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APPLIED RESEARCH JOURNAL

RESEARCH ARTICLE

ISSN: 2423-4796

Applied Research Journal *Vol.1, Issue, 2, pp.71-79, April, 2015*

APPLICATION OF NON-LINEAR PROGRAMMING OPTIMIZATION TECHNIQUE IN POWER TRANSFORMER DESIGN

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ARTICLE INFO

Article History:

Received: 15, March, 2015
Final Accepted: 20, April, 2015
Published Online: 25, April, 2015

Keywords:
Power Transformer, Transformer Design, Optimization, Nonlinear Programming.

ABSTRACT

Due to huge number of power transformers yearly consumed and installed in the utility networks, it is always required and targeted to build transformers with the most reasonable cost. Achieving the guaranteed characteristics of transformers is an important factor that should be considered knowing that transformer design task is time consuming. In this work, a successful attempt for designing large size power transformer using non-linear programming (NLP) technique was presented. The mathematical transformer design formulation is explained in a systematic way for a typical power transformer. Optimization methodologies and implementation of results were also presented. The results showed the effectiveness of the proposed mathematical formulation of transformer design problem and the reduction of total cost when compared to conventional designs.

1. INTRODUCTION

This paper is based on power transformer design optimization. The design optimization of transformers is majorly determined by the minimization of the overall transformer cost and system losses cost with considerations to constraints connected to international technical specifications as well as transformer user needs that seek constrained minimum cost solution by optimally setting the transformer geometry parameters with relevant electrical and magnetic quantities [1].

This difficulty in achieving the optimum balance between transformer cost and performance has become even more complicated nowadays, as the active materials used in transformer manufacturing (copper and aluminum for transformer windings, and iron for magnetic circuit) are variable stock exchange commodities whose prices are modified on daily basis. Techniques that include mathematical models employing analytical formulas, based on design constants and approximations for the calculation of the transformer parameters are often the basis of the design procedures adopted by transformer manufacturers [2].

It is also noted that the overall transformer manufacturing cost minimization is scarcely addressed in the technical literature. On the other hand, the main approaches dealt with the minimization of specific transformer cost components, such as cost of magnetic material [3], [4], or that of active part cost [5]. This paper introduces the application of a non-linear programming (NLP) an optimization technique in power transformer design optimization problem. The beauty of the proposed techniques can be categorized as

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follows: i). NLP technique is successfully applied to the overall cost minimization of transformer active and mechanical part, ii). Transformer design variables such as the conductors' cross-section and windings are added to the optimization algorithm for an enlarged and transverse optimum transformer designs. The proposed methods find acceptable optimum transformer design by minimizing either the overall transformer material cost (i.e. the transformer active part cost plus mechanical part cost) or the overall transformer materials and operating cost taking into consideration proper loss evaluation factors, while simultaneously satisfying all the constraints imposed by international standards and transformer user needs, instead of focusing on the optimization of only one parameter of transformer performance (e. g no-load losses or short circuit impedance). Using the proposed technique, a graphic user interface (GUI) software package is developed that combine's transformer design with analysis, optimization and visualization tools, useful for both design optimization and educational use. The technique is applied to the design of power transformers of several ratings and loss. Categories and the results are compared with transformer design optimization method (which is already used by transformer industry), resulting to significant cost savings.

2. MATERIAL AND METHODS

2.1. Nonlinear Program (NLP) Optimization Technique

In the transformer design optimization region, NLP techniques are very suitable and effective due to the fact that the design variables can assume not only continuous values but also integer values (e.g., number of winding turns). In this framework, this paper proposes an optimization algorithm adapted to a NLP formulation, completing previous research [5]. NLP refers to mathematical programming with continuous and discrete variables and nonlinearities in the objective function and constraints. A general NLP can be stated as

Find x = $(x_1, x_2, \ldots\ldots, x_n)$ such that $F(x)$ is a minimum

Subject to $g_i(x)$ $\{\leq=0$, $i = 1, 2, \ldots, m$ $\{ll_1, ll_2, M, \ldots ll_n\} \leq \{x_1, x_2, M, \ldots . x_n\} \leq \{ul_1, ul_2, M, . \; ., ul_n\}$

with x ≥ 0 being a non-negative solution.

x = $x_1, x_2, \ldots\ldots, x_n$ = independent design variables.

$F(x)$ = nonlinear objective function i.e. cost function

$g_i(x)$ = nonlinear inequality constraint functions i.e. geometry and performance characteristics.

ll &*ul* = set of lower and upper limits of design variables, respectively [11].

The function $f(x)$ can be classified as linear, nonlinear, integer, zero one, depending on the terms of it. It could also be linear or nonlinear. Solving with the exterior penalty function method, the augmented P(x,r) is formulated as:

$$P(x, r) = F(X) + r\sum_{i=0}^{m}[g_i(x)]^q, \quad r \geq 0 \tag{1}$$

Where $g_i(x)$ is defined as max $[g_i(x), 0]$ and q has a popular value of 2, although other values are possible. By Powell's method with an initial value of x_1, and r_1, minimize P (x_1, r_1). A new function for x_2 is formulated with $r_{2=}\, cr_1$, c > 1 such that

$$P(x, r_2) = F(x) + r_2 \sum_{i=0}^{m}[g_i(x)]^q, \quad r \geq 0 \tag{2}$$

This process of minimization continues and as $r_{k \to}\infty$, it can be proved that:

$$\underset{k \to \infty}{\text{Min}}\, P(x, r_k) = \text{Min}\, F(x) \tag{3}$$

NLP [6], [7] constitute a well-known approach for solving optimization problems to optimality. The technique uses an embedded enumeration scheme for exploring the search space in an "intelligent" way. This is done by partitioning the search space and producing upper and lower limits of the solutions attainable in each partition. Thus, the search performed by the algorithm can be represented as a tree that is traversed in a certain way. The most efficient (in terms of the number of iterations required to find the optimum and prove its optimality) is to use a depth-first traversal. The proposed algorithm solves continuous optimization problems, while constraining some variables into sets of standard values, which may consist of discrete or integer values. The associated discrete programming problem is recursively divided into two sub-problems, by fixing the discrete variables to the closest above and below standard values. The search starts by solving a nonlinear programming (NLP) relaxation, and using the solution as the lower limit of the problem. If the solutions of the discrete variables are all equal to the values defined at the standard discrete set, then the optimum solution is reached and the search is stopped. Otherwise, the search branches on the first discrete variable that has non-standard solution. The closest discrete values above and below the current solution are identified. If both above and below values exist, the NLP with the fixed above values becomes the first sub-problem. The first discrete variable with non-standard solution is identified. Subsequently, a new equality constraint to fix this variable to the above value is added to the original constraints, and the NLP sub-problem subject to the updated constraints is solved. If the NLP sub-problem converges, and yields the superior solution over the existing lower limit, then this solution becomes the new lower limit. The branching continues recursively to the next discrete value with non-standard solution. Otherwise, the node is fathom. If this happens, the algorithm backtracks to the ascendant node, and then resumes branching at the sub-problem associated with new values.In this paper, NLP technique with penalty [7], which enforces early detection and termination of infeasible or inferior NLP solutions for solving power transformer design optimization.

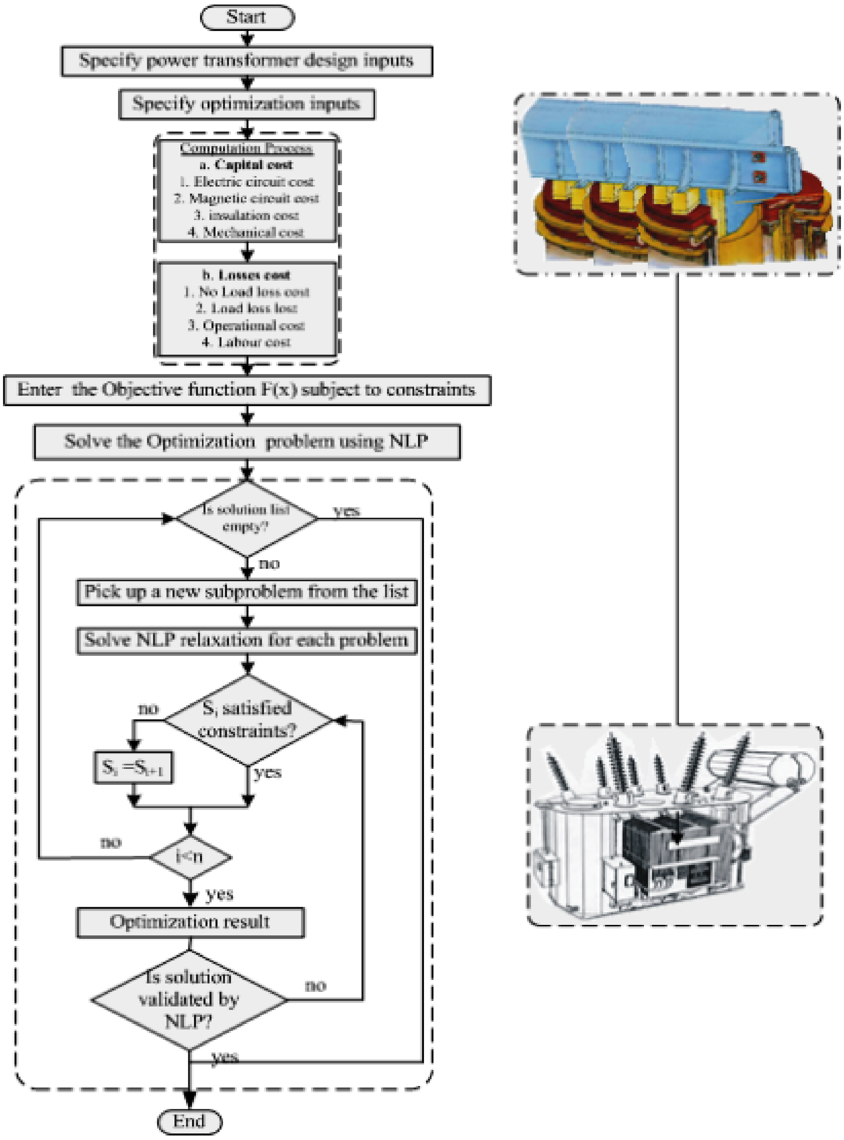


Figure 1 Flowchart of the proposed technique.

2.2. Formulation of Design Problem

This section presents the mathematical formulation of the objective function, design variables, and design constraints for power transformer design optimization (PTDO). This technique is integrated in Matlab environment, using suitable graphical user interface (GUI). The proposed method is shown in the flowchart of Fig.1. The transformer inputs (Fig. 1) involves design parameters, such as rated power, voltages, etc., while the 12 NLP inputs (Fig. 1) comprises of the upper/lower limits and the initial value of the design matrix. An NLP for optimizing the transformer design is based on the minimization of the overall transformer cost function.

2.3. Objective Function, Variables and Constraint

The objective function for the minimization of power transformer design problem is formulated with capital cost and losses cost. Losses cost is made of no-load loss cost, load loss cost and demand charge cost while capital cost consist cost stampings and windings.

Mathematically, the objective function for power transformer design optimization problem is set as:

$$min\ \mathcal{F}(x) = min \sum_{i=1}^{n} c_i f_i\ (x) \tag{3}$$

Where c_i and f_i are the units cost (naira/kg) and the weight (kg) of each component (active and mechanical part, Fig. 1), and $\underline{x}$ are the independent design variables as maximum flux density in the core (x_1), Wb/mm^2, current density in the primary winding (HV),(x_2) A/mm^2, current density in the secondary winding (LV),(x_3) A/mm^2, height of the windings (x_4),m; width of the windings(x_5), m; voltage per turn(x_6),$Volts$; and distance between core centers(x_7), m.

The minimization of the objective function is subject to the following constraints:

$$\mathrm{P_o + P_k - 1.10.(GP_o + GP_k) < 0} \tag{4}$$

$$\mathrm{P_o - 1.15.P_k < 0} \tag{5}$$

$$\mathrm{P_k - 1.15.P_k < 0} \tag{6}$$

$$\mathrm{0.90.Z_k < Z_{sc} < 1.10.Z_k} \tag{7}$$

$$\mathrm{P_o + P_k < \theta_C} \tag{8}$$

$$\mathrm{TCL - (3H_w + 2CY)\ \leq 0} \tag{9}$$

$$\mathrm{D - W_w - d \quad \leq 0} \tag{10}$$

$$\mathrm{TPV}(LP)^{0.5} - \mathrm{K} \leq 0 \tag{11}$$

$$\mathrm{P_o - [A_i k1_i k2_i\ (3H_w + 2\ t_u)] k3_i\ k4_i < 0} \tag{12}$$

$$ll_\mathrm{i} \leq \mathrm{x_i} \leq \mathrm{ul_i},\ \ \mathrm{i} = 1, 2, \ldots, \mathrm{n} \tag{13}$$

$$\mathrm{x_i} \geq 0, \mathrm{i} = 1, 2, \ldots, \mathrm{n} \tag{14}$$

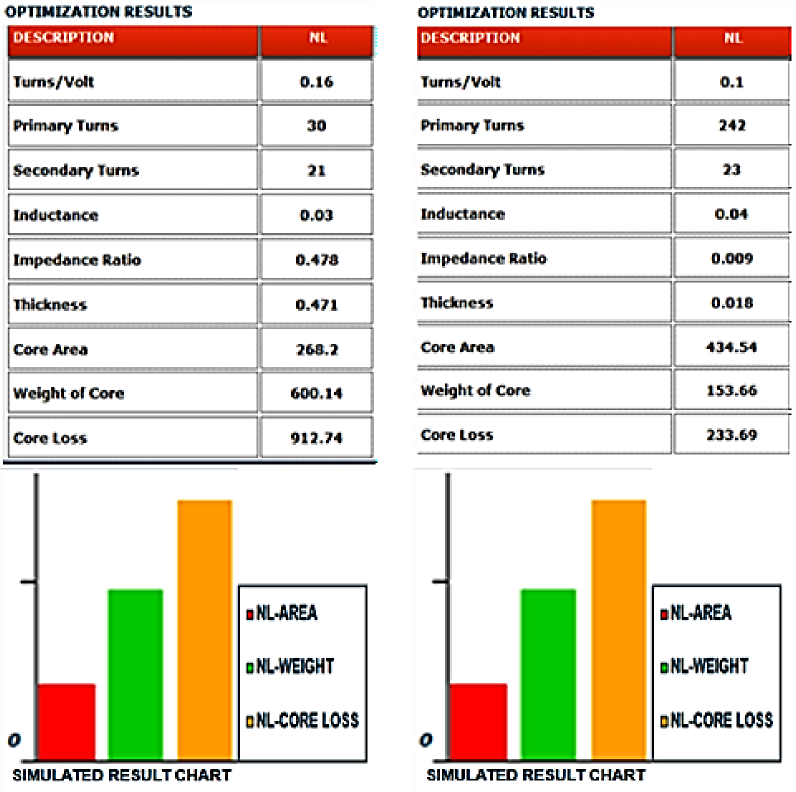
Where $\mathrm{P_o}$ denotes the designed no-load loss (W), $\mathrm{P_k}$ the designed load loss (W), Z_{sc} the designed short-circuit impedance (%), GP_o the guaranteed no-load loss (W), GP_k the guaranteed load loss (W), Z_k the guaranteed short-circuit impedance (%), θ_C is the heat dissipated (by convection) through the transformer cooling system (W), $\mathrm{k1_i}$ the building factor, $\mathrm{k2_i}$ the stacking factor, $\mathrm{k3_i}$ the specific density of iron, and $\mathrm{k4_i}$ the Loss/kg, while H_w,W_w, t_u, D, d, $\mathrm{A_i}$ are the geometric characteristics of the active part (Fig. 1), and ul, and ll are lower and upper limits of x. The coefficients appearing in (4)–(7) are based on the tolerances specified by IEC 60076-1, while the respective coefficients in (8)–(12) are based on the transformer manufacturer specifications.

Upon user selection, the transformer capital and losses cost can also be integrated into (3) enabling the optimization design based on the overall cost i.e. (capital cost and losses cost).

$$min_{xi} F_{PTDO} = min_{xi}\left(\sum_{i=1}^{n} c_i x_i + 0.01r.C_p + C_e N(P_o + \alpha^2 P_k) + C_d(P_o + P_k)\right) \tag{15}$$

Where C_c denotes capital cost and computed as $C_c = C_p * \frac{r}{100} = min \sum_{i=1}^{n} c_i f_i \,(x)$ in (Naira), C_{po} denotes no load loss cost rate of the transformer in (Naira/Watt), C_{pk} denotes load loss cost rate of the transformer in (Naira/Watt), C_d denotes demand charges cost in (Naira/Watt), $C_p$denotes transformer manufacture and purchase price in (Naira/Watt), N denotes operational hours per year, C_e denotes energy charges in (Amount/kwh), P_o denotes no-Load or Iron loss in (kw), P_k denotes copper loss in (kw), P_t denotes total loss in (kw) which is the sum of $(P_o + P_k)$, C_d denotes demand charges in (Amount/kw.yr), α denotes constant operation load/rated load and is 0.8 < α<1, r denotes $p * q^n(q^n - 1)$, q denotes $\frac{p}{100+1}$, p denotes interest rate in percent per annum, q is the interest factor, n is the depreciation period in years, r is the depreciation factor and is given as 13.39. The strong point of the proposed software is that the designer can define the demand charges cost for ((P_o and P_k) using 1) and easy-to-use user friendly GUI.

One of the crucial design variables during the transformer design optimization is the calculation of the conductors' cross-section. The conductors' cross-section derived from the current density of the high voltage (HV) and low voltage (LV) winding, which consist crucial design parameters, dependent on the transformer rating and loss category. In the proposed method, three new approaches are proposed with the aim of successfully defining the values of the HV and LV winding current density (in A/mm), denoted as x_2 and x_3, respectively. At the first approach, the transformer designer can define directly the value of the x_2 and x_3. The main drawback of this approach is that the transformer designer should be quite experienced in order to correctly set this value and direct the method to the optimal solution. At the second approach, an interval with a set of discrete and values for the LV and HV winding, respectively, can be defined. In this case, the proposed method will calculate optimum transformer designs, and finally will keep the best optimum transformer design among them. Although this approach is time-consuming, it assures a global optimum design. At the third approach, the designer can increase the vector of the four design variables into six. In particular, the correct definition of the current density value is under the rules (supervision) of the NLP optimization method. In this way, the transformer designer defines the initial, the upper and the lower value of the proposed method and finds an optimum transformer design, designating the values of the six variables of the design vector.



OPTIMIZATION RESULTS

DESCRIPTION	NL
Turns/Volt	0.16
Primary Turns	30
Secondary Turns	21
Inductance	0.03
Impedance Ratio	0.478
Thickness	0.471
Core Area	268.2
Weight of Core	600.14
Core Loss	912.74

OPTIMIZATION RESULTS

DESCRIPTION	NL
Turns/Volt	0.1
Primary Turns	242
Secondary Turns	23
Inductance	0.04
Impedance Ratio	0.009
Thickness	0.018
Core Area	434.54
Weight of Core	153.66
Core Loss	233.69

Figure 2 Average cost difference

Average cost difference shows the average difference between the costs of the optimum transformer designs produced by the proposed method versus the current method employed in the manufacturing industry, for each KVA category considered in the study.

3. RESULT AND DISCUSSION

The robustness of the proposed method is presented in comparison with that of current method [1] that is already applied in a transformer manufacturing industry.

The proposed method minimizes the overall transformer cost (1), subject to the constraints (2)–(11) by seeking the optimum settings of the design variables, namely, the core constructional parameters H_w,W_w, t_u, D, d, A_i shown in Fig. 1 (continuous variables), the magnetic induction (continuous variable), and the number of turns (integer variable). Two more design variables can be optionally added: x_2 and x_3 (continuous variables).

The proposed method has been applied in a wide spectrum of actual transformers, of different voltage ratings and loss categories. In particular, 188 optimum transformer designs were created and compared with the current method [6].

Fig. 2 depicts the results. It should be noted that experiments were carried out using constant x_2 andx_3, values (1st approach for the current density determination, described in Section II-C) because the current heuristic technique [6] could not support the other two approaches.

3.1. NLP Optimization Result

In addition to the 300MVA transformer simulation using NL optimization algorithms, the results obtained from the use of Nonlinear Programme (NLP)) optimization on different power transformer ratings, voltages, etc. and utilizing various core materials are shown in Table 1.

Table 1 Different Transformer Designs Using NLP Technique

Characteristics of power transformers design optimization	**Ex. No. 1**	**Ex. No. 2**	**Ex. No. 3**	**Ex. No. 4**
Rating (MVA)	150	300	650	1250
Primary Voltage (KVolt)	138	138	330	11
Secondary Voltage (KVolt)	33	36	132	0.415
Frequency	50	50	50	50
Primary/Secondary connection	D/S	D/S	D/S	D/S
Building Factor	1.1	1.15	1.2	1.2
% Z	4	4	5	6
Reference Temp	85	75	75	85
Temp. Rise	60	50	45	60
Average Ambient Temp	30	30	30	30
Core Material	DKH	M0H	M4	M3
Core Price (₦/kg)	9	7.5	5.5	6.5
Copper Price (₦/kg)	15	14.5	16	13
No Load Loss Evaluation price (₦/W)	13	11	11	9
Load Loss Evaluation price (₦/W)	3	4	4	3
Average Cost (%)	10	9.3	9.1	7.9

Table 1 shows the result of the application of the technique to different transformers of 150, 300, 650, and 1250 MVA.

The transformer has rated primary and secondary voltages as 138/33KV, the vector group is Dyn11, the frequency is 50Hz, the impedance is 4%, the building factor is 1.1 and the core material is DKH which yielded and optimum design with an average cost of 10%. Similarly, the procedure was repeated for the second, third and fourth transformers with the corresponding average cost of 9.3%, 9.1% and 7.9% respectively.

Table 2 Results of Objective Function Optimization Using NLP

Optimization method	**Results**				
	NL				
	Ex. No. 1	**Ex. No. 2**	**Ex. No. 3**	**Ex. No. 4**	**Average**
% Z	4.158	3.916	5.195	5.859	4.782
No Load Loss (W)	269	574	1394	1506	935.8
Load Loss (W)	2080	2338	3683	9606	4427
Capital Cost	135	135	135	135	135
Demand Charge Cost	5400	5400	5400	5400	5400
Total cost (₦)	**14802**	**23238**	**43201**	**63111**	**36088**

Finally, a comparison of the optimization results incorporating the transformer operating cost, using (13), has been conducted, for the same case study of the 150, 300, 650, and 1250MVA transformer. Table II shows the results of the proposed method using **NLP**.

It can be seen that NL programming algorithm designs are better than those obtained by conventional method. This can be attributed to the availability of constraints in the transformer design problem. Constraints are difficult to incorporate into the conventional program as generally it is left to the fitness function to manage and quantify possible infeasibility. In general, conventional methods should not be regarded as a replacement for NL programming algorithm, but as another optimization approach that can be used.

According to the results obtained from this thesis, the following can be concluded: i.) A new formulation of an oil-immersed power transformer has been proposed. A computer based design program was successfully developed. The proposed design formulation takes into consideration the practical constraints in the art of transformer design and manufacturing. ii). Results of the constructed transformer using the proposed formulation were presented to illustrate and confirm the reliability and effectiveness of proposed mathematical design formulation. iii). A nonlinear programming algorithm has been implemented and applied to the transformer design problem. The results obtained using the NL programming algorithm was encouraging when compared to the design made by practical experience. iv). NL programming showed good results in most of the application cases.

In addition to the achievement of thesis objectives, it is worth mentioning that one important outcome is effectiveness and success rather than suitability of using of optimization techniques (Nonlinear) in the field of transformer design. Using such techniques in transformer area, with no doubt, is of a high value added from the point of view of saving money, effort and time.

4. CONCLUSION

The proposed method is very effective because of its robustness, its high execution speed and its ability to effectively search the large solution space. Positive and encouraging results have been achieved using the NL programming optimization techniques. The validity of this method is illustrated by its application to a wide spectrum of actual transformers, of different power ratings and losses, resulting to optimum designs with an average cost saving of 9.1%.

Future work with optimization technique using advanced step in the field of power transformer optimization is recommended. Such a new algorithm may starts with execution of GA subprogram which does not require any initial values for the designed variables and then feed the GA optimized values as initial values to the NL program.

In addition, a mathematical formulation of power transformers design problem could lead to more economical designs when optimized through Artificial Intelligence (AI) and NL techniques.

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