RESEARCH ARTICLE OPEN ACCESS

Optimal Kinematic Synthesis of Planar Mechanism, Part I: Offset Crank-Slider Mechanism

Galal Ali Hassaan
Department of Mechanical Design & Production, Faculty of Engineering, Cairo University, Giza, Egypt

----------------------------*************************-------------------------

Abstract:

Optimal synthesis of mechanisms is a successful approach for mechanism design to satisfy all the desired characteristics of the designed mechanism. The crank – slider mechanism has wide industrial applications and providing an offset feature provides a design with a stroke greater than its crank length and a time ratio greater than one.

The optimal design problem in this case is a constrained multi-dimensional problem. Powell optimization technique is used to minimize a special objective function combining the mechanism stroke and time ratio. 2 functional constraint functions are used for the minimum and maximum transmission angle of the mechanism.

The optimal results are fitted in a proper nonlinear model relating the normalized mechanism dimensions to the normalized stroke and time ratio. A comparison is conducted between the optimal design and the non-optimal one illustrating the advantages of the optimal approach.

Keywords:- **Kinematic synthesis of planar mechanisms, offset crank-slider mechanism, optimal synthesis, computer applications.**

----------------------------*************************-------------------------

I. INTRODUCTION

Mechanisms represent the skeleton of machinery. Successful synthesis of mechanisms leads to a successful machine design. On the other hand classical mechanism synthesis techniques lead to mechanisms satisfying some kinematic requirements such as stroke, time ratio, specific link positions, specific function generation etc.

To satisfy all the kinematical requirement more advanced approaches are required such as optimal design methodology. In this case it would be possible to satisfy all the requirements in the form of objective function and functional constraints.

The optimal approach of mechanism design is handled by a number of researchers through the last decades. Suh & Mecklenburg (1973) used Powell's optimization technique for optimal synthesis of spatial mechanisms. They used a least square objective function [1]. Rao and Lakshminarayana (1984) studied the optimal design of RSSR rocker mechanisms. Their objective function was the minimum transmission angle [2]. Sadler & Yang (1990) studied the optimal design of RSSR crank-rocker mechanisms for a unit time ratio and various oscillation angles [3]. Krishnamurty & Turcic (1992) presented the application of multiple objective optimization techniques based on the methods of nonlinear goal programming to perform optimal synthesis of general planner mechanisms [4]. Gutkowski, Bauer & Putresza (1995) studied the optimal design of multi-arm mechanism elements. They considered hinge reactions as an objective to be minimized. They applied their approach to excavator arms [5]. Shibaik (1995) a rationalized approach to design a micro – mechanism focusing on configuration, material and fabrication processes [6]. Zhang, Zhou & Ye (2000) optimized the mechanism design using a sum of squares objective function and linear constraints [7].

Lau, Du & Lim (2001) used a function based on functional specifications to solve for the topology of complaint mechanisms [8]. Haulin, Lakis & Vinet (2001) studied the optimal synthesis of a planar 4-link mechanism with reference to n positions of the output and coupling bars. They investigated the effect of the number of positions on the optimal dimensions [9]. Zheu & Cheung (2001) introduced the concept of orientation structural error of the fixed link and presented an optimal synthesis of crank-rocker linkages for path generation [10]. Cabrera, Simon & Prado (2002) studied the optimal synthesis of planar mechanisms using genetic algorithms based on evolutionary techniques and the type of goal function. They teste their technique using 4 – bar mechanisms [11]. Mermerta (2004) studied the optimal kinematic design of a planar manipulator with 4-bar mechanism. The objective function was the local mobility index to maximize the manipulator performance [12].

Hao & Merlet (2005) used an approach based on interval analysis allowing determining almost all possible mechanism geometries such that all compulsory requirements are satisfied simultaneously [13]. Chen & Yang (2005) applied a multidisciplinary design optimization to generate optimum mechanisms. The optimized mechanisms satisfied mechanism and structural constraints [14]. Liu, Wang & Pritschow (2006) the optimal kinematic design of a PRRRP mechanism having 2 degrees of freedom. They assigned a performance chart with indices for workspace, control accuracy, velocity, payload capability and stiffness [15]. Martin, Russel & Sodhi (2007) presented an algorithm for selecting planar 4-bar motion generators with respect to Grashof conditions, transmission angle and having the minimum perimeter value [16]. Mundo & Yan (2007) proposed a method for the kinematic optimization of transmission mechanisms where non-circular gears are used to perform a mechanical control on the output motion. A ball – screw transmission mechanism was investigated and their objective was to lower the screw acceleration [17]. Wu, Liu & Wang (2007) developed closed form solutions to optimize the kinematics design of a 2 DOF planar manipulator based on the work space [18]. Gatti & Mundo (2007) proposed a method for the optimal synthesis of cam – integrated 6-bar linkages for tasks of exact rigid-body guidance. They used an optimization technique based on the evolutionary theory. Genetic algorithm was employed as an optimum searching procedure [19]. Liu & Wang (2007) applied the performance chart based on design methodology to parallel mechanisms as an optimal kinematic design methodology [20].

Chen & Liu (2007) studied the optimal design of Stewart platform safety mechanism considering the singular points and joints rotational angles [21]. Xie & others (2009) presented a kinematic optimization design of a 4R 2-DOF parallel mechanism considering the force transmissibility [22]. Martin, Alonso & Castillo (2009) studied the path synthesis of crank-rocker mechanisms using a wavelet-based neural network [23]. Savsani, Rao & Vakharia (2010) studied the problem of minimum weight design of simple and multi – stage gear trains. They used the particle swarm optimization and simulated annealing to find the optimal design parameters [24]. Jiang, He & Tong (2010) studied the optimal design of the Gough-Stewart platform manipulators based on dynamic isotropy [25]. Peng & Sodhi (2010) developed an optimal synthesis method for multi – phase continuous path generation of adjustable planar 4-bar linkage [26].

Daivagna and Balli (2011) suggested a method for the synthesis of a five-bar offset slider mechanism. They claimed that their approach is simple and accurate than the graphical techniques [27]. Moubarak, Tzvi, Ma and Alvarez (2012) presented an optimal synthesis for kinematic design and dynamic analysis of a dual-rod slider rocker mechanism. They presented a case-study application in modular robotic docking [28]. Dutta and Naskar (2013) presented a technique to design an adjustable offset slider-crank mechanism to generate a function and a path simultaneously with the lengths of the input link and offset varying. They used n-degree polynomial to design the contours of the guiding slots with n representing the number of precision points [29]. Jiguang, Chuanyn and Weiyang (2014) presented an analytical synthesis method of a crank slider mechanism with time ratio and a selecting range of design variables.

They solved the synthesis problem of having minimum maximum transmission angle when the time ratio is given [30].

II. ANALYSIS

The mechanism performance functions used in the optimal design of the mechanism are:

- Mechanism stroke, S.
- Mechanism time ratio, TR.
- Mechanism minimum transmission angle, TA_{min}.
- Mechanism maximum transmission angle, TA_{max}.

Fig.1 shows an offset crank-slider mechanism in its limiting positions.

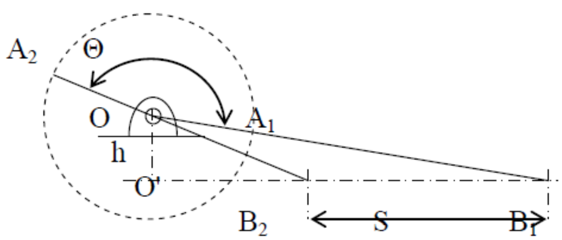

Fig.1 Mechanism limiting positions.

Stroke:

Let: $r_2 = OA$, $r_3 = AB$, $h = OO'$

Geometrically, the mechanism stroke, S is given by:

$$S = \sqrt{(r_3 + r_2)^2 - h^2} - \sqrt{(r_3 - r_2)^2 - h^2} \quad (1)$$

The normalized stroke, S_n is obtained by dividing the 2 sides of Eq.1 by r_2. That is:

$$S_n = \sqrt{(r_{3n}+1)^2 - h_n^2} - \sqrt{(r_{3n} - 1)^2 - h_n^2} \quad (2)$$

Where: r_{3n} = normalized connecting rod length $= r_3/r_2$

h_n = normalized offset $= h/r_2$

Time ratio:

Let: ψ = angle B_1OB_2

$$= \cos^{-1}\{h/(r_3+r_2)\} - \cos^{-1}\{h/(r_3-r_2)\} \quad (3)$$

Using the normalized parameters, Eq.3 becomes:

$$\psi = \cos^{-1}\{h_n/(r_{3n}+1)\} - \cos^{-1}\{h_n/(r_{3n}-1)\} \quad (4)$$

Angle, Θ:

$$\Theta = 180 - \psi \quad (5)$$

Time ratio, TR:

$$TR = (360 - \Theta) / \Theta \quad (6)$$

Minimum and maximum transmission angle:

The position of the offset crank-slider mechanism in the minimum and maximum transmission angle positions is shown in Fig.2.

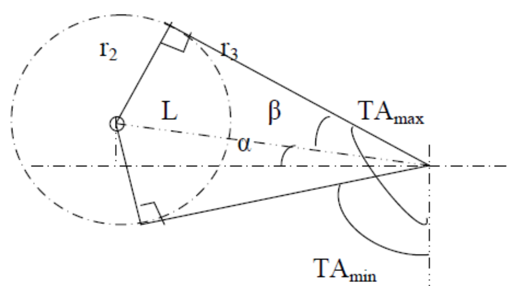

Fig.2 Minimum and maximum transmission angles.

Angle β:

$$\beta = \tan^{-1}(r_2/r_3) = \tan^{-1}(1/r_{3n}) \quad (7)$$

Angle α:

$$\alpha = \sin^{-1}(h/L) = \sin^{-1}(h_n/L_n) \quad (8)$$

where:

$$L = \sqrt{(r_2^2 + r_3^2)}$$

and $L_n = L/r_2 = \sqrt{(1 + r_{3n}^2)}$

Minimum transmission angle, TA_{min}:

$$TA_{min} = 90 - (\beta - \alpha) \quad (9)$$

Maximum transmission angle, TA_{max}:

$$TA_{max} = 90 + \beta + \alpha \quad (10)$$

III. OPTIMAL DESIGN OBJECTIVES

The objectives of the optimal design of the offset crank slider mechanism are:

(a) Attain a specific stroke.
(b) Attain a specific time ratio.
(c) Not to violate the recommended transmission angle during operation.

IV. OPTIMAL DESIGN OBJECTIVES

(i) Design parameters:

The kinematic design parameters of the offset crank-slider mechanism are:

- Normalized offset, h_n.
- Normalized connecting rod length, r_{3n}.

(ii) Objective function:

An objective function is selected to satisfy the first requirements of stroke and time ratio. That is:

$$F = \delta_1(S_n - S_{ndes})^2 + \delta_2(TR - TR_{des})^2 \quad (11)$$

Where:

δ_1 and δ_2 are weighing constants.

(iii) Functional constraints:

Functional constraint 1, F_{c1}:

$$F_{c1} = TA_{min} \quad (12)$$

Functional constraint 2, F_{c2}:

$$F_{c2} = TA_{max} \quad (13)$$

Functional constraint 3, F_{c3}:

This constraint is set to avoid imaginary stroke as in Eq.2. That is:

$$F_{c3} = (r_{3n} - 1)^2 - h_n^2 \quad (14)$$

(iv) Design parameters constraints:

$$0.1 \leq h_n \leq 3 \quad (15)$$
$$1.05 \leq r_{3n} \leq 10 \quad (16)$$

(v) Limits of functional constraints:

$$45^o \leq F_{c1} \leq 90^o \quad (17)$$
$$90^o \leq F_{c2} \leq 135^o \quad (18)$$
$$0.05 \leq F_{c3} \leq 10 \quad (19)$$

(vi) Technique:

The Powell's conjugate gradient optimization technique for multi variables unconstrained minimization is used in this work [31]. This technique is popular since it does not need any derivatives for the minimized function.

To use this technique the constrained optimization problem has to be transferred to an unconstrained one. This is achieved using the principle of variables transformation for variables transformation originated by Box [32] and the penalty functions for functional constraints [33].

- Constrained parameters transformation [31]:

$$Y_i = \sin^{-1} [(X_i - L_{1i}) / (H_{1i} - L_{1i})] \quad (20)$$

Where: Y_i = transformed unconstrained design parameter.

X_i = constrained design parameter

L_{1i} = lower limit of parameter i

H_{1i} = upper limit of parameter i

- Modified objective function using penalty functions [33]:

$$F_m = F + \sum_{j=1}^{3} [S\,(\Delta F_{cj})^2 / K_j] \quad (21)$$

where: $S = 1$ if $\Delta F_{cj} > 0$
$= 0$ if $\Delta F_{cj} \leq 0$

K_j = penalty constant (small value) associated with the functional constraint F_{cj}.

ΔF_{cj} = difference between the functional constraint and the limits G_{2j} and H_{2j} on those constraints given by:

$$\Delta F_{cj} = G_{2j} - F_{cj} \quad \text{if} \quad F_{cj} \leq G_{2j}$$
$$= F_{cj} - H_{2j} \quad \text{if} \quad F_{cj} \geq H_{2j}$$

V. OPTIMAL MECHANISM DESIGN

The objective function given by Eq.21 is minimized with respect to the unconstrained parameters given by Eq.20.

Some selected values for mechanism stroke and time ratio are selected as desired values. A prepared computer program is used producing the optimal mechanism parameters and the corresponding stroke, time ratio, minimum transmission angle and maximum transmission angle.

The program outputs are presented graphically in Fig.3 for the optimal stroke and Fig.4 for the optimal time ratio.

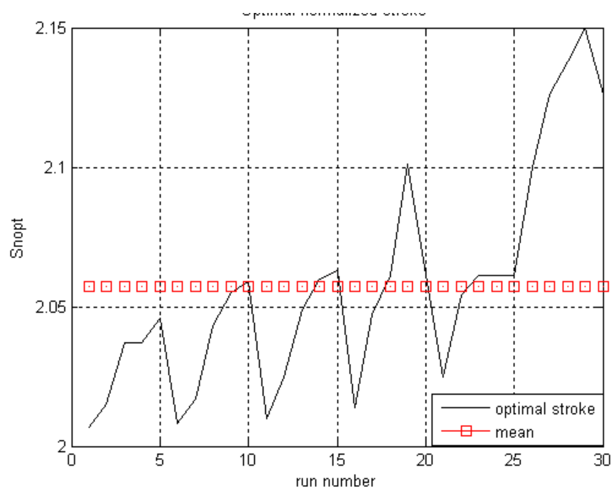

Fig.3 Optimal normalized stroke.

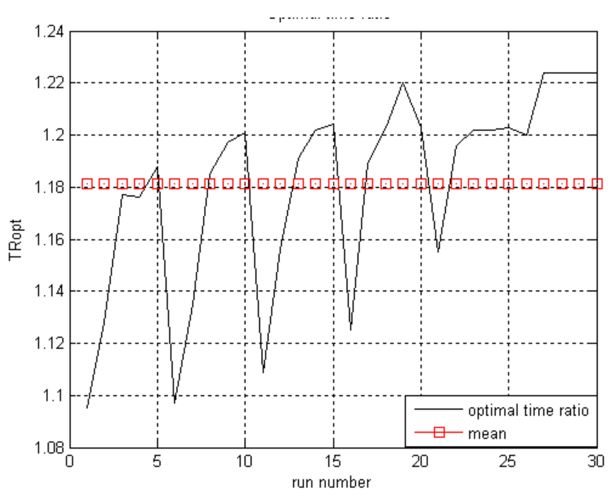

Fig.4 Optimal time ratio.

The optimal normalized stroke has the characteristics:

<u>Range:</u>

$$2.008 \leq S_{nopt} \leq 2.150$$

<u>Mean:</u>

$S_{nopt,m} = 2.057$ with 0.0391 standard deviation.

<u>Deviation from mean:</u>

$$-0.049 \leq \text{Deviation} \leq 0.093$$

The optimal time ratio has the characteristics:

<u>Range:</u>

$$1.094 \leq TR_{opt} \leq 1.224$$

<u>Mean:</u>

$TR_{opt,m} = 1.181$ with 0.0384 standard deviation.

<u>Deviation from mean:</u>

$$-0.087 \leq \text{Deviation} \leq 0.043$$

Optimal mechanism normalized dimensions:

To facilitate computer aided design of mechanisms, the normalized mechanism dimensions are presented in the form of second order 2 independent variables models. The independent variables are the desired stroke and time ratio in the ranges associated by the optimal process. The models take the form:

$$h_n = a_{01}+a_{11}S_n+a_{21}TR+a_{31}S_n^2+a_{41}TR^2+a_{51}S_nTR \quad (22)$$

$$r_{3n}= a_{02}+a_{12}S_n+a_{22}TR+a_{32}S_n^2+a_{42}TR^2+a_{52}S_nTR \quad (23)$$

The parameters of the 2 models of Eqs.22 and 23 are:

a_{01}= -78.17080688477 ; a_{11}= 42.1585998535
a_{21}= 48.89712905884 ; a_{31}= - 2.0545334816
a_{41}= -1.86831796169 ; a_{51}=-22.46410369873
a_{02}= -230.653076172 ; a_{12}=144.45611572266
a_{22}= 124.905418396 ; a_{32}= -15.91238498688
a_{42}= - 6.7251915932 ; a_{52}= -55.66365814209

VI. CASE STUDY

The objective of this case study is to investigate the validity of the optimal design approach through the comparison with a conventionally designed offset crank slider mechanism.

<u>Design requirements :</u>

r_2 = 100 mm , S = 250 mm , TR = 1.5

<u>Requirements:</u>

h (offset) and r_3 (connecting rod length).

<u>Optimal dimensions:</u>

Normalized desired stroke, S_n = 2.5.

Both S_n and TR are outside the range associated by the optimal design approach.

The optimal stroke and time ratio using the results of this research are:

$S_{opt} = 2.12$

and $TR_{opt} = 1.15$

The corresponding normalized mechanism dimensions using the models in Eqs.22 and 23 are:

$h_{nopt} = 0.9649$

and $r_{3n} = 3.1164$

Thus, the optimal mechanism dimensions are:

$h = 96.49$ mm

and $r_3 = 311.64$ mm

Non-optimal dimensions:

With $r_2 = 100$ mm, S = 250 mm, and TR = 1.5, Eqns.1 and 6 have only 2 unknowns: h and r_3. The equations are nonlinear. Solving the equations gives h and r_3 as:

$h = 138.5$ mm

and $r_3 = 262.5$ mm

The mechanism performance in both optimal and non-optimal design is as follows:

Slider displacement:

The slider displacement with both non-optimal and optimal dimensions is shown in Fig.5 as generated using MATLAB for one revolution of the crank.

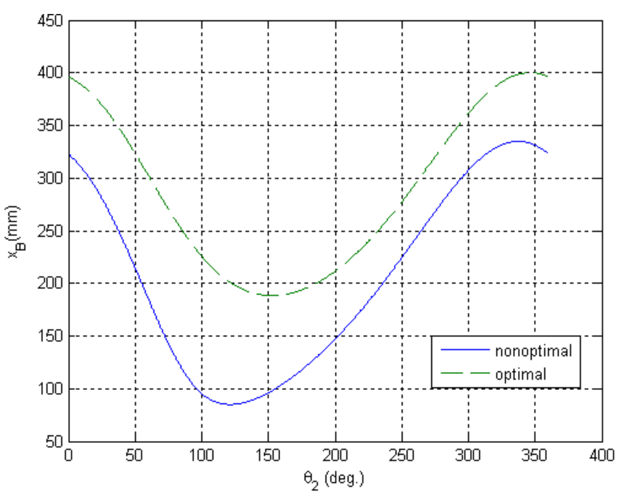

Fig.5 Slider displacement.

Transmission angle:

The transmission angle of the mechanism with both non-optimal and optimal dimensions is shown in Fig.6 for revolution of the mechanism crank.

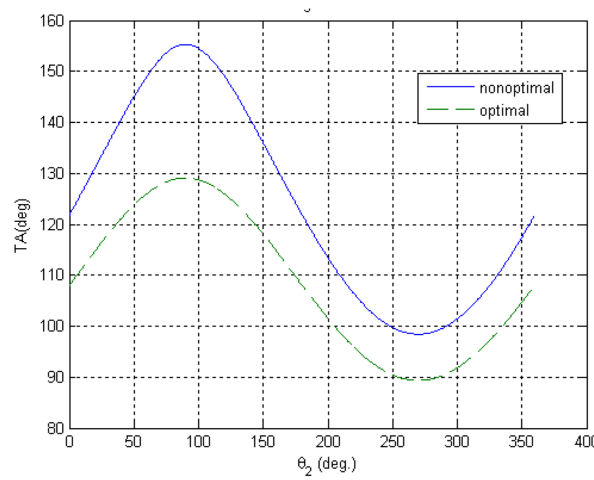

Fig.6 Transmission angle Transmission angle range:

$89.355 \leq TA \leq 129.087$ degrees (optimal design)

$98.434 \leq TA \leq 155.309$ degrees (non-optimal design)

VII. CONCLUSIONS

^ Optimization is a powerful technique which leads to successful kinematic design of machinery.

^ It tries to satisfy all the kinematic constraints assigned by the designer.

^ The optimal design process was reduced to the assignment of the desired stroke and time ratio within an estimated range, and a simple code using regression models reveals the mechanism normalized offset and connecting rod length.

^ The optimal normalized stroke was about 2.06.

^ The optimal time ratio of this type of mechanisms was ≤ 1.224.

^ The classical analytical design produced a mechanism transmission angle outside the recommended range.

^ The optimal design has kept the transmission angle within the recommended range sacrificing the desired stroke and time ratio.

REFERENCES

[1] C. Suh and W. Mecklenburg, "Optimal design of mechanisms with the use of matrices and least squares", *Mechanism and Machine Theory*, vol. 8, Issue 4, p. 479, 1973.
[2] L. Rao and K. Lakshminarayana, "Optimal designs of the RSSR crank-rocker mechanism—I. General time ratio", *Mechanism and Machine Theory*, vol. 19, Issues 4-5, p.431, 1984.
[3] J. Sadler and Z. Yang, "Optimal design of cam-linkage mechanisms for dynamic-force characteristics", *Mechanism and Machine Theory*, vol. 25, Issue 1, p.41, 1990.
[4] S. Krishnamurty and D. Turcic, "Optimal synthesis of mechanisms using nonlinear goal programming techniques", *Mechanism and Machine Theory*, vol. 27, Issue 5, p.599, September 1992.
[5] W. Gutkowski, J. Baues and J.Putresza, "Multi-arm mechanism design minimizing hinge reactions between arms", *Mechanism and Machine Theory*, vol. 30, Issue 6, p.829, August 1995.
[6] N. Shibaike, "Design of micro-mechanisms focusing on configuration, materials and processes", *Materials & Design*, vol.16, Issue 2, p.83, 1995.
[7] X. Zhang, J. Zhou and Y. Ye, "Optimal mechanism design using interior-point methods", Mechanism and Machine Theory, vol. 35, Issue 1, p.83, January 2000.
[8] G. Lau, H. Du and M. Lim, "Use of functional specifications as objective functions in topological optimization of compliant mechanism", *Computer Methods in Applied Mechanics and Engineering*, vol.190, Issue 34, p.4421, May 2001.
[9] E. Haulin, A. Lakis and L. Vinet, "Optimal synthesis of a planar four-link mechanism used in a hand prosthesis", *Mechanism and Machine Theory*, vol. 36, Issues 11-12, p.1203, November 2001.
[10] H. Zhou and E. Cheung, "Optimal synthesis of crank–rocker linkages for path generation using the orientation structural error of the fixed link", *Mechanism and Machine Theory*, vol. 36, Issue 8, p.973, August 2001.
[11] J. Cabrera, A. Simom and M. Prado, "Optimal synthesis of mechanisms with genetic algorithms", *Mechanism and Machine Theory*, vol. 37, Issue 10, p.1165, October 2002.
[12] V. Mermerta, "Optimal design of manipulator with four-bar mechanism", *Mechanism and Machine Theory*, vol.e 39, Issue 5, p.545, May 2004.
[13] F. Hao and J. Merlet, "Multi-criteria optimal design of parallel manipulators based on interval analysis", *Mechanism and Machine Theory*, vol. 40, Issue 2, p.157, February 2005.
[14] T. Chen and C. Yang, "Multidisciplinary design optimization of mechanisms", *Advances in Engineering Software*, vol. 36, Issue 5, p.301, May 2005.
[15] X. Liu, J. Wang and G. Pritschow, "On the optimal kinematic design of the PRRRP 2-DOF parallel mechanism", *Mechanism and Machine Theory*, vol. 41, Issue 9, p.1111, September 2006.
[16] P. Martin, K. Russel and R. Sodhi, "On mechanism design optimization for motion generation", *Mechanism and Machine Theory*, vol. 42, Issue 10, p.1251, October 2007.
[17] D. Mundo and H. Yan, "Kinematic optimization of ball-screw transmission mechanisms", *Mechanism and Machine Theory*, vol. 42, Issue 1, p.34, January 2007.
[18] J. Wu,T. Li and L. Wang, "Optimal Kinematic Design of a 2-DOF Planar Parallel Manipulator", *Tsinghua Science & Technology*, vol. 12, Issue 3, p.269, June 2007.
[19] G. Gatti and D. Mundo, "Optimal synthesis of six-bar cammed-linkages for exact rigid-body guidance", *Mechanism and Machine Theory*, vol.42, Issue 9, p.1069, September 2007.
[20] X. Liu and J. Wang, "A new methodology for optimal kinematic design of parallel mechanisms", *Mechanism and Machine Theory*, vol. 42, Issue 9, p.1210, September 2007.
[21] H. Chen , W. Chen and J. Liu, "Optimal Design of Stewart Platform Safety Mechanism", *Chinese Journal of Aeronautics*, vol. 20, Issue 4, p.370. August 2007.
[22] F. Xie , X. Liu, L. Wang and J. Wang, "Optimum Kinematic Design of the 4R 2-DOF Parallel Mechanism", *Tsinghua Science & Technology*, vol. 14, Issue 5, p.663, October 2009.
[23] G. Martin, F. Alonso and J. Castillo, "Shape optimization for path synthesis of crank-rocker mechanisms using a wavelet-based neural network", *Mechanism and Machine Theory*, vol. 44, Issue 6, p.1132, June 2009.
[24] V. Savsani, R. Rao and D. Vakharia, "Optimal weight design of a gear train using particle swarm optimization and simulated annealing algorithms", *Mechanism and Machine Theory*, vol. 45, Issue 3, p.531, March 2010.
[25] H. Jiang, J. He and Z.Tong, "Characteristics analysis of joint space inverse mass matrix for the optimal design of a 6-DOF parallel manipulator", *Mechanism and Machine Theory*, vol. 45, Issue 5, p.722, May 2010.
[26] C. Peng and R. Sodhi, "Optimal synthesis of adjustable mechanisms generating multi-phase approximate paths", *Mechanism and Machine Theory*, vol. 45, Issue 7, p.989, July 2010.
[27] U. Daivagna and S. Balli, "Synthesis of five-bar slider mechanism with variable topology", *Advances in Mechanical Engineering*, vol.2011, Paper ID 697316, 10 pages, 2011.
[28] P. Moubarak, P. Tzvi, Z. Ma and E. Alvarez, "Kinematic synthesis and dynamic analysis of the dual rod slider rocker mechanism: An application to modular robotics", *Proceedings of the ASME 2012 International Design Engineering Technical Conference and Computers and Information in Engineering*, Chicago, USA, August 12-15, 6 pages, 2012.
[29] S. Dutta and T. Naskar, "Synthesis of adjustable offset slider-crank mechanism for simultaneous generation of function and path using variable length links", *Proceedings of the 1st International and 16th National Conference of Machines and Mechanisms*, IIT Roorkee, India, December 18-20, p.465, 2013.
[30] H. Jiguang, Z. Chuanyn and Z. Weiyang, "Slider crank mechanism design with time ratio and minimum transmission angle", *International Journal of Control and Automation*, vol.7, No.11, p.163, 2014.

[31] M. Powell, "An efficient method for finding the minimum of several functions without calculating derivatives", *The Computer Journal*, vol.7, p.155, 1964.
[32] M. Box, "A comparison of several current optimization methods and the use of transformation in constrained problems", *The Computer Journal*,, vol.9, p.67, 1966.
[33] G. Beveridge and R. Schechter, "Optimization: theory and practice", *McGraw – Hill*, 1970.

BIOGRAPHY

Galal Ali Hassaan

- Emeritus Professor of System Dynamics and Automatic Control.
- Has got his Ph.D. in 1979 from Bradford University, UK under the supervision of Late Prof. John Parnaby.
- Now with the Faculty of Engineering, Cairo University, EGYPT.
- Research on Automatic Control, Mechanical Vibrations , Mechanism Synthesis and History of Mechanical Engineering.
- Published 10’s of research papers in international journal and conferences.
- Author of books on Experimental Systems Control, Experimental Vibrations and Evolution of Mechanical Engineering.
- Reviewer of a number of international journals including IJCT.