cross-section with eight geometrical design variables denoted by X_1 through X_8 as shown in Fig. 1. The variables X_7 and X_8 are calculated as a function of m, which is the slope of the top and bottom flanges of the cross-section, as shown in Fig. 1.

If the m value is chosen to be zero, the cross-section of the beam turns into an idealized I-beam. Also, additional design variables X_9 and X_{10} are considered, which represent the eccentricity and the prestressing force, respectively. In the figure, Y_t and Y_b are the distances of the top and bottom fibers of the cross-section from the center of gravity of concrete section (*c.g.c.*), and h denotes the total depth of the cross-section.

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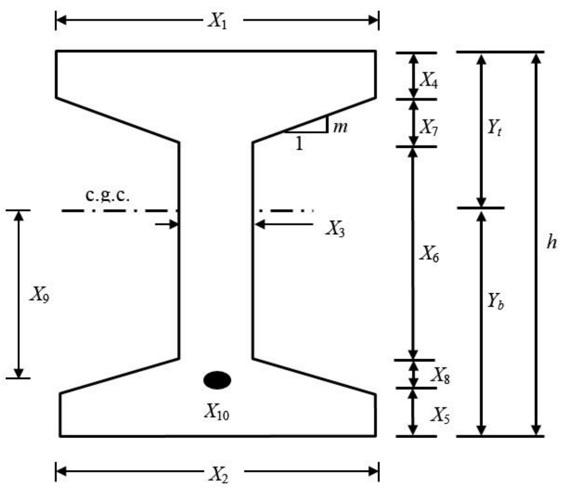

Fig. 1. Cross-section of general I-shaped PC beam.

The objective function to be minimized is denoted by $f(X)$ and the constraint functions by $g_i(X)$. The optimization problem can thus be defined as:

Minimize $\quad f(X) = A_p \qquad (1)$

Subject to $\quad g_i(X \leq 0)\,, \geq 0\,, (i = 1, 2, \ldots, k) \qquad (2)$

where A_p is the cross-sectional area of prestressed concrete beam and k denotes the number of constraints. The aims of the objective function are to minimize both the cross-sectional area of prestressed concrete and find the minimum prestressing force corresponding to maximum eccentricity. To obtain the minimum prestressing force corresponding to maximum eccentricity, Magnel's graphical method has been used (Magnel, 1948).

The following constraints were considered for the optimization problem: flexural stresses, prestressing force and eccentricity, cross-sectional dimensions and ultimate moment.

2.1. Flexural stresses

There are two stages of loading to be considered. The first stage is at the transfer of prestressing force to the beam. For computing the flexural stresses at the top and bottom of the beam, following equations can be written:

$$f_{tt} = \frac{P}{A} - \frac{P\,e\,Y_t}{I} + f_{td}\,, \tag{3}$$

$$f_{tt} = \frac{P}{A} - \frac{P\,e\,Y_b}{I} - f_{bd}\,, \tag{4}$$

where f_{tt} and f_{tb} are the flexural stresses at the top and bottom of the beam and P and e are the prestressing force and the eccentricity, respectively. I and A denote the gross second moment of area and the gross area of the cross-section, respectively. f_{td} and f_{bd} are the flexural stresses at the top and bottom of the beam due to the dead load.

The second stage is at the service condition when the beam carries live loads in addition to its own weight. The following equations can be written:

$$f_{st} = \frac{P\,\alpha}{A} - \frac{P\,\alpha\,e\,Y_t}{I} + f_{td} + f_{tl}\,, \tag{5}$$

$$f_{sb} = \frac{P\alpha}{A} + \frac{P\,\alpha\,e\,Y_b}{I} - f_{bd} + f_{bl}\,, \tag{6}$$

where f_{st} and f_{sb} are the flexural stresses at the top and bottom of the beam, respectively; α is the loss factor; f_{tl} and f_{bl} are the stresses due to the live load at the top and bottom fibers, respectively.

The stresses computed by Eqs. (3-6) should not exceed the allowable stresses, which are specified in design codes. Rearranging Eqs. (3-6), and considering the allowable stresses, the following inequalities are obtained:

$$\left(-\frac{1}{A} + \frac{e\,Y_t}{I}\right)\frac{1}{(f_{IT}+f_{td})} \leq \frac{1}{P}\,, \tag{7}$$

$$\left(\frac{1}{A} + \frac{e\,Y_b}{I}\right)\frac{1}{(f_{IC}+f_{bd})} \leq \frac{1}{P}\,, \tag{8}$$

$$\left(\frac{1}{A} - \frac{e\,Y_t}{I}\right)\frac{\alpha}{(f_{FC}-f_{td}-f_{tl})} \leq \frac{1}{P}\,, \tag{9}$$

$$\left(\frac{1}{A} + \frac{e\,Y_b}{I}\right)\frac{\alpha}{(-f_{FT}+f_{bd}+f_{bl})} \geq \frac{1}{P}\,, \tag{10}$$

where f_{IT} and f_{IC} are the allowable tensile and compressive stresses in the concrete at transfer condition and f_{FC} and f_{FT} are the allowable compressive and tensile stresses in the concrete at service condition, respectively.

2.2. Prestressing force and eccentricity

For the selection of prestressing force and eccentricity, Magnel's graphical technique (Magnel, 1948) has been used and the following inequalities are obtained:

$$E_3 \geq E_1\,, \tag{11}$$

$$E_4 \geq E_2\,, \tag{12}$$

in which

$$E_1 = \frac{1}{(f_{IT}+f_{td})}\,, \tag{13}$$

$$E_2 = \frac{1}{(f_{IC}+f_{bd})}\,, \tag{14}$$

$$E_3 = \frac{\alpha}{(-f_{FC}+f_{td}+f_{tl})}\,, \tag{15}$$

$$E_4 = \frac{\alpha}{(-f_{FT}+f_{bd}+f_{bl})}\,. \tag{16}$$

If Eqs. (11-12) are satisfied, and then the cross-sectional area is adequate under the given loading condition. It is assumed that for maximum efficiency, the eccentricity takes its largest allowable value at mid-span (with minimum cover). Therefore,

$$e \leq Y_b - e_{cover}\,, \tag{17}$$

in which e_{cover} is the required concrete cover.

2.3. Cross-sectional dimensions

Aspect ratios of the web and flanges cannot exceed a prescribed limiting value without any conditions concerning slenderness. This limit is presently taken as 8.0.

Further constraints have also been introduced for the cross-sectional dimensions as follows:

$$(X_i)_{min} \leq X_i \leq (X_i)_{max} \quad (i = 1, 2, \ldots, n)\,, \tag{18}$$

where n is the number of variables.

2.4. Ultimate moment

Ultimate moment design is based on the solution of the equations of equilibrium, using the equivalent rectangular stress block shown in Fig. 2. An equivalent rectangular stress block is used with ease and without loss of accuracy to calculate the compressive force and hence the flexural moment strength of the section.

Design requirement should meet the following condition:

$$\phi M_n \geq M_u\,, \tag{19}$$

where M_u is the applied factored moment at a section; ϕ is the strength reduction factor which is taken to be as 0.9.

3. Optimization Procedure

The algorithm described below was developed by Çağatay (1996) and is a modification of the one given by Walsh (1975). This modified algorithm, which follows, is going to be used in this study.

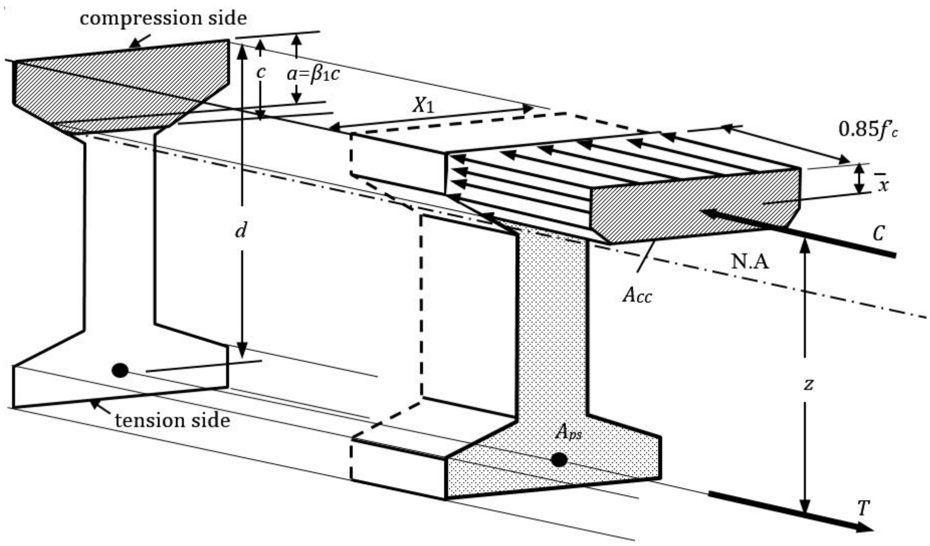

Fig. 2. Assumed stress distribution in the cross-section of the beam at ultimate limit state.

The algorithm described below was developed by Çağatay (1996) and is a modification of the one given by Walsh (1975). This modified algorithm, which follows, is going to be used in this study.

1. Start with the solution region defined by Eq. (18), in which $X_i (i = 1,2, \ldots, n)$, are the design variables, $(X_i)_{min}$ and $(X_i)_{max}$ are the minimum and maximum values of the corresponding variables, respectively. Take three values for each variable in the region, two of them at the ends and the third at the midpoint. Make a search for the optimum among all possible combinations of computation points, which satisfies the constraints.
2. Reduce the solution region to half the width of the previous one. Take the range for each variable to be equal to half the range of the previous step, with two additional computation points on the two sides of the previous optimizing computation point while making sure to remain within the initial solution range.
3. Repeat step 2 until the variable set is obtained with the desired accuracy. For the accuracy criterion, different options can be used; a fixed percentage accuracy for each variable or a percentage accuracy in the least squares sense, etc. In the present study, fixed percentage accuracy criterion has been employed. That is to say, computation is continued until

$$\varepsilon_i = \frac{(X_i)_{K+1} - (X_i)_K}{(X_i)_K}\,, \tag{20}$$

$$\varepsilon_i \leq (\varepsilon_i)_{accuracy} \quad (i = 1, 2, \ldots, n)\,, \tag{21}$$

where ε_i and $(\varepsilon_i)_{accuracy}$ are the limits of the respective variables.

4. Numerical Examples

4.1. Example 1

Design a simply supported beam of 16460 mm span subjected to an applied load of 23.34 kN/m. Assume the unit weight of concrete is 24 kN/m^3, allowable stresses for compression are at transfer 16.55 MPa, at service 15.51 MPa, those for tension are at transfer –1.31 MPa, at service -2.93 MPa and loss factor is 0.85, f_c'=34 MPa, f_{pu}=1862 MPa, and e_{cover}=50 mm. For this problem, also assume that, because of clearance requirements, the overall depth of the beam, h, cannot exceed 914.4 mm.

This example was discussed first by Khachaturian and Gurfinkel (1969), and later by Taylor and Amirebrahimi (1987). The solutions and the comparison of the results are presented in Tables 1-2 and Table 3, respectively.

Table 1. Variables, initial values and results for Example 1.

Variables	Minimum Values (mm)	Maximum Values (mm)	Initial Values (mm)	Optimum Results (mm)
X_1	101.6	750.0	425.8	587.9
X_2	101.6	750.0	425.8	506.8
X_3	101.6	400.0	250.8	101.6
X_4	101.6	400.0	250.8	101.6
X_5	50.0	150.0	100.0	124.6
X_6	400.0	900.0	650.0	681.2

Table 2. Optimum results, the stresses at transfer and service for Example 1.

Stage	Location	Stress (MPa)	Allowable Stress (MPa)	P (kN)	E (mm)	Cross-sectional area (mm²)
Transfer	Top	-1.31	-1.31	1424.0	409.7	192103.0
	Bottom	16.16	16.55			
Service	Top	15.51	15.51			
	Bottom	-2.93	-2.93			

Table 3. Comparison of the results.

	P (kN)	E (mm)	A (mm²)
Khachaturian and Gurfinkel	1650	360.6	219999
Taylor and Amirebrahimi	1673	370.8	205160
This study	1424	409.7	192103

As seen from Table 3, the program gives the minimum cross-sectional area and the minimum prestressing force. The cross-sectional area is found 14% less than the solution of Khachaturian and Gurfinkel (1969), and 6% less than the solution given by Taylor and Amirebrahimi (1987). The overall depth of the beam is found to be 907.4 mm, which had to be less than 914.4 mm. But, Taylor and Amirebrahimi (1987) obtained this value as exceeded the aforementioned constraint.

In the present study, the prestressing force is found to be 15% less than the solution of Khachaturian and Gurfinkel (1969), and 17% less than the solution given by Taylor and Amirebrahimi (1987).

4.2. Example 2

This example has the same span and loading as Example 1, but the cross-section has a general I shape as seen in Fig. 2.

To obtain optimum solution, m is chosen in the range from 0 to 1.5. The optimum values are obtained when m=0.5. The cross-sectional area with eight dimensions is found to be 209158 mm², which is 8% more than the solution of idealized section considered in Example 1. The prestressing force is 1530 kN which is also 7% more than the value obtained for the idealized cross-section.

Table 4. Optimum results for Example 2.

X_1 (mm)	X_2 (mm)	X_3 (mm)	X_4 (mm)	X_5 (mm)	X_6 (mm)	X_7 (mm)	X_8 (mm)	P (kN)	e (mm)	Cross-sectional area (mm²)
506.8	405.5	101.6	101.6	143.1	664.6	103.3	75.9	1530	401	209158

5. Conclusions

A computer program that is capable of finding the minimum weight of a prestressed concrete beam satisfying the given constraints including flexural stresses, cover requirement, the aspect ratios for top and bottom flanges and web part of a beam and ultimate moment, based on modified grid search optimization technique, is developed and evaluated. The computer program offers a rapid, practical, and interactive method for realizing optimum design of beams of general I sections.

The program finds the optimum solution in a few iterations. Thus, a considerable saving is obtained in computational work. The program has also graphical output, which indicates Magnel diagram with feasible region, which helps in determining the prestressing force and the eccentricity values.

Two examples, one of which is available in the literature and the other is modified form of it, have been solved for minimum cross-sectional area designs and the results were found to be in good agreement.

The idealized I beam section with six dimensions gives about 10% smaller value compared with general I-shaped beam section with eight dimensions. In the case of general I section, taking 0.5 for the slope, m, yields the minimum cross-sectional area.

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