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Adjustment of the Force Transmissibility of a SDOF Vibrating System; Part II: Fraction Transmissibility

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Abstract:

The purpose of this paper is to present an efficient technique for the adjustment of the transmissibility of a SDOF system through the optimal selection of the damping ratio of the SDOF vibrating system. As an application a case of fraction transmissibility. Different objective function relationships are presented and the best one is assigned leading to the desired unit force transmissibility and a frequency spectrum of the system vibration amplitude ratio having a settling frequency ratio above 4.2. Five objective function expressions (ISE, IAE, ITAE, ITSE and ISTSE) are considered and their impact on the force transmissibility and other related parameters is investigated. Desired force transmissibility in the range 0.1 to 0.5 is considered. The force transmissibility of the vibrating system is adjusted around the mean value with standard deviation less than 0.085. System optimal damping ratio of the system is less than 1.3 depending on the desired force transmissibility and the objective function used. The optimum vibration amplitude ratio settles within a 0.05 band at an exciting frequency ratio greater than 4.2.

Keywords **— vibration isolators design, transmissibility adjustments, fraction transmissibility, settling frequency ratio, optimum system dynamics, performance indices.**

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I. INTRODUCTION

Adjustment of force transmissibility belongs to the subject of vibration control. Force transmissibility plays an important role in structural dynamics since it excites structure vibration and noise in near-by locations. Research activities are focused on this aspect over years. Here, I will present some of the research activities regarding this aspect during the last 15 years.

Maia, Silva and Ribeiro (2001) presented a generalization of the transmissibility concept to MDOF systems. They presented some properties such as the validity of the transmissibility for grounded structures and the existence of an inverse transmissibility matrix [1]. Johnson (2002) analysed data from a 3-story building structure and found the transmissibility-based damage feature to provide better and consistent results in terms of detection, location and quantification [2]. Park and Gu (2003) presented an experimental methodology to examine the chassis force transmissibility from a fully assembled vehicle. They utilized the spindle force/moment estimation and the conventional noise path analysis techniques to calculate the transmissibility [3]. Fontul, Ribeira, Silva and Maia (2004) generalized the transmissibility concept to MDOF systems with multiple random excitations. They defined the transmissibility matrix relating two sets of responses [4]. El-Wahed (2005) carried out an experimental investigation of the performance of a homogeneous ER fluid in an oscillatory squeeze flow. The variation in the force transmitted through the fluid in response to changes in the oscillation frequency was studied [5]. Hill, Stallaert and Sas (2006) solved the problem of noise radiation from a structure housing a rotating device by reducing the force transmission in the path. They presented some of the design requirements to integrate an active bearing, control approaches and preliminary experimental data [6]. Shimone, Katsura and Takei (2007) proposed a micro-macro control for standardized modal space. Force and position controls were able to be realized simultaneously. The transmission of force sensation from the micro environment was achieved [7]. Rustighi and Elliott (2008) addressed a general method of computing the force transmissibility for complex or statically indeterminate mechanical system with more than one support. They addressed the results of an oscillator travelling over a beam on three simple supports [8]. Waters (2009) considered the force transmissibility of a quasi-zero-stiffness isolator. He derived the force transmissibility of the system and compared with that of a linear system. He derived also expressions for the maximum force transmissibility and the jump-down frequencies [9]. Liu, Wang, Wu and Bonev (2010) studied the optimal kinematic design of a 3-axis tool head with 3-PRS parallel kinematics. They considered the orientation capability and the motion/force transmission [10].

Xie, Liu and Wang (2011) proposed indices to estimate motion/force transmissibility to evaluate the performance of redundant paralle kinematic manipulators. They carried out transmissibility comparisons between redundant and non-reduntant parallel kinematical manipulators [11]. Guo, Lang and Peng (2012) derived analytical relationships to evaluate the transmissibility of a SDOF displacement vibration isolation and force vibration isolation systems where a nonlinear viscous damper is used. They proposed a procedure for the design of nonlinear viscous dampers for vibration isolation purposes [12]. Gao and Chen (2013) investigated the primary resonance, stability and design methodology of a piecewise bilinear system under cubic velocity feedback control with a designed time delay. They assessed the effects of the feedback parameters on vibration transmissibility [13]. Lage, Maia and Neves (2014) proposed the reconstruction of forces based on the direct and reverse problems of transmissibility in MDOF systems. They used the force transmissibility to calculate the reactions for a given applied loads and vice versa [14]. Hassaan (2015) presented an efficient simple technique for the adjustment of the force transmissibility of a SDOF vibrating system through the optimal selection of the damping ratio of the vibrating system. He considered a desirable unit force transmissibility using various objective functions. He could get a frequency response of the vibrating system with a newly defined settling frequency ratio as low as 0.03[15].

II. ANALYSIS

A single-degree of freedom vibrating system excited by harmonic excitation is shown in Fig.1.

The harmonic force is $F_o \sin\omega t$, the isolators have equivalent stiffness k and equivalent damping coefficient c. The system lumped mass is M and its dynamic motion is x.

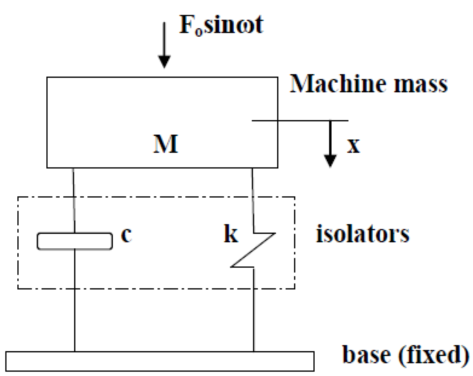

Fig.1 SDOF vibrating system excited by a harmonic force.

The differential equation of the vibrating mass M is:

$$Mx'' + cx' + kx = F_o \sin\omega t \qquad (1)$$

The steady-state vibration response using Eq.1 is:

$$x = X\sin(\omega t - \varphi) \qquad (2)$$

where X is the peak amplitude of the mass vibration and φ is the phase angle between the exciting force and mass displacement.

The peak amplitude ratio X/Xo as function of the frequency ratio (r=ω/ωn) and damping ratio of the system ζ is given by [16]:

$$X/X_o = 1 / \sqrt{\{(1 - r^2)^2 + (2\zeta r)^2\}} \qquad (3)$$

The force transmissibility of the system, TR is given by [17,18]:

$$TR = \sqrt{\{1 + (2\zeta r)^2\}} / \sqrt{\{(1 - r^2)^2 + (2\zeta r)^2\}} \qquad (4)$$

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III. OPTIMAL DYNAMIC SYSTEM PERFORMANCE

The objective of this study is to provide force transmissibility TR very close to a unit value. This dynamically means that the transmitted force to the base of Fig.1 is equal to the amplitude of the exciting force F_o independent of the exciting frequency ratio. To achieve this objective, we define as error function e as the difference between the force transmissibility TR at any frequency ratio and the desired fraction value of the transmissibility TR_{des}. That is:

$$e = TR - TR_{des} \qquad (5)$$

Now, we define an objective function f which depends mainly on the error function e of Eq.5 as follows:

A. *Integral of Square Error (ISE)*

In the integral of square error, the objective function f_{ISE} is defined as [19,20]:

$$f_{ISE} = \int e^2 dr \qquad (6)$$

The objective function of Eq.6 is minimized using the MATLAB command 'fmincon' with one unknown system parameter which is the damping ratio. The results depend on the desired value of the force transmissibility. Preliminary application of the suggested technique showed that the damping ratio of the vibrating system has a value < 1 for $0.1 \leq TR_{des} \leq 0.4$. This means that force transmissibility > 1 is expected about resonance. Therefore, it was supposed that this vibrating system will run at exciting frequencies above resonance , say in the range $3 \leq r \leq 6$. Knowing the excitation pattern of the exciting force and using the isolator stiffness and system mass, the system natural frequency can be adjusted to provide a frequency ratio starting from 3. This assumption facilitates having force transmissibility with low standard deviation about the mean fraction value. The optimal results of the damping ratio and vibration performance parameters are given in Table 1 when using the ISE objective function for a desired force transmissibility in the range: $0.1 \leq TR_{des} \leq 0.5$.

TABLE 1
OPTIMAL TRANSMISSIBILITY ADJUSTMENT USING ISE OBJECTIVE FUNCTION

TR_{des}	ζ_{opt}	X/X_o	r_{set}	TR_{mean}	SD
0.10	0.1523	3.3215	4.577	0.0958	0.0296
0.15	0.2675	1.9395	4.566	0.1434	0.0373
0.20	0.3779	1.4288	4.549	0.1918	0.0460
0.25	0.4902	1.1703	4.526	0.2408	0.0546
0.30	0.6073	1.0363	4.500	0.2904	0.0626
0.35	0.7318	1	4.460	0.3408	0.0699

0.40	0.8660	1	4.413	0.3919	0.0760
0.45	1.0130	1	4.351	0.4439	0.0808
0.50	1.1763	1	4.275	0.4964	0.0839

The performance of the vibrating system using the optimal damping ratio is measured by the optimal force transmissibility and the amplitude ratio of the vibrating system. Both are shown in Figs.2 and 3 for a desired force transmissibility of 0.1.

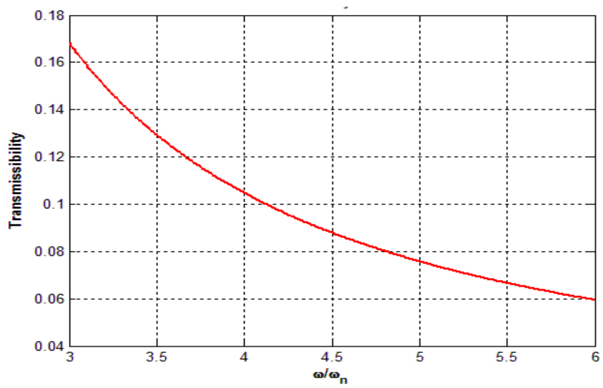

Fig.2 Force transmissibility for 0.1 desired transmissibility using ISE function.

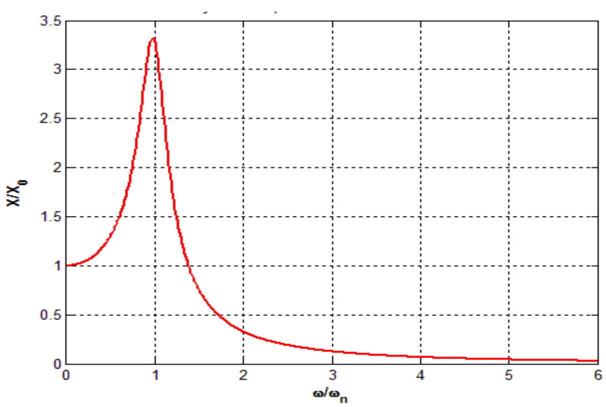

Fig.3 Amplitude ratio for 0.1 desired transmissibility using ISE function.

B. Integral of Absolute Error (IAE)

In the integral of absolute error, the objective function f_{IAE} is defined as [19,21]:

$$f_{IAE} = \int |e|dr \quad (7)$$

The objective function of Eq.7 is minimized using the MATLAB command 'fmincon' with one unknown system parameter which is the damping ratio. The results are as given in Table 2.

TABLE 2

OPTIMAL TRANSMISSIBILITY ADJUSTMENT USING IAE OBJECTIVE FUNCTION

TR_{des}	ζ_{opt}	X/X_o	r_{set}	TR_{mean}	SD
0.10	0.1685	3.0112	4.580	0.1021	0.0305
0.15	0.2823	1.8461	4.570	0.1495	0.0385
0.20	0.3942	1.3802	4.550	0.1989	0.0473
0.25	0.5083	1.1423	4.525	0.2485	0.0559
0.30	0.6279	1.0232	4.490	0.2989	0.0639
0.35	0.7558	1	4.450	0.3502	0.0711
0.40	0.8927	1	4.400	0.4017	0.0770
0.45	1.0436	1	4.330	0.4541	0.0815
0.50	1.2128	1	4.250	0.5075	0.0843

Figs.4 and 5 show the optimal force transmissibility and amplitude ratio of the vibrating system for an 0.1 desired transmissibility using the IAE objective function.

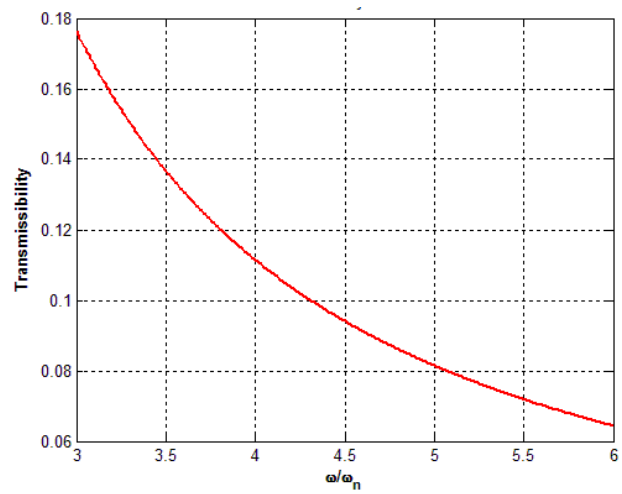

Fig.4 Force transmissibility for 0.1 desired transmissibility using IAE function.

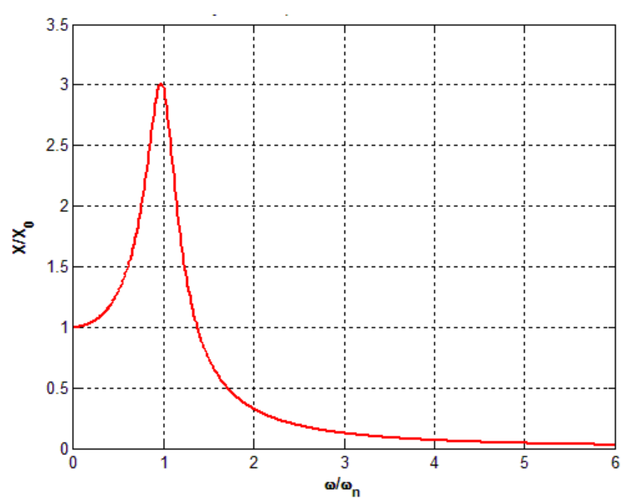

Fig.5 Amplitude ratio for 0.1 desired transmissibility using IAE function.

C. Integral of Time Multiplied by Absolute Error (ITAE)

In the integral of time multiplied by absolute error, the objective function f_{ITAE} is defined as [19,20]:

$$f_{ITAE} = \int r|e|dr \quad (8)$$

The objective function of Eq.8 is minimized using the MATLAB command 'fmincon' with one unknown system parameter which is the damping ratio. The results are given in Table 3.

TABLE 3
OPTIMAL TRANSMISSIBILITY ADJUSTMENT USING ITAE OBJECTIVE FUNCTION

TR_{des}	ζ_{opt}	X/X_o	r_{set}	TR_{mean}	SD
0.10	0.1875	2.7149	4.573	0.1097	0.0316
0.15	0.3072	1.7101	4.560	0.1607	0.0404
0.20	0.4251	1.2994	4.540	0.2125	0.4970
0.25	0.5460	1.0931	4.513	0.2646	0.0586
0.30	0.6745	1.0041	4.478	0.3179	0.0667
0.35	0.8085	1	4.435	0.3704	0.0736
0.40	0.9560	1	4.376	0.4242	0.0791
0.45	1.1145	1	4.305	0.4772	0.0829
0.50	1.2926	1	4.215	0.5307	0.0849

Figs.6 and 7 show the optimal force transmissibility and amplitude ratio of the vibrating system for an 0.1 desired transmissibility using the ITAE objective function.

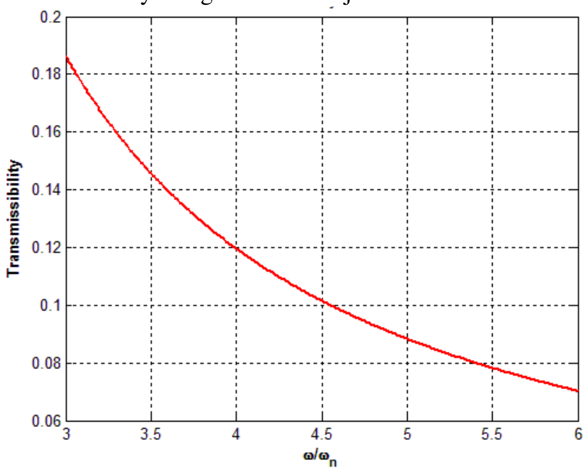

Fig.6 Amplitude ratio for 0.1 desired transmissibility using ITAE function.

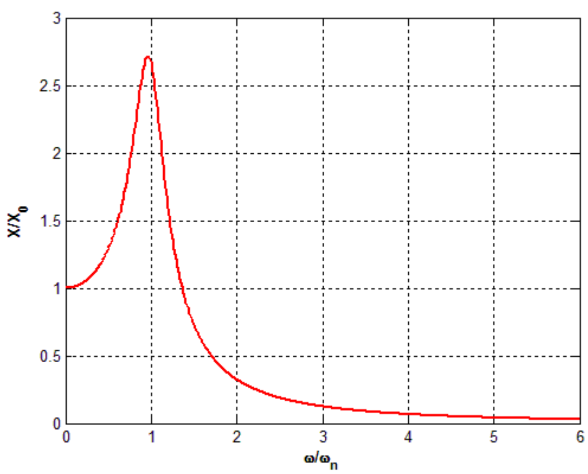

Fig.7 Amplitude ratio for 0.1 desired transmissibility using ITAE function

D. Integral of Time Multiplied by Square Error (ITSE)

In the integral of time multiplied by square error, the objective function f_{ITSE} is defined as [20,22]:

$$f_{ITSE} = \int re^2dr \quad (9)$$

The objective function of Eqs.9 is minimized using the MATLAB command 'fmincon' with one unknown system parameter which is the damping ratio. The results are given in Table 4.

TABLE 4
OPTIMAL TRANSMISSIBILITY ADJUSTMENT USING ITSE OBJECTIVE FUNCTION

TR_{des}	ζ_{opt}	X/X_o	r_{set}	TR_{mean}	SD
0.10	0.1670	3.0366	4.575	0.1015	0.0304
0.15	0.2846	1.8325	4.562	0.1509	0.0387
0.20	0.3991	1.3664	4.545	0.2011	0.0477
0.25	0.5160	1.1313	4.520	0.2518	0.0565
0.30	0.6383	1.0176	4.490	0.3032	0.0646
0.35	0.7684	1	4.448	0.3551	0.0717
0.40	0.9087	1	4.396	0.4075	0.0776
0.45	1.0623	1	4.330	0.4603	0.0819
0.50	1.2326	1	4.246	0.5133	0.0845

Figs.8 and 9 show the optimal force transmissibility and amplitude ratio of the vibrating system for an 0.1 desired transmissibility using the ITSE objective function.

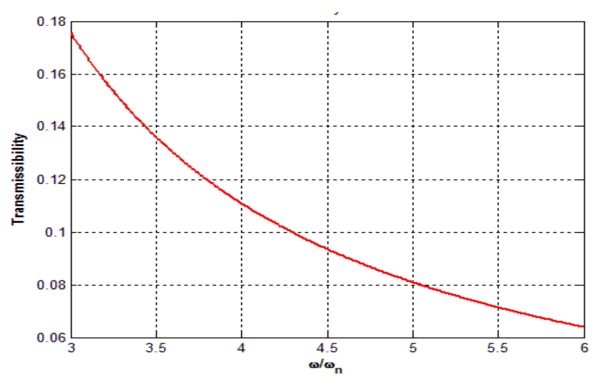

Fig.8 Force transmissibility for 0.1 desired transmissibility using ITSE function.

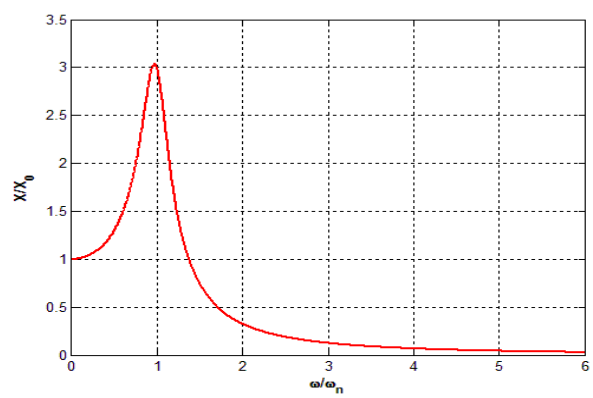

Fig.9 Amplitude ratio for 0.1 desired transmissibility *using ITSE function.*

E. Integral of Square Time Multiplied by Square Error (ISTSE)

In the integral of square time multiplied by square error, the objective function f_{ISTSE} is defined as [23]:

$$f_{ISTSE} = \int r^2 e^2 dr \qquad (10)$$

The objective function of Eqs.10 is minimized using the MATLAB command 'fmincon' with one unknown system parameter which is the damping ratio. The results are given in Table 5.

TABLE 5
OPTIMAL TRANSMISSIBILITY ADJUSTMENT USING ISTSE OBJECTIVE FUNCTION

TR_{des}	ζ_{opt}	X/X_o	r_{set}	TR_{mean}	SD
0.10	0.1809	2.8107	4.575	0.1070	0.0312
0.15	0.3016	1.7390	4.560	0.1582	0.0400
0.20	0.4201	1.3114	4.540	0.2103	0.0493
0.25	0.5417	1.0981	4.515	0.2628	0.0583
0.30	0.6690	1.0055	4.480	0.3157	0.0664
0.35	0.8045	1	4.435	0.3689	0.0734
0.40	0.9504	1	4.380	0.4223	0.0790
0.45	1.1098	1	4.307	0.4757	0.0829
0.50	1.2861	1	4.217	0.5288	0.0849

Figs.10 and 11 show the optimal force transmissibility and amplitude ratio of the vibrating system for an 0.1 desired transmissibility using the ISTSE objective function.

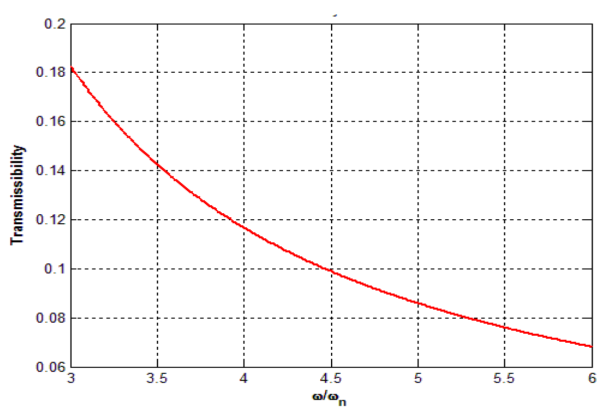

Fig.10 Force transmissibility for 0.1 desired transmissibility using ISTSE function.

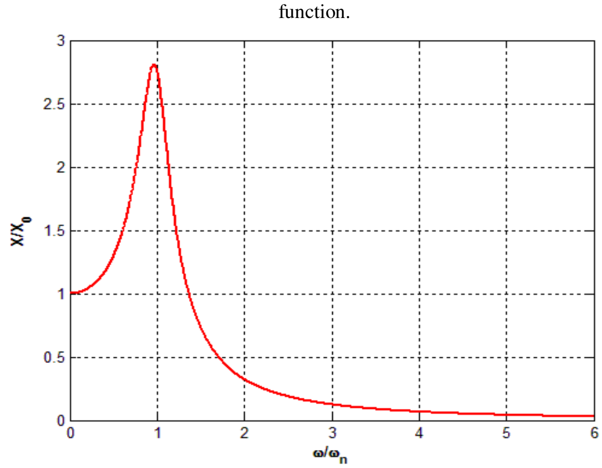

Fig.11 Amplitude ratio for 0.1 desired transmissibility using ISTSE function.

IV. COMPARISON OF TRANSMISSIBILITY OPTIMAL PARAMETERS

The transmissibility optimal parameters are the optimal damping ratio, transmissibility itself, maximum vibration amplitude ratio, and settling frequency ration of the vibration amplitude ratio. Those parameters depend on the objective function used in the optimization process and the desired force transmissibility.

A. Optimal Damping Ratio

The optimal damping ratio of the vibrating system for different objective functions and desired transmissibility in the range $0.1 \le TD_{des} \le 0.5$ is shown in Fig.12.

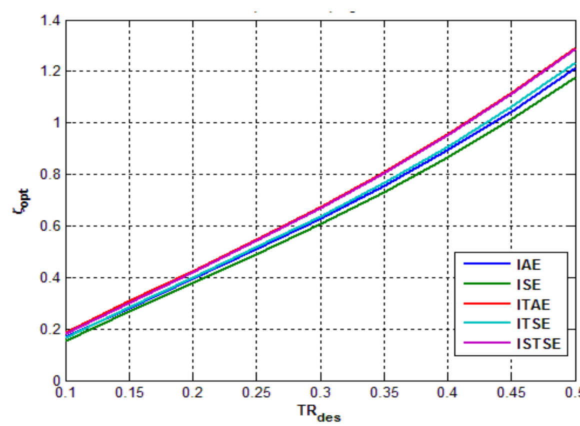

Fig.12 Optimal damping ration for different objective functions.

B. Optimal Maximum Amplitude Ratio

The optimal maximum amplitude ratio of the vibrating system for different objective functions in the frequency ratio range $3 \le r \le 6$ is shown in Fig.13.

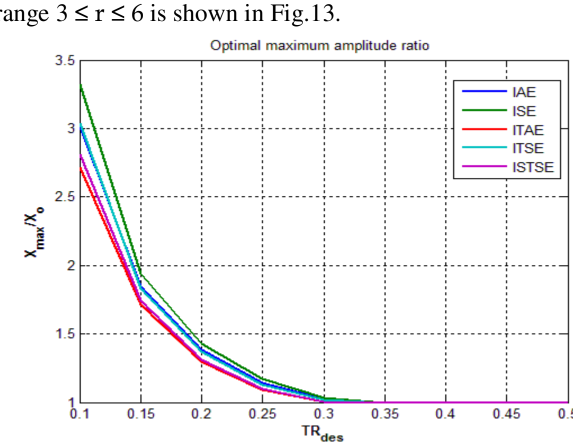

Fig.13 Optimal maximum amplitude ratio for different objective functions.

C. Optimal Settling Frequency Ratio for the Amplitude Ratio

The optimal settling frequency ratio of the optimal amplitude ratio of the dynamic system mass for different objective functions is shown in Fig.14.

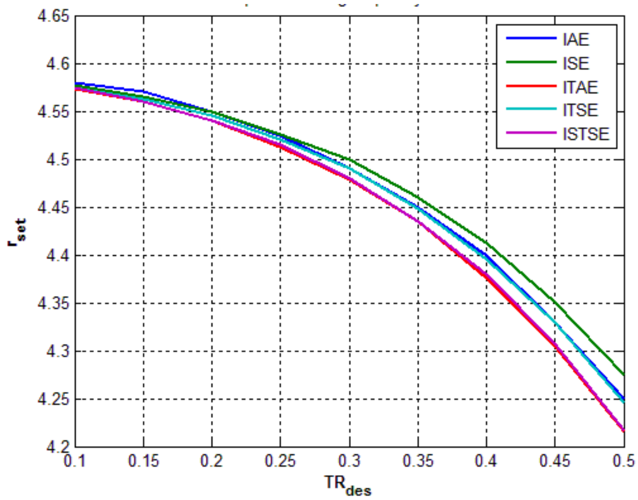

Fig.14 Optimal settling frequency ratio for different objective functions.

D. Optimal Mean Force Transmissibility

The optimal mean force transmissibility of the dynamic system for different objective functions during the exciting frequency ratio range $3 \le r \le 6$ is shown in Fig.15.

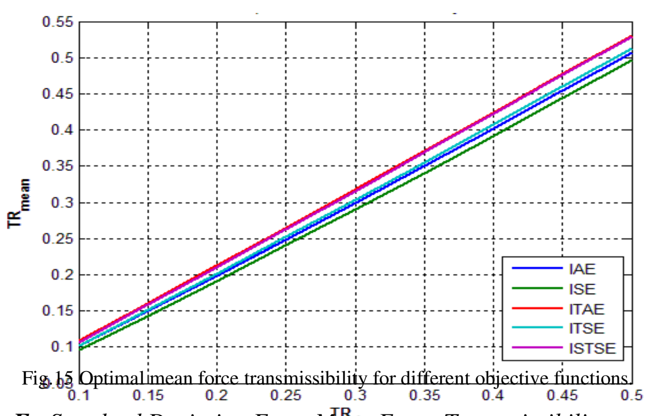

Fig.15 Optimal mean force transmissibility for different objective functions.

E. Standard Deviation From Mean Force Transmissibility

The standard deviation from the mean force transmissibility of the dynamic system for different objective functions during the exciting frequency ratio range $3 \le r \le 6$ is shown in Fig.16.

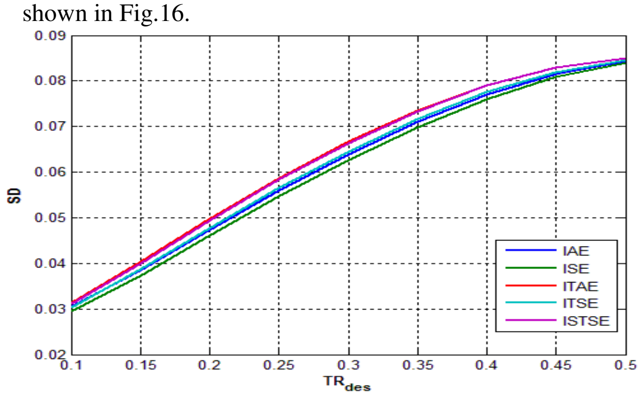

Fig.16 Optimal standard deviation for different objective functions.

V. CONCLUSIONS

- It was possible to attain a mean force transmissibility at a desired level.
- The optimization toolbox of the MATLAB was used to achieve the transmissibility adjustment.
- The ISE, IAE, ITAE, ITSE and ISTSE objective functions were used to provide the optimal adjustment of the force transmissibility of a SDOF vibrating system.
- A desired transmissibility range from 0.1 to 0.5 was investigated.
- It was possible to adjust the mean force transmissibility to a specific desired value with standard deviation (SD) in the range 0.0296 to. 0.0849.
- The optimal damping ratio required to adjust the force transmissibility was in the range 0.153 to 1.2926.
- The maximum peak amplitude ratio for an exciting frequency range from 0 to 6 was in the range 1 to 3.3215.
- The settling frequency ratio was used as a measure for the forced response settling within 0 to 0.05 band.
- The settling frequency ratio was in the range 4.2152 to 4.5800.
- The optimal mean force transmissibility was in the range 0.0958 to 0.5307. The vibration engineer can decide the desired mean force transmissibility according to his application and exciting frequency range.
- The ITSE objective function ha given a mean force transmissibility very close to the desired value with deviation from the mean value in the range 0.0010 to 0.0133 depending on the desired transmissibility.
- Less damping ratio was required by the ISE objective function and maximum damping ratio was required by the ITAE objective function.
- The ISE objective function provided the minimum standard deviation values at small desired values of the force transmissibility.
- The ITAE objective function provided the largest standard deviation around the mean force transmissibility.

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DEDICATION

- I dedicate this research work to Prof. Ibrahim Fawzi.
- Professor Fawzi is the Emeritus Professor of Mechanical Vibrations in the Department of Mechanical Design and Production, Faculty of Engineering Cairo University, Egypt.
- He taught me a course on Mechanical Vibrations in 1971.
- He was an excellent instructor who made me love Mechanical Vibrations.
- His teaching methodology had a clear impact on my teaching experience over more than 40 years.
- Thanks dear professor.

BIOGRAPHY

Galal Ali Hassaan

- Emeritus Professor of System Dynamics and Automatic Control.
- Has got his B.Sc. and M.Sc. from Cairo University in 1970 and 1974.
- Has got his Ph.D. in 1979 from Bradford University, UK under the supervision of Late Prof. John Parnaby.
- Now with the Faculty of Engineering, Cairo University, EGYPT.
- Research on Automatic Control, Mechanical Vibrations , Mechanism Synthesis and History of Mechanical Engineering.
- Published 10's of research papers in international journals and conferences.
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- Scholars interested in my publications can visit: http://scholar.cu.edu.eg/galal