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Published in the Russian Federation

European Journal of Contemporary Education

ISSN 2219-8229

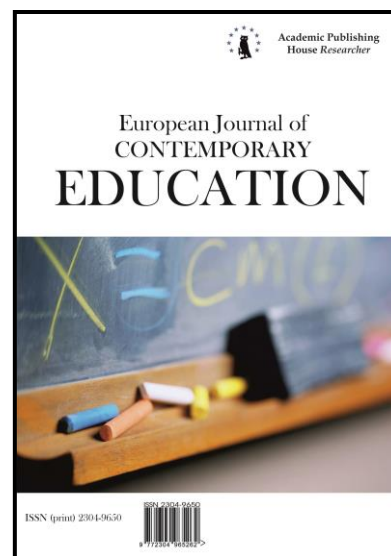
E-ISSN 2224-0136

Vol. 13, Is. 3, pp. 198-212, 2015

DOI: 10.13187/ejced.2015.13.198

[www.ejournal1.com](http://www.ejournal1.com)

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UDC 004.94: 37.02

## Computer-Assisted Simulation Methods of Learning Process

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### Abstract

In this article we analyse: 1) one–component models of training; 2) the multi–component models considering transition of weak knowledge in strong and vice versa; 3) the models considering change of working efficiency of the pupil during the day. The results of imitating modeling are presented, graphs of dependences of the pupil’s knowledge on time are provided.

**Keywords:** computer modeling, didactics, education, mathematical methods, pedagogy, pupil, simulations, teacher.

### Introduction

One of the directions of development of the modern theory of training consists in studying didactic systems by the methods of mathematical [1–7] and imitating (or computer) modelling [8–10]. Using the method of imitating modelling, scientists can investigate complicated objects and processes in those cases when real experiments with them are impossible or inexpedient. The essence of this method consists in creation of computer model of real system and carrying out a series of computing experiments for the purpose of understanding the system’s behavior or an estimation of various strategies of management providing its functioning [11, p. 12]. High speed of modern computers allows to process large amounts of information and quickly enough to carry out computer imitation. Changing initial data and parameters of the model it is possible to investigate ways of the system development, and to define its state at the end of training. This is the advantage of this approach in comparison with the method of the qualitative analysis. Therefore, researching various mathematical and computer models of process of training has a great importance for the development of didactics.

R. Shannon calls the process of creating the imitating model "intuitive art" or "modelling art" which "consists in ability to analyse a problem, to mark out its essential features from it by abstraction, to choose and modify properly the main assumptions characterizing the system and then to fulfill and improve the model until it begins to yield results, useful for practice" [11, p. 34]. In certain cases for studying didactic systems imitating models based on the solution of system of differential equations, discrete models when the pupil is modelled as the probabilistic automats, multi-agent modelling, when each pupil is replaced with the program agent functioning irrespective to other agents, are used [8–10, 12–14].

Let us formulate the main task of imitating modelling the process of training: knowing the parameters of pupils, characteristics of the used methods and the training program (the distribution of educational information), it is necessary to define the amount of knowledge or the level of pupil's skill formation at any stage of training [14]. Also it is possible to solve the optimizing task consisting in finding the distribution of the training material, the level of the teacher's requirements, the duration of lessons for the pupil's amount of knowledge to reach the preset or maximum value at the end of training, and for the process of training to satisfy the restrictions imposed on it.

Methodological basis of the research are ideas and works by N. Winer, K. Shannon, F. Rosenblatt, V. M. Glushkov, D. A. Pospelov (cybernetics, the theory of information), R. Atkinson [1], G. Bauer of [1], O. G. Gohmann [4], L. B. Itel'son [3], L. P. Leontyev [4], Je. Kroters [1], D. A. Novikov [15], F. S. Roberts [5] (mathematical modelling of training), B. Skinner, N. Krauder, S. I. Arkhangel'sky, V. P. Bepalko, E. I. Mashbits, V. E. Firstov [7], V. S. Avanesov, I. V. Robert (cybernetic approach in pedagogy, the programmed training and the automated training systems). The mathematical model of didactic transitional process offered by V. V. Mayer [16, p. 129–140] has great influence too.

The main idea of the research consists in that it really makes sense to use the imitating models based on the solution of differential equations for studying didactic systems in those situations when it is inexpedient or impossible to make pedagogical experiments with people. So, the real pupil is replaced by some abstract model which behavior is described by one or several equations. According to the principle of plurality of the description, any complex system can be modelled by a large number of ways. Therefore, discussing the problem designated above, we will speak about a set or hierarchy of models, each of which is development and specification of the previous. The task consists in researching behavior of this or that abstract model of the pupil or all didactic systems in various situations.

### 1. One-component model of training

In the most rough approach it is possible to consider that the training material is uniform, that is consists of elements, independent and equal in complexity, which are all equally easily acquired, and at the end of training are forgotten with an identical speed. In this case we have a one-component model of training result of which is characterized by the level of pupil's knowledge. Let us formulate its cornerstone principles:

1. The information (knowledge) given to pupils is a set of elements of learning material (ELM) untied among themselves while their number is proportional to its quantity. All ELMs are remembered almost equally and forgotten with an identical speed.

2. The teacher demands of the pupil to acquire all given information  $I(t)$ , that is the level of requirements is  $L = I(t)$ . The speed of change of pupil's knowledge quantity is equal to a difference of speeds of assimilation and forgetting.

3. The speed of increase in knowledge is proportional to the product of amount of pupil's knowledge  $Z$  raised to power  $b$  ( $0 \leq b \leq 1$ ) and amounts of the efforts  $F$  made by the pupil in a unit of time:  $dZ/dt = \alpha F Z^b$ . The more the pupil knows, the easier he acquires new knowledge because of the formed associative links with the existing knowledge. On the other hand, the lower the student's motivation, the less effort  $F$  he is making, and the lower the rate of increase in his knowledge. In the case when the growth of pupil's knowledge  $\Delta Z$  is much less than

his total knowledge  $Z$  (training during one or several lessons), it is possible to consider that  $Z$  almost doesn't increase and  $b = 0$ .

4. The pupil's efforts  $F$  (or motivation  $M$ ) are directly proportional to a difference  $D$  between the level  $L$  of the teacher's requirements (amount of information which the pupil has to acquire) and the quantity of his knowledge  $Z$ :  $F = k(L - Z) = kD$ . In the case when  $D = L - Z$  exceeds some limit  $C$ , the pupil stops to make efforts:  $F = 0$ .

5. In the absence of training the quantity of the pupil's knowledge decreases because of forgetting according to the exponential law. The speed of forgetting is proportional to quantity of the available knowledge:  $dZ/dt = -\gamma Z$ ,  $Z(t) = Z_0 \exp(-\gamma \cdot t)$ . The forgetting coefficient is  $\gamma = 1/\tau$ , where  $\tau$  – the time of knowledge reduction in  $e = 2,72\dots$  times.

Basing on the listed above reasons, we receive that during training the pupil's knowledge increase speed is equal to:

$$\frac{dZ}{dt} = \begin{cases} \alpha \cdot (L - Z)Z^b - \gamma \cdot Z, & \text{if } Z < L \leq Z + C, \\ -\gamma \cdot Z, & \text{if } L > Z + C. \end{cases}$$

When  $Z$  isn't big, the growth rate of knowledge level is low due to the lack of possibility to form associative links. With increasing knowledge  $Z$  the rate grows, but at  $Z \rightarrow L$  it decreases due to the decrease in efforts  $F$  (the pupil's motivation  $M$ ). If  $L$  exceeds  $Z$  by an amount which is greater than the critical value  $C$ , the pupil stops to study. In the considered equations, time is an independent variable and can be measured in days, months or years. It is also convenient to measure it in the conventional units of time (CUT), and coefficients of assimilation and forgetting – in  $\text{CUT}^{-1}$ . The state of the didactic system is characterized by quantities of this or that type of the knowledge reported by the teacher and acquired by the pupils; these quantities are proportional to the number of the studied notions, formulas and other ELMs and can be measured in conventional units too. The coefficients of these and subsequent models are selected so that the results of modelling could correspond to common sense and teaching.

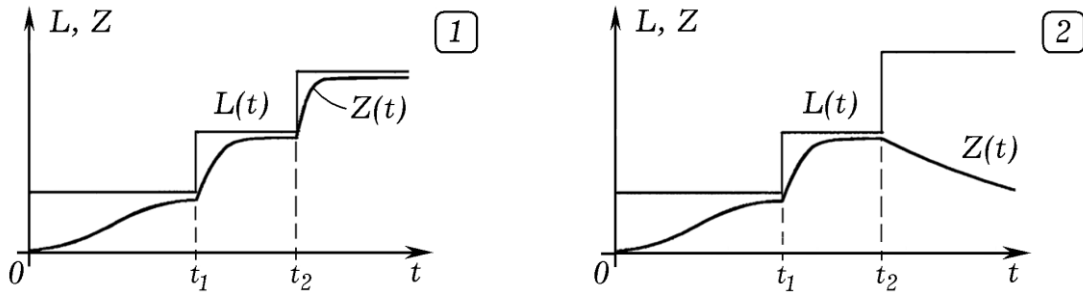
Also other approaches are possible. For example, it is possible to consider that the efforts  $F$  made by the pupil are connected with the level of teacher's requirements  $L$  as follows:

$$F = \begin{cases} L - Z, & \text{if } Z < L \leq Z + C, \\ C \exp[-\beta(L - Z - C)], & \text{if } L > Z + C. \end{cases}$$

That is if  $L - Z < C$ , then  $F = L - Z$ , and in the case when  $L \geq Z + C$  the motivation gradually decreases exponentially:  $F = C \exp[-\beta(L - Z - C)]$ . By means of this model it is possible to prove the well-known principle "from simple to complicated" [17]. Let us say, at first a complicated theme is studied, and then a simple one, that is at first the level of teacher's requirements are high, and then – low ( $L_1 > L_2$ ). If  $L_1$  strongly surpasses the quantity of the pupil's knowledge  $Z$ , the motivation to training decreases, and the level of pupil's knowledge doesn't grow (the pupil can't simply acquire the material). If the pupil has acquired a difficult theme, and after studying the next simpler theme, the growth rate of knowledge is low because the requirements level  $L$  slightly surpasses the quantity of the pupil's knowledge  $Z$ , and he doesn't make many efforts. Therefore, it is expedient to study the simple theme first and then a difficult (or complicated) one.

Pic. 1 shows the considered above training model when the level  $L$  of the teacher's requirements (or amount of the knowledge reported by the teacher) increases in steps. Initially the teacher offers pupils rather simple educational material, and when they master it the teacher increases the level of requirements, offering more difficult material. To ensure increasing of the pupil's knowledge, it is necessary to provide not a very big difference between  $Z$  and  $L$ .

Too sharp increase in the level of requirements  $L$  (complexity and the amount of new material) leads to decrease in motivation and reduction of quantity of knowledge owing to forgetting (pic. 1.2). At the moment  $t_2$  the pupil "comes off" or gets behind the teacher, ceasing to acquire the information given to him. If at first too difficult tasks (requirements level is high) are offered, and then – simple ones, results of training will be low. For the best of educational process it is necessary to select the level of requirements so that motivation to training should stay high.



Pic. 1. Training at intermittent (step-wise) increase of requirements level

The particular interest is represented by the case when the speed of assimilation of educational information is constant and equal to the maximum value  $v_m$ . That is what a skilled teacher aims to reach. If the pupil's initial knowledge is  $Z_0$  then:

$$\frac{dZ}{dt} = v_m - \gamma Z, \quad \int_{Z_0}^{Z(t)} \frac{dZ}{v_m - \gamma Z} = \int_0^t dt, \quad Z(t) = \frac{v_m}{\gamma} (1 - \exp(-\gamma t)) + Z_0 \exp(-\gamma t).$$

At the end of training, speed of assimilation is equal 0,  $dZ/dt = -\gamma Z$ ; the amount of knowledge decreases exponentially.

## 2. Results of one-component model use

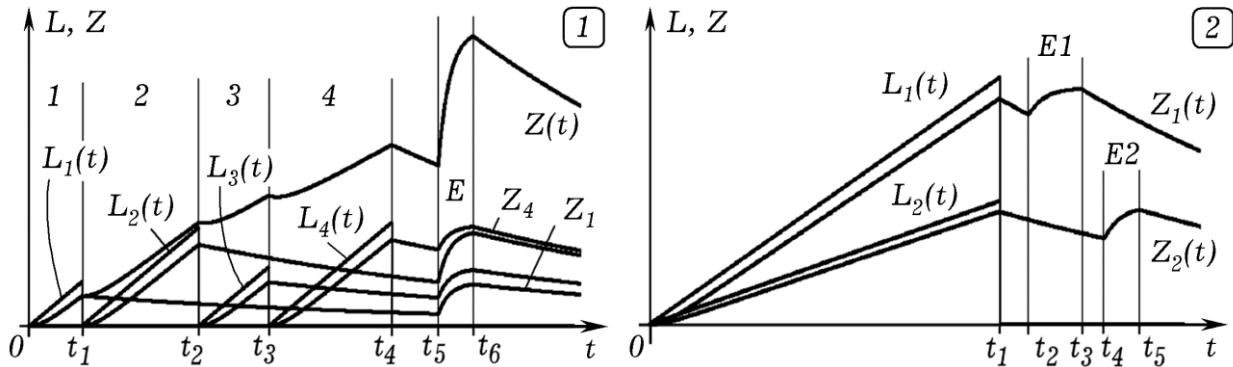
With the help of the considered model let us analyse studying independent themes of various complexity which aren't connected among themselves, that is mastering of one theme doesn't influence mastering of another.

**Situation 1.** A group of pupils studies a subject of four independent themes. Each theme is finished with a test, and at the end of the course there is an examination while preparing for which the pupil has to learn all ELMs. Requirements level for each theme grows according to the law  $L_i = 0,05(t - t_{0i})$ , and time of studying each theme is equal  $T_i = \{ 90; 200; 120; 210 \}$  ( $i = 1, 2, 3, 4$ ). We get the graph of pupil's knowledge dependence on time. The coefficients of the pupil's assimilation (mastering) and forgetting  $\alpha = 0,035 \text{ CUT}^{-1}$ ,  $\gamma = 0,0015 \text{ CUT}^{-1}$ . For the solution of this task a special computer program created with the help of Free Pascal is used. It contains a cycle on time which calculates the quantity of the pupil's knowledge in consecutive timepoints and draws the graphs  $Z_i(t)$ . If  $\Delta\tau$  – step on time, then we have the equations:

$$dZ_i / dt = \alpha \cdot (L_i - Z_i) - \gamma \cdot Z_i, \quad Z_i^{t+1} = Z_i^t + (\alpha \cdot (L_i^t - Z_i^t) - \gamma \cdot Z_i^t) \Delta\tau.$$

The received graphs are shown in pic. 2. It is obvious that when studying each theme the level of requirements  $L_i = L_i(t)$  grows in proportion to time. At each of the following lessons within one theme the teacher requires mastering new knowledge and preservation of the knowledge obtained earlier. Studying the theme is finished with a test; then the teacher reminds about it only at the examination. During studying the  $i$ -th theme, the quantity of the pupil's knowledge  $Z_i$  of this theme increases, and after its termination  $Z_i$  decreases exponentially because of forgetting.

During preparation for the examination E (an interval  $[t_5; t_6]$ ) the pupils revise and recall the material of all themes. As they use more efforts and study all free time, their knowledge considerably increases. After the examination levels of knowledge  $Z_i$  decrease owing to forgetting.



Pic. 2. Studying of the training courses which are finished with examinations

**Situation 2.** Within several months at the same time (that is in parallel) the pupil studies two courses. At each subsequent lesson the teacher requires the knowledge of all previous material, and requirements levels  $L_1$  and  $L_2$  grow in proportion to time  $t$ . At the end of the term the examinations in all themes are provided.

The program modelling the training process should contain a cycle on time  $t$  in which the speed of knowledge increase is calculated, amounts of the pupil's knowledge  $Z_1(t)$  and  $Z_2(t)$  in the following timepoint  $t + \Delta t$  are defined; the results are displayed on the screen. Then everything is repeated again. The received graphs  $Z_1(t)$  and  $Z_2(t)$  are shown in pic. 2.2. Parameters of the model are selected so that reasonable results should be obtained. During the term (from 0 to  $t_1$ ) the pupil's knowledge level monotonously grows, then decreases a little (pic. 2.2). While revising (preparing) for examinations (from  $t_2$  to  $t_3$  and from  $t_4$  to  $t_5$ ) the level of the corresponding knowledge increases again, and after passing the examination – decreases because of forgetting.

Now let us consider the situation when themes have various complexity  $S_i$  and are not independent, that is mastering one theme demands understanding some other themes. When studying each theme the level of requirements increases; pupils prepare for the test paper consisting of several tasks. At the end of the course an examination is held. The time  $T_i$  ( $i = 1, 2, \dots, n$ ) corresponding to each theme is given. Let us construct a computer model of the training process.

We take into account that the difficulty, or the subjective complexity  $S_l$  of the studied  $l$ -th theme of the course, can depend on the pupil's knowledge quantity  $Z_k$  of the  $k$ -th theme. Let this dependence be expressed as:  $S_l = a + b \exp(-cZ_k)$ , where  $a, b, c > 0$ ,  $a + b \leq 1$ . The difficulty of the theme lies in the range of  $[0, 1]$ ; if  $Z_k$  is growing, the difficulty  $S_l$  decreases to  $a$ . The minimum difficulty  $S = 0$  corresponds to a very simple (easy) theme, maximum  $S = 1$  – to the theme which the pupil can not understand in principle (very much time is required for this purpose). The equation is:

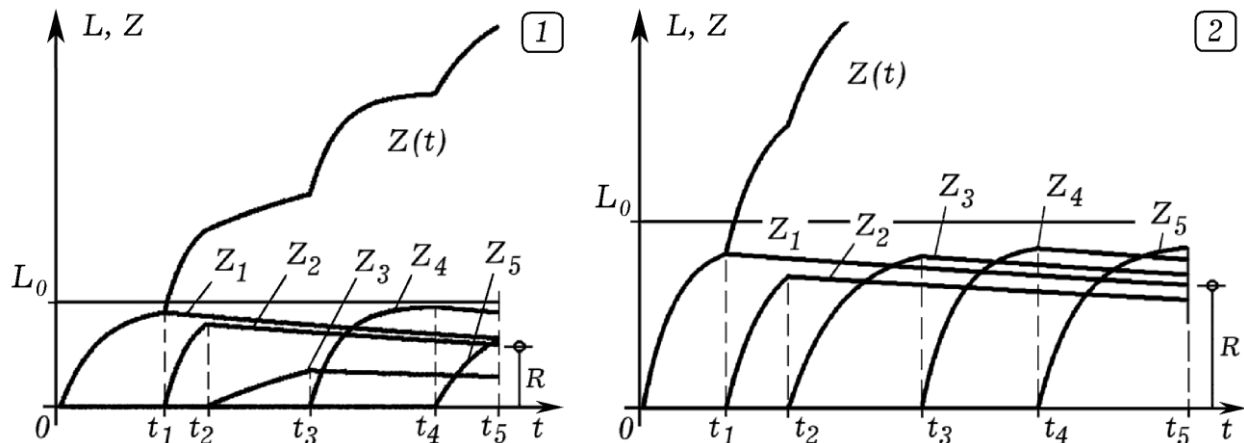
$$\frac{dZ_i}{dt} = \alpha(1 - S_i)(L_i - Z_i) - \gamma \cdot Z_i; \quad Z_i > 0; \quad L_i > Z_i; \quad \alpha, \gamma > 0,$$

where  $Z_i$  – the level of the pupil’s knowledge of the  $i$  –th theme,  $L_i$  – the level of teacher’s requirements, that is the knowledge amount of the  $i$  –th theme which the pupil should to acquire. We receive in the final differences:

$$Z_i^{t+1} = Z_i^t + (\alpha(1 - S_i)(L_i - Z_i^t) - \gamma \cdot Z_i^t) \Delta \tau.$$

After studying the course the teacher holds an examination (test) of  $m$  tasks  $K = \{z_1(1), z_2(1,2), \dots, z_m(4,5)\}$ . If for the solution of task  $z_k(i)$  it is enough to have the knowledge of the  $i$  –th theme, the probability of its solution is equal to the level of the pupil’s mastering of this theme:  $p_k = Z_i / L_i$ ; so  $Z_i \leq L_i$ , then  $0 < p_k \leq 1$ . Let us use the law of multiplication of probabilities. If the  $k$  –th task  $z_k(r, s)$  is of the combined type and demands the knowledge of the  $r$  –th and  $s$  –th themes, then the probability of its solution is  $p_k = (Z_r / L_r)(Z_s / L_s)$ .

**Situation 3.** The student studies the course consisting of five themes which are not connected among themselves and presented consistently one after another: 1, 2, 3, 4, 5. The difficulties  $S_i$  of the themes and the time  $T_i$  given for studying are set by two matrixes:  $S = (0,3; 0,1; 0,4; 0,7; 0,2)$  and  $T = (1,2; 1,7; 1,5; 1,8; 2,4)$ . At the end of the course the test of five tasks is held:  $K = \{z_1(1), z_2(1,2), z_3(2,3), z_4(3,4), z_5(3,5)\}$ . Let us create the imitating model of this process and calculate the result of fulfilling the test.



Pic. 3. Modelling of studying the themes with various complexities (difficulties).

The used computer program contains the cycle on time which values  $Z_i$  for every  $i$  –th theme ( $i = 1, 2, \dots, 5$ ), and the total level of knowledge is defined. The results are displayed in the graph (pic. 3.1). The grade for the examination is calculated with a formula:  $R = (Z_1 + Z_1Z_2 + Z_2Z_3 + Z_3Z_4 + Z_3Z_5) / 5$ . In pic. 3.1 the mark  $R$  corresponds to the segment in the right part of the graph. At other values of  $S_i$  and  $T_i$  we have the graphs presented in pic. 3.2.

### 3. Two-component model of training of the first type

To increase the accuracy of results let us consider that durability of various mastered ELMs is not identical. We divide all the pupil’s knowledge into two categories: strong (or solid) and weak (poor). Strong knowledge is more involved in the pupil’s educational activity and therefore is forgotten significantly slower than the weak knowledge. Let us consider a two-component model of the pupil, and let us divide all acquired information into two categories: 1) knowledge  $K_{n-1}$  which is used daily and therefore is hardly forgotten (reading, writing, arithmetic operations, the simple facts, etc.); 2) knowledge  $K_{n-2}$  which is seldom used and therefore is forgotten quickly (difficult

ideas, principles, facts, theories). The offered two-component model of training is expressed by the system of the equations:

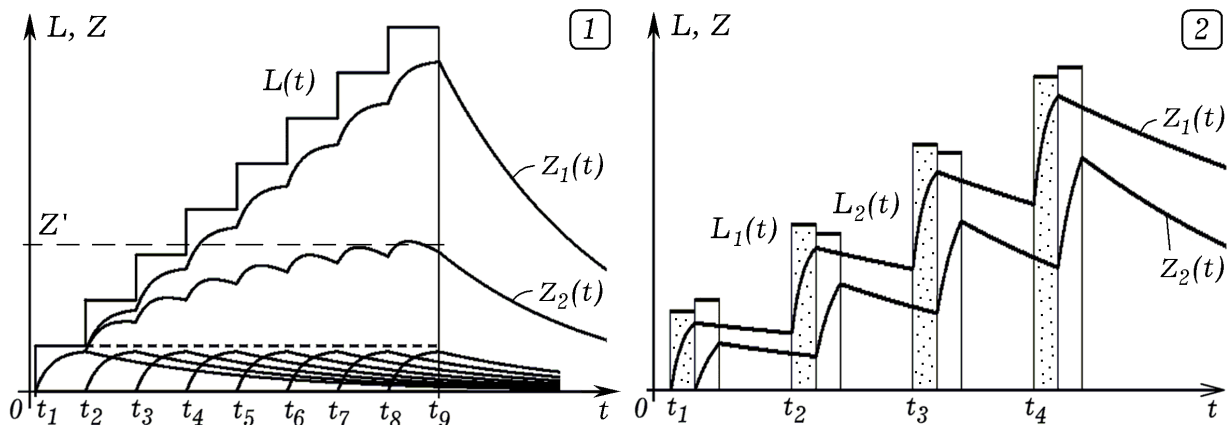
$$dZ_1 / dt = k\alpha_1(L_1 - Z_1) - \gamma_1 Z_1, \quad dZ_2 / dt = k\alpha_2(L_2 - Z_2) - \gamma_2 Z_2, \quad Z = Z_1 + Z_2.$$

Here  $L_1$  and  $L_2$  – are levels of the teacher’s requirements, corresponding to Kn–1 and Kn–2, amount of which is equal  $Z_1$  and  $Z_2$ ;  $Z$  – is the pupil’s total knowledge. While training  $k = 1$ , else  $k = 0$ .

**Situation 4.** When studying some theme during eight lessons pupils gain knowledge of two types: 1) knowledge Kn–1 which after studying is used at the subsequent lessons; 2) knowledge Kn–2 which is studied once and is not used any more. The requirement levels equal to quantity of knowledge which must be acquired at each lesson, are known:  $L_{1i} = (30, 60, 90, 120, 150, 180, 210, 240)$ ,  $L_{2i} = (30, 30, 30, 30, 30, 30, 30, 30)$ . Let us simulate the training process.

At the  $i$ –th lesson the teacher reports  $L_{1i}$  knowledge Kn–1 and  $L_{2i}$  knowledge Kn–2, requiring full mastering of each piece. Requirements level for knowledge Kn–1 every week increases in steps: the pupil should remember the information received at previous and present (current) lessons. The requirements level to knowledge Kn–2 in the process of studying of the course remains constant and is equal 70; the teacher requires pupils to master knowledge Kn–2 studied only at the present  $i$ –th lesson.

The results of modelling are presented in pic. 4.1. It is obvious that the quantity of knowledge Kn–1 in the process of studying the course monotonously increases while the amount of knowledge Kn–2 at first grows, and then, having reached  $Z'$ , fluctuates relative to this value. Since  $t_8$  there comes dynamic balance: the average amount of knowledge Kn–2 acquired by the pupil during rather a long time is equal to the amount of the knowledge forgotten by him during the same time. At the end of training forgetting begins.



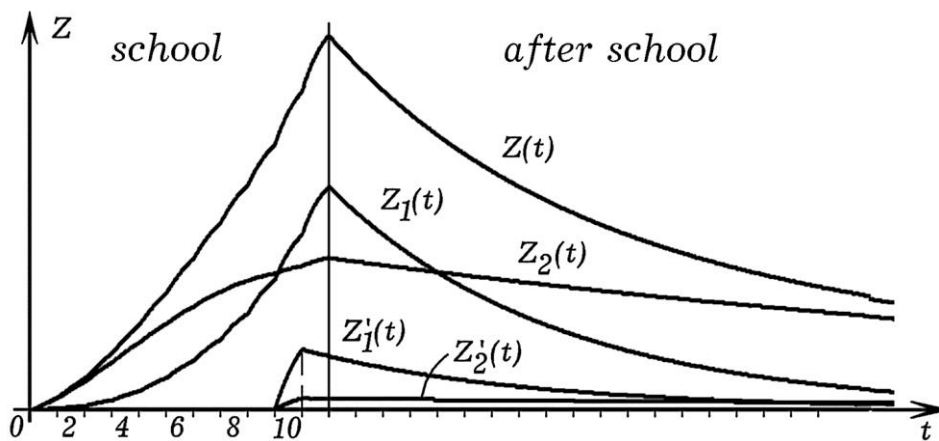
Pic. 4. Results of modelling: situations 4 and 5.

**Situation 5.** Within four weeks the pupil visits lessons of subjects 1 and 2 (for example, English and German), following each another once a week. The quantity of knowledge  $L_i$  ( $i = 1, 2$ ) which the pupil should acquire, is given. The pupil’s coefficients of assimilation and forgetting for subject 1 are equal  $\alpha_1 = 0,025 \text{ CUT}^{-1}$  and  $\gamma_1 = 0,0005 \text{ CUT}^{-1}$ , and for subject 2 –  $\alpha_2 = 0,012 \text{ CUT}^{-1}$  and  $\gamma_2 = 0,001 \text{ CUT}^{-1}$ . It is necessary to investigate the change of the pupil’s knowledge level in the process of studying both education courses.

Modelling is carried out similarly. In the cycle on time  $Z_1$  and  $Z_2$  are separately calculated; the results are displayed in the form of graphs (pic. 4.2). It is obvious that during lessons the knowledge quantity of subject 1 and subject 2 increases. In breaks between lessons the knowledge level decreases owing to forgetting.

**Situation 6.** The pupil studies at school for 11 years. While training the coefficient of assimilation of information increases and is set by the matrix  $\alpha_i = (0,01; 0,015; 0,02; 0,025; 0,03; 0,035; 0,04; 0,045; 0,05; 0,055; 0,06)$ . The requirements levels of the teacher is corresponding to knowledge  $Kn-1$  and  $Kn-2$  to be acquired in the  $i$ -th class are set by the matrixes:  $L_1 = (50, 46, 42, 36, 30, 25, 20, 15, 10, 10, 10)$  and  $L_2 = (4, 8, 14, 18, 24, 28, 33, 38, 46, 58, 62)$ . Forgetting coefficients of  $Kn-1$  and  $Kn-2$  are equal  $\gamma_1 = 0.002 \text{ CUT}^{-1}$  and  $\gamma_2 = 0.01 \text{ CUT}^{-1}$ . It is necessary to calculate the total level of the pupil's knowledge and quantity of knowledge  $Kn-1$  and  $Kn-2$  at various moments  $t$ .

The results of modelling are given in pic. 5. It presents: 1) graphs of  $Z_1(t)$  and  $Z_2(t)$  dependences of knowledge quantity  $Kn-1$  and  $Kn-2$  on time; 2) the graph  $Z(t) = Z_1 + Z_2$  of dependence of total of knowledge on time; 3) graphs  $Z_1'(t)$  and  $Z_2'(t)$  dependences of knowledge  $Kn-1$  and  $Kn-2$  acquired by the pupil in the 10-th class on time. It is obvious that during training the total knowledge quantity, and also levels of knowledge  $Kn-1$  and  $Kn-2$  monotonously increase in school, and after training decrease owing to forgetting. The pupil's knowledge  $Kn-1$  is forgotten significantly quicker, than  $Kn-2$ . The parameters of the model and requirements levels  $L_1(t)$ ,  $L_2(t)$  are selected so that the model approximately corresponds the typical situation which is found in practice.



Pic. 5. Change of knowledge quantity when training at school.

**4. Multi-component model of training of the second type**

It is known that process of mastering (assimilation) and remembering of the given information consists in establishing associative links between new and existing knowledge. As a result, the acquired knowledge becomes stronger and is forgotten much slower. Let us consider a multi-component model of training which takes into account transformation of weak (poor) knowledge into the strong (solid) knowledge:

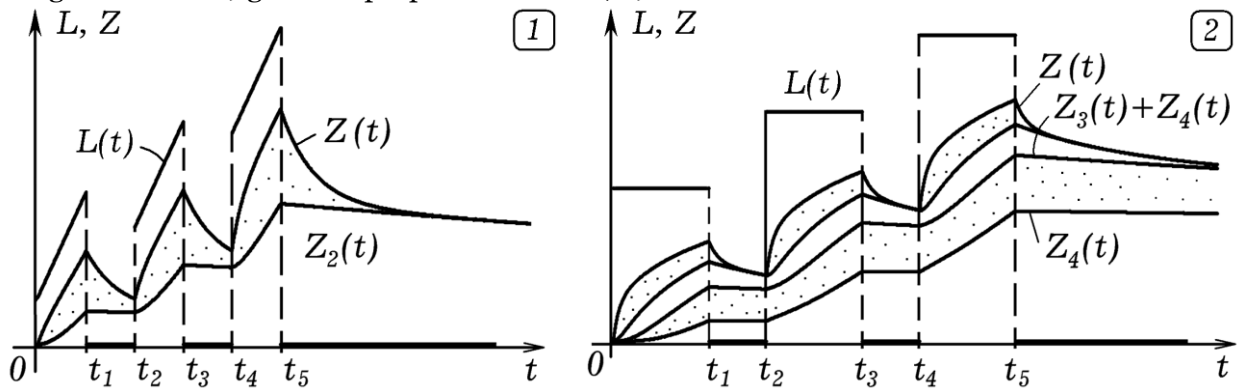
$$dZ_1 / dt = k\alpha(L - Z)Z^b - k\alpha_1 Z_1 - \gamma_1 Z_1, \quad dZ_2 / dt = k\alpha_1 Z_1 - k\alpha_2 Z_2 - \gamma_2 Z_2, \\ dZ_3 / dt = k\alpha_2 Z_2 - k\alpha_3 Z_3 - \gamma_3 Z_3, \quad dZ_4 / dt = k\alpha_3 Z_3 - \gamma_4 Z_4,$$

where  $L$  – the requirements level equal to the knowledge quantity  $I(t)$  presented by the teacher,  $Z$  – the pupil's total knowledge,  $Z_1$  – the weakest knowledge of the first category (type) with high coefficient of forgetting  $\gamma_1$ , and  $Z_4$  – the strongest knowledge of the fourth category (or type) with low  $\gamma_4$  ( $\gamma_4 < \gamma_3 < \gamma_2 < \gamma_1$ ). Coefficients of assimilation  $\alpha_i$  characterize the transformation speed of knowledge of  $i$ -th category into the knowledge of  $(i + 1)$ -th category.



While training,  $k = 1$ , and when it stops  $k = 0$ . If the increase of the pupil's knowledge is significantly less than their total knowledge, then  $b = 0$ . The forgetting coefficient is  $\gamma_i = 1/\tau_i$ , where  $\tau_i$  – time during which the knowledge quantity of  $i$ -th category decreases by  $e = 2,72\dots$ . The result of training is characterized by the total level of the acquired knowledge  $Z = Z_1 + Z_2 + Z_3 + Z_4$  and the durability coefficient  $K_D = (Z_2/4 + Z_3/2 + Z_4)/Z$  which is within the interval  $[0; 1]$ . In the course of training the amount of weak knowledge  $Z_1$  and  $Z_2$  grows, weak knowledge transforms into strong, the quantity of strong knowledge  $Z_3$  and  $Z_4$  increases, durability  $K_D$  grows.

**Situation 7.** The teacher gives three lessons. Let us analyse the training process of the pupil by means of two and four-component models in the cases, when the requirements level  $L(t)$  during the lesson: 1) grows in proportion to time; 2) remains constants.



Pic. 6. Two- and four-component model of training.

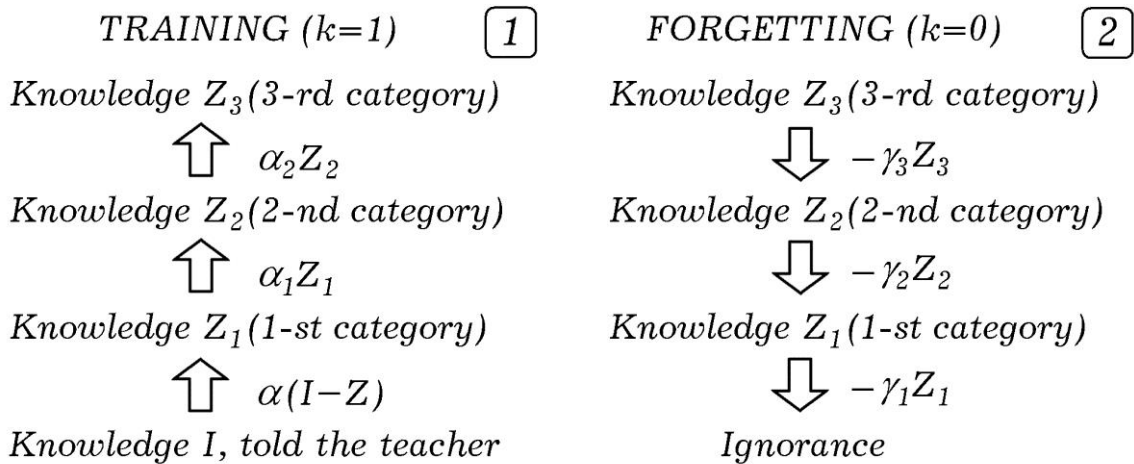
The two-component model of training is expressed by the equations:

$$\begin{aligned} dZ_1 / dt &= k\alpha(L - Z) - k\alpha_1 Z_1 - \gamma_1 Z_1, \\ dZ_2 / dt &= k\alpha_1 Z_1 - \gamma_2 Z_2, \quad Z = Z_1 + Z_2. \end{aligned}$$

The results of imitating modelling are shown in pic. 6.1. The teacher gives three lessons during which the requirements level grows in proportion to time:  $L = a(t - t_0) + b$ . It is obvious that during breaks and after training the quantity of weak (poor) pupil's knowledge  $Z_1$  decreases quickly, and strong knowledge  $Z_2$  is forgotten significantly slower. When using four-component model of training similar results (pic. 6.2) turn out. It is considered that the requirements level  $L(t)$  during lessons remains constant.

### 5. Multi-component model of training of the third type

The pupil's total knowledge  $Z$  includes weak knowledge of the first category (or type)  $Z_1$ , stronger knowledge of the second category (know-how or ability)  $Z_2$  and very strong knowledge of the third category (skills)  $Z_3$ :  $Z = Z_1 + Z_2 + Z_3$ . In the course of training ( $k = 1$ ) at first information given by the teacher turns into knowledge of the first category, and then as a result of its use when performing educational tasks – into knowledge of the second and third category (pic. 7). So, durability of the acquired material gradually increases. The speed of transformation (or transition) of weak knowledge into category of stronger knowledge is characterized by coefficients of assimilation  $\alpha$ ,  $\alpha_1$  and  $\alpha_2$ .



Pic. 7. Change of durability of the acquired knowledge during training and forgetting.

With no training ( $k = 0$ ) there is the back transition (pic. 7): a part of strong knowledge of the third category gradually becomes less strong knowledge of the second category, then partially turns into the category of weak knowledge of the first category and is forgotten. Transformation speeds of strong knowledge into weak and into ignorance while forgetting are characterized by coefficients of forgetting  $\gamma_1$ ,  $\gamma_2$  and  $\gamma_3$ . So, the following principles are the cornerstone of the offered model:

1. In the course of training the pupil operates with the information which is available for him, performing various educational tasks. Thus the knowledge reported by the teacher at first is acquired as weak or fragile (become knowledge of the first category), then in the process of their revision and use – is stronger (turn into knowledge of the second category), and then becomes strong (knowledge of the third category).

2. The increase speed of pupil’s weak knowledge in the course of training is proportional to a difference between the level of the teacher’s requirements  $L$  (the quantity of the reported knowledge) and the pupil’s total knowledge  $Z = Z_1 + Z_2 + Z_3$  and is equal  $\alpha(L - Z)$ .

3. While training the speed of transformation of weak knowledge  $Z_i$  into stronger knowledge  $Z_{i+1}$  is proportional to the quantity of weak knowledge  $Z_i$  and is equal  $\alpha_i Z_i$  ( $i = 1, 2$ ). Thus the quantity of forgotten information is negligible.

4. With no training there is forgetting: the pupil’s knowledge becomes less strong, and then turns into ignorance. The speed of transformation of strong pupil’s knowledge  $Z_i$  into less strong knowledge  $Z_{i-1}$  or into ignorance is proportional to quantity  $Z_i$  and is equal  $-\gamma_i Z_i$  ( $i = 1, 2, 3$ ).

The result of training is characterized by the total level of the acquired pupil’s knowledge  $Z$  and durability coefficient  $K_D = (Z_2 / 2 + Z_3) / Z$ . If all knowledge acquired by the pupil during studies is weak ( $Z_1 = Z$ ,  $Z_2 = Z_3 = 0$ ), the durability coefficient  $K_D = 0$ . It is necessary to aspire to a situation, when all acquired knowledge is strong ( $Z_3 = Z$ ,  $Z_1 = Z_2 = 0$ ), then  $K_D = 1$ . With long studying of one theme the knowledge level  $Z$  increases to  $L$ , along with it there is a share increase of strong knowledge  $Z_3 / Z$ , durability  $K_D$  grows, tending to 1.

The offered three–component model of training is expressed by the system of the equations (when training  $k = 1$ ; while forgetting  $k = 0$ ):

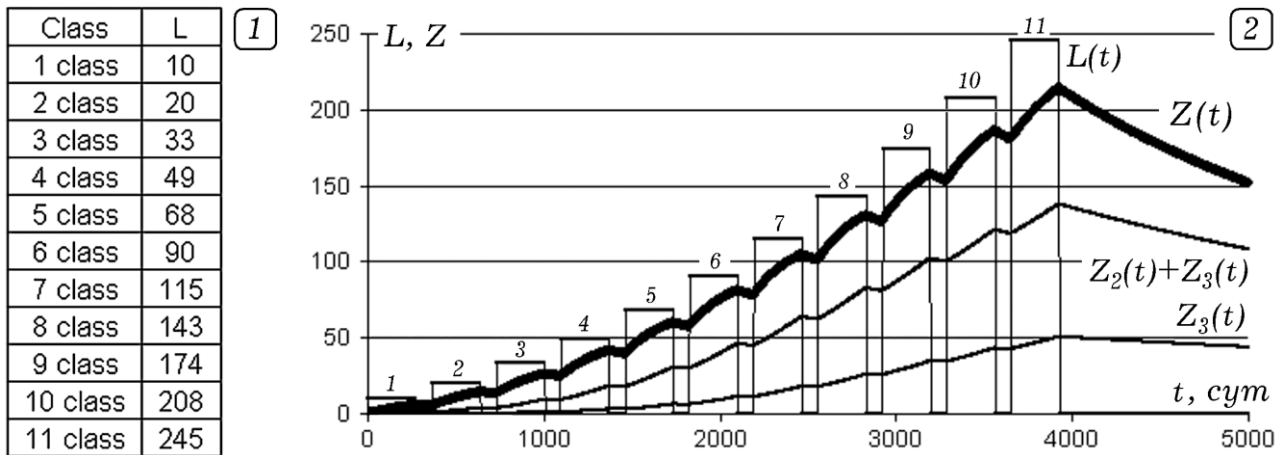
$$\begin{aligned} dZ_1 / dt &= k(\alpha(L - Z) - \alpha_1 Z_1) - (1 - k)(\gamma_1 Z_1 - \gamma_2 Z_2), \\ dZ_2 / dt &= k(\alpha_1 Z_1 - \alpha_2 Z_2) - (1 - k)(\gamma_2 Z_2 - \gamma_3 Z_3), \end{aligned}$$

$$dZ_3 / dt = k\alpha_2 Z_2 - (1-k)\gamma_3 Z_3, \quad Z = Z_1 + Z_2 + Z_3.$$

$$\alpha = (0,003 + 0,01 \cdot (1 - \exp(-Z/100))) \cdot (1 - 0,07 \cdot S_j), \quad \alpha_1 = \alpha / e,$$

$$\alpha_2 = \alpha_1 / e, \quad \gamma_1 = 0,001, \quad \gamma_2 = \gamma_1 / e, \quad \gamma_3 = \gamma_2 / e, \quad e = 2,72 \dots$$

To solve this system of the equations with the help of the numerical method there is a special computer program. Thanks to it is possible to make the imitational model of training at 11-year school. The table (pic. 8.1) shows tentative (or estimated) values of the teacher's requirements level  $L_j$  ( $j = 1, 2, \dots, 11$ ) for each class; the complexity  $S_j$  of training material is given as:  $S_j = 0,07 j$ . It is considered that within a year the pupil studies for 275 days, and has a rest during 90 days of summer vacation. Coefficients of assimilation and forgetting are selected so that the graph of the total knowledge would approximately correspond to a rather successful pupil who acquires 70–90 percent of the required information (pic. 8). Abscissa axis shows the time in days from the moment of the pupil arrival at school in the first form. It is obvious that eventually the quantity of total knowledge  $Z(t)$  and levels of formation of abilities (know-how) and skills increase. After the course of training forgetting begins; first of all, the pupil loses weak knowledge which is not demanded in practice. Failures in graphics  $Z(t)$  correspond to three-month vacation.



Pic. 8. Results of imitating modelling of training at school.

### 6. Accounting of change of the pupil's efficiency during the day

It is known that the pupil's efficiency (working capacity) during the day gradually decreases and leads to reduction of speed (or deceleration) of assimilating knowledge. Let us consider that the speed of the pupil's knowledge increase is proportional to his coefficient of learning  $\alpha$ , working efficiency coefficient  $r$ , the applied efforts  $F$  (or motivation  $M$ ) and quantity of knowledge  $Z$  in degree  $b$  ( $0 \leq b \leq 1$ ):  $dZ / dt = r\alpha F Z^b - \gamma \cdot Z$  where  $\gamma$  – the forgetting coefficient. When the pupil does the work, at first value  $r$  is equal  $r_0$  ( $0 < r_0 \leq 1$ ), and then smoothly decreases to 0 according to the law:  $r = r_0 / (1 + \exp(k_1(P - P_0)))$ . Here  $P_0$  – the work (product of activity) done by the pupil at the lesson where its working capacity decreases from  $r_0 = 1$  to  $r = 0,5$ . While training the level of teacher's requirements (the knowledge he told to pupil) is more than the level of pupil's knowledge ( $L > Z$ ), and the study work done by the pupil (the number of the fulfilled tasks) depends on the applied efforts (intensity of mental activity) and training duration. The pupil's efforts  $F$  are proportional to the difference between the level of the teacher's requirements  $L$  and quantity of pupil's knowledge  $Z$ . We receive that:

$$F = L - Z, \Delta P = k_2 F \Delta t = k_2 (L - Z) \Delta t, P = \sum_{i=1}^N (k_2 F_i + k_2') \Delta t.$$

Here  $N$  – the number of elementary periods into which the lesson is divided. If the level of teacher’s requirements is low ( $L \leq Z$ ), that is the pupil is occupied with the solution of tasks, simple for him, the work made by him are proportional to time:  $P = k_2' t$ . It allows to consider the appearance of fatigue and decrease in working capacity of the pupil even in the case when he does simple tasks for a long time. In breaks between lessons the pupil has a rest, his working capacity is restored according to the exponential law:

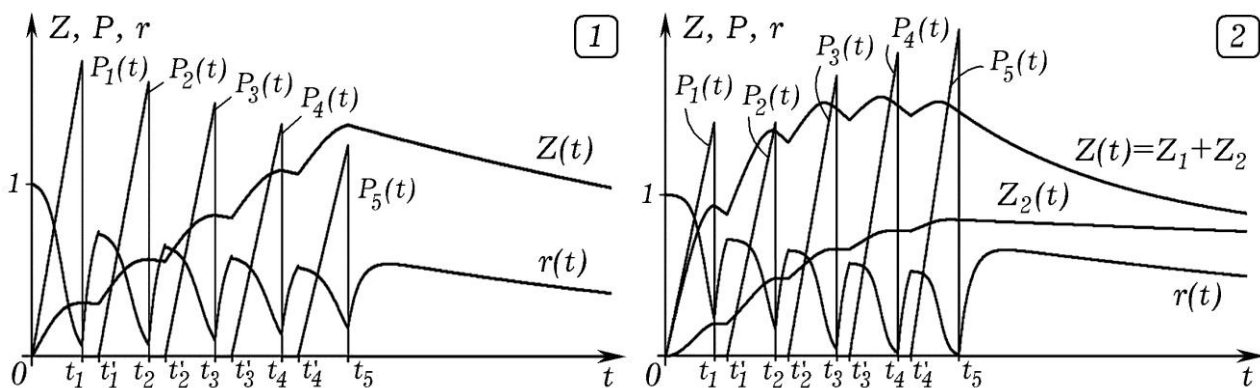
$$dr/dt = k_3(r_{\max} - r), \quad r(t) = r_{\max} - (r_{\max} - r_0) \exp(-k_3(t - t_0)),$$

where  $r_0 = r(t_0)$  – the working capacity at the moment of beginning of rest  $t_0$ , where  $r_{\max}$  – the maximum efficiency of the pupil at the given time  $t$  of the school day. It smoothly decreases according to the law  $r_{\max} = \exp(-k_4 t)$ . With other things being equal, the speed of knowledge increase is higher when the subjective complexity (difficulty of understanding)  $S$  of the studied material is less:  $dZ/dt = \alpha \cdot r(1 - S)FZ^b$ . The complexity of the training material  $S$  lies within the interval  $[0; 1]$  and generally depends on the level of studying (or understanding) other ELMs. So, the one–component model of training looks like:

During training ( $L > Z$ ): 
$$\frac{dZ}{dt} = \frac{\alpha(1 - S)(L - Z)Z^b}{1 + \exp(k_1(P - P_0))} - \gamma Z.$$

During break ( $L = 0$ ): 
$$dZ/dt = -\gamma Z.$$

Let the teacher organize the training process so that during the day pupils work with the maximum tension  $F = L - Z = const$ . Five lessons of identical duration  $T_l = t_1 = t_2 - t'_1 = \dots = t_5 - t'_4$  divided by breaks with the duration  $T_b = t'_1 - t_1 = t'_2 - t_2 = \dots = t'_4 - t_4$  are conducted (given). The results of imitating modelling with reasonable parameters of the model are presented in pic. 9.1. In the interval from 0 to  $t_5$  the coefficient of working capacity  $r$  oscillates relative by to smoothly decreasing value. When breaks between lessons are shortened, pupils don't manage to restore the working capacity, and the results of training decrease.



Pic. 9. One – and two–component models considering changes of the pupil’s efficiency

Let us consider the multi–component model of training taking into account various complexity of the studied themes and change of the pupil’s efficiency during the school day. Let  $Z$  be the pupil’s total knowledge,  $Z_1$  – the weakest knowledge of the first category with high

coefficient of forgetting  $\gamma_1$ ,  $Z_2$  – knowledge of the second category with smaller coefficient of forgetting  $\gamma_2$ , ..., and  $Z_n$  – the strongest knowledge of the  $n$  –th categories with low  $\gamma_n$  ( $\gamma_1 > \gamma_2 > \dots > \gamma_n$ ). Coefficients of assimilation  $\alpha_i$  characterize the speed of transition (or transformation) of the knowledge of the  $(i - 1)$  –th categories into stronger knowledge of  $i$  –th categories. Training is characterized not only by the quantity of the acquired knowledge  $Z = Z_1 + Z_2 + \dots + Z_n$ , but also by the durability coefficient:  $K_D = (Z_2 / 2^{n-2} + \dots + Z_{n-1} / 2 + Z_n) / Z$ .

When studying one theme, at first the level of knowledge  $Z$  grows, then there is an increase in the share of strong knowledge  $Z_n$ , and the durability  $K_D$  increases. The author offers the generalized training model which doesn't have any analogs in the literature known to him. Let the pupil's initial efficiency be  $r_0 = 1$ . At any moment  $Z(t) = Z_1(t) + \dots + Z_n(t)$ . The mathematical model is expressed by the equations:

During training:  $F = L - Z > 0$ ,

$$r = r_0 / (1 + \exp(k_1(P - P_0))), \quad P = k_2 \int_{t_0}^t (1 + S)(L - Z) dt,$$

$$dZ_1 / dt = r(1 - S)(\alpha F Z^b - \alpha_1 Z_1) - \gamma_1 Z_1,$$

$$dZ_2 / dt = r(1 - S)(\alpha_1 Z_1 - \alpha_2 Z_2) - \gamma_2 Z_2,$$

$$\dots, \quad dZ_n / dt = r(1 - S)\alpha_{n-1} Z_{n-1} - \gamma_n Z_n.$$

If complexity  $S = 0$  then pupil's work is equal  $P = k_2 \int_{t_0}^t (L - Z) dt > 0$ .

During the break:  $L = 0$ ,  $dr / dt = k_3(r_{\max} - r)$ ,  $r_{\max} = \exp(-k_4 t)$ ,

$$dZ_1 / dt = -\gamma_1 Z_1, \quad dZ_2 / dt = -\gamma_2 Z_2, \quad \dots, \quad dZ_n / dt = -\gamma_n Z_n.$$

The results of using the two-component model ( $n = 2$ ) are given in pic. 9.2. The quantity of strong knowledge  $Z_2$  during training grows, and after its completing – remains almost stable. After the end of training weak knowledge  $Z_1 = Z - Z_2$  is forgotten significantly quicker. The pupil's efficiency during the lesson smoothly decreases, and during breaks – increases up to the amount which gradually decreases during the day because of the accumulative fatigue.

### Conclusion

The present article is devoted to the research of various models of the didactic systems demanding the numerical solution of the differential equations. It analyses: 1) one-component models of training; 2) the multi-component models considering transformation of weak knowledge into strong and vice versa; 3) the models considering change of the pupil's efficiency during the day. They form the sequence in which each following model is formed by complicating the previous one at the expense of accounting of some additional factors.

The known models of training process [1–10] are based on the assumption that all ELMs are acquired and forgotten equally easily. The multi-component models of didactic systems which are analysed in the article, consider that in the course of training a part of the pupil's knowledge becomes strong and is forgotten slower. The process of increase of durability of the acquired

knowledge while its using by the pupil in every day activity is the cornerstone of formation of abilities (know-hows) and skills which remain for a long time [17, p. 211–212].

Also there is a certain interest to the models considering the decrease of the pupil's working capacity during the day. For example, with their help we can prove that changing types of activities, alternation of studying the theory and fulfilling practical tasks lead to increase of the pupil's knowledge at the end of training [12]. Thus, the imitating models of didactic systems based on the solution of the differential equations allow to analyse the training process, to reveal its features, to establish links between the level of the pupil's knowledge at the end of training, distribution of educational information and the pupil's parameters, and help to plan ways to improve or optimize training.

### Acknowledgments

The author is grateful to professors of the Glazov State Pedagogical Institute V.V. Mayer and V.A. Saranin for the discussion of the principles of modelling didactic systems and methods of the differential equations solution on the computer.

### References

1. Atkinson R., Baujer G., Kroters Je. Vvedenie v matematicheskiju teoriju obuchenija [Introduction to the mathematical theory of training]. M: Mir, 1969, 486 s.
2. Dobrynina N.F. Matematicheskie modeli rasprostraneniya znaniy i upravleniya processom obuchenija studentov [Mathematical models of the spread of knowledge and learning management students]: Basic research, 2009, N 7.
3. Itel'son L. B. Matematicheskie i kiberneticheskie metody v pedagogike [Mathematical and cybernetic methods in pedagogy], M.: Prosveshhenie, 1964, 248 s.
4. Leont'ev L.P., Gohman O.G. Problemy upravleniya uchebnym processom: Matematicheskie modeli [Problems Training Management: Mathematical model], Riga, 1984, 239 p.
5. Roberts F.S. Diskretnye matematicheskie modeli s prilozhenijami k social'nym, biologicheskim i jekologicheskim zadacham [Discrete mathematical models with annexes to social, biological and ecological tasks], M.: Nauka, Gl. red. fiz.–mat. lit., 1986, 496 s.
6. Solovov A.V., Men'shikov A.A. Diskretnye matematicheskie modeli v issledovanijah processov avtomatizirovannogo obuchenija [Discrete mathematical models in the study of processes automated training], Educational Technology & Society, 2001, N 4, s. 205 – 210.
7. Firstov V.E. Matematicheskie modeli upravlenija didakticheskimi processami pri obuchenii matematike v srednej shkole na osnove kiberneticheskogo podhoda [Mathematical models of control didactic process of teaching mathematics in secondary schools on the basis of the cybernetic approach]: diss. ... doc. of pedagogical sciences. – St. Petersburg, 2011, 460 s.
8. Dorrer A.G., Ivanilova T.N. Modelirovanie interaktivnogo adaptivnogo obuchajushhego kursa [Modeling of interactive adaptive course of learning: Modern problems of science and education], 2007, N 5.
9. Ivashkin Ju. A., Nazojkin E.A. Mul'tiagentnoe imitacionnoe modelirovanie processa nakoplenija znaniy [Multi-agent simulation of the process of accumulation of knowledge]: Software products and systems, 2011, N 1, pp. 47 – 52.
10. Kudrjavcev V.B., Vashik K., Strogalov A.S., Alisejchik P.A., Peretruhin V.V. Ob avtomatnom modelirovanii processa obuchenija [On the automaton model of the learning process]: Discrete Mathematics, 1996 , T. 8. , Vol. 4 , pp. 3 – 10.
11. Shannon R. Imitacionnoe modelirovanie sistem: iskusstvo i nauka [Imitating modeling of systems: art and science], M.: Mir, 1978, 302 s.
12. Mayer R.V. Vlijanie smeny vidov dejatel'nosti na rezul'tat obuchenija: Modelirovanie na PJeVM [Influence of change of kinds of activity on result of training: Modeling on computer] // Nauchnye issledovanija i razrabotki. Social'no-gumanitarnye issledovanija i tehnologii, 2015, N 2(11), s. 38-41. DOI: 10.12737/11936
13. Mayer R.V. Komp'juternaja dvuhkomponentnaja verojatnostnaja model' izuchenija discipliny [Computer two-component probabilistic model of studying of discipline] // Sovremennoe obrazovanie, 2015, № 1, S. 42 – 52. DOI: 10. 7256/2409-8736.2015.1.13701. URL: [http://e-notabene.ru/pp/article\\_13701.html](http://e-notabene.ru/pp/article_13701.html)

14. Mayer R.V. Osnovnaja zadacha matematicheskoj teorii obuchenija i ee reshenie metodom imitacionnogo modelirovanija [Main problem of mathematical theory of training its decision with using method of imitating modeling], *Perspective of Science and Education*, 2013, N 5, pp. 95–100.
15. Novikov D.A. Zakonomernosti iterativnogo nauchenija [Laws of iterative learning], M.: Institut problem upravljenja RAN, 1998, 77 s.
16. Razumovskij V.G., Mayer V.V. Fizika v shkole: Nauchnyj metod poznaniya i obuchenie [The physics in school: The scientific method of learning and teaching], M.: Gumanitar. izd. centr VLADOS, 2004, 463 s.
17. Fridman L.M., Kulagina I.Ju. Psihologicheskij spravocnik uchitelja [Psychological handbook teachers], M.: Prosveshhenie, 1991, 288 s.

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for acceptance by Rushan Ziatdinov.