Transformation To Dual Physical Problems for The Visualization of Abstract Topics

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*Abstract –*Abstract topics are often very difficult to be visualized for the undergraduate students in mathematics, physics and engineering.Those problems could be represented by four fundamental examples; basic vector algebra, formation of standing waves, polarization of waves and optical filtering. In this work, the performance of teaching abstract topics using two different approaches are comparatively examined; the classical mathematical derivations and the transformation of abstract topics to visual dual physical problems. Efficiency of methodsare analysed statistically.Third year electrical engineering students' exam results are examined using the empirical cumulative distribution function (ECDF). The transformation method is observed to be helpful, especially for below the average students. Formations of healthy normally distributed exam scores are observed.

Keywords – electromagnetics, empirical cumulative distribution function, optical filtering, performance measurement.

1. Introduction

Current undergraduate curriculum for mathematics, physics and engineering offer series of basic concepts for students. All these basics must be comprehended and be placed in the long term memories of each individual student to guarantee success in further education. More advanced lectures rely on the efficient utilization of those basics. Some problems are harder to be visualized than others due to the differences between their domains. The variance of the performance of a classroom increases due to the differences of each individual's learning approach. Teaching abstract concepts requires understanding of the relationships of components of a set without the help of the basic senses of touching, seeing and hearing. This relationship of a set becomes even harder to be visualized in three dimensions when the process requiresrotations, projections and transpositions which could all be successively related. This is a common challengein teaching abstract topics to a classroom with a performance of high variance.
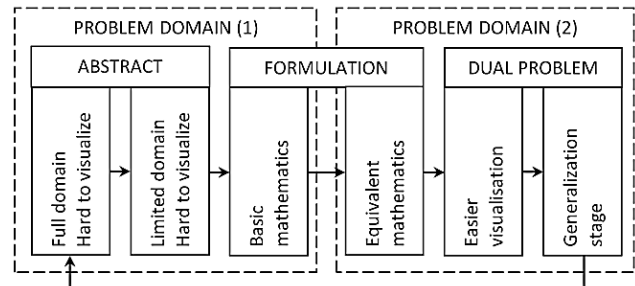

Fig. 1.Transformation of an abstract problem into a visual dual problem.

Some teachers are forced to aim the highest 70% of the classroom for success, leaving at least 30% just at the start. Some might think that this is fair enough, not to 'waste the time' of the majority. New programs are being developed to keep the whole classroom in contact; through interactions between classmates, utilizing visual simulation programs and supporting lectures with laboratories. Some examples are: UC Berkeley's webcast, Open Yale Courses,MIT's Open Courseware,Indian Institutes of Technology,and the Carnegie Mellon's Open Learning Initiative [1–6]. Today, E-learningand mobile learning are educational research technologies of today [7–10].Those supporting materials are very helpful. Unfortunately, they can be used in "as is" format. Customizations are not possible due to their copyrights. It is not always possible to develop similar lecture materials due to the limitations oftime, effort and money. Visualizations are often ignored while teaching abstract topics due to the unavailability of practical solutions.Today, educational priorities and goals for curriculum development are being analyzedfor balancing the traditional and technological methods [11–13].

In this work,the method of visualization of abstract topics by transformation to dual physical problems is examined. The exam scores obtained by different teaching methods are analyzed. The empirical cumulative distribution function (ECDF) is used for comparisons. Four canonical examples are selected from the field of electromagnetic wave theory.

2. Transformation method for problem visualization

The problem of transforming an abstract problem into a visual one is not always an easy task. It is not possible to use the same practical solution for all problems. Thus, different approaches are necessary. A practical and systematic methodology is required.

In this work, a general procedure is proposed. It is summarized in five steps as shown in Fig. 1. First, the abstract problem domain is narrowed down to a single problem element. This allows easier handling of the abstract concept. In the second step, the problem is formulated using simpler mathematics. Third stage is the critical innovation stage where the instructor should be able to jump into a different domain using equivalent mathematics. If this is possible, simplified mathematics is used to introduce the new visual and hopefully more interestingproblem in the fourth step. Transformation of domains ishelpful since similar formulations are used. The instructor can spend more time on the formulation of a much easierproblem. This way leads to being understood by the whole classroom. Fourth step would be more successful if also supported by interactions between classmates. At the final stage, returning to the original problem requires much less concentration and background when compared to the original hard to handle abstract concept.When this transformation method is used several times, students are expected to follow more quickly and effectively.

3. Canonical Examples

Four canonical examples are selected in the field of fundamental electromagnetic wave theory; (1) basic vector decomposition using unit vectors, (2) formation of standing waves, (3) definitions of polarization, and (4) optical filtering using polarizing filters. The transformation method proposed herein is illustrated in the following examples.

a. Example 1 – Basic Vector Algebra

First example is the starting point invector algebra. It is widely selected by almost all universities around the world, since vectors are one of the most important basic tools in mathematics, physics and engineering. Vector algebra offers solutions in various fields in multidimensional problems including; statistical mathematics, control theory, electromagnetics, signal processing, power engineering, etc. Some drawings on the board are often found hard to be visualized by the undergraduate students, since often unskillfully drawn figuresare limited to two dimensions.
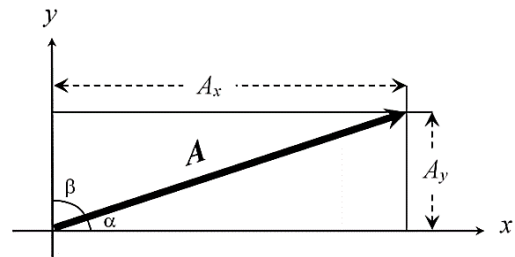
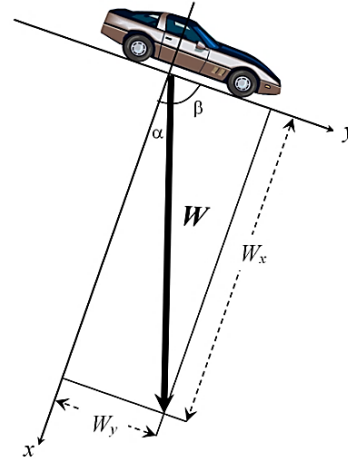*Fig. 2.Projection of a vector A onto the Cartesian coordinates, x and y.*

Fig. 3. A car parked on a slope, its weight is the sum of parallel and perpendicular components with respect to the road's surface.

Drawing in perspective is not always possible. This is an important drawback in understanding of further topics.

Let us now analyze the problem of decomposition of a vector in space. Any vector A is equal to the sum of its projections defined by the three orthogonal unit vectors for some coordinate system. If Cartesiancoordinates are selected, vector A can be written as

$$A = u_x A_x + u_y A_y + u_z A_z \qquad (1)$$

whereu_x, u_y, and u_z are the unit vectors, and A_x, A_y, and A_z are the scalar projection values along the *x, y* and *z*-axis[14, 15]. The projections can be found using the dot product operator

$$A_x = A \bullet u_x = A \cos(\alpha) \qquad (2)$$

$$A_y = A \bullet u_y = A \cos(\beta) \qquad (3)$$

$$A_z = A \bullet u_z = A \cos(\chi) \qquad (4)$$

whereα, β and χ are the directional cosines, $A=|A|$ is the length of vector A. Now let us narrow our problem by assuming $A$to be on the *x-y* plane defined by

$$A = u_x A_x + u_y A_y. \qquad (5)$$

The graphical representation for (5) is given in Fig.2.Once the problem is simplified and formulated, it is now easier to transfer to a different problem domain. Students could easily examine forces acting on a car parked at a slope as shown in Fig. 3.The instructor could discuss the forward force component acting on the car. The visualization could be emphasized by examining the limiting cases; flat surface ($\alpha = 0$)and the free fall ($\beta = 0$). Projections in three dimensions can be analyzed at the final generalization case.

b.   Example 2 – Formation of Standing Waves

Propagation of a single electromagnetic (EM) wave is regarded as one of the hardest topics for an undergraduate student. Interactions of two EM waves propagating in opposite directions near a reflection point is far more complex to cope with. The dynamics are explained using mathematics, and sinusoidal waveforms are drawn for a stationary observer. EM waves travel at the speed of light, and must be slowed down. Further details force a dead end for visualizations. It is obvious that transfering to a different domain is necessary. Alternatively, thesound or water waves can be studied. They both have very similar dynamics; their propagation and standing wave mechanisms are almost equivalent. Sound wave alternative opens up whole new opportunities for the instructor. A guitar's string, a flute or the harmonica which is frequently used in Baskent University.

Simple demonstration tools can be developed; a stretched rope with one end tied to a wall, and the other is hold by hand. Up and down oscillationsapplied by the hand causes formation of standing waves in one dimension.Further, two dimensional standing wave formations can be observed by placing a$50\ cm$x$50\ cm$square metal plate on a comparable size speaker, or a simple excitation by a bow as illustrated in Fig. 4. For a list of similar examples visit [16–18].

If one sprinkles some amount of salt on the plate, salt particles are pushed towards the stationary nodes of the standing wave patterns, forming highly visual forms. It provides unforgettable experience for the students to observe various standing wave patterns forming instantaneously at different frequencies. Switching from the electromagnetic waves to sound waves enables students to develop very practical tools for visualizations. Musical instruments like; violin, guitar and drums are also good examples of standing wave generators. Numerous visualization solutions are possible.

Fig. 4.Formation of standing waves by the excitation of a metal plate by a bow.Note that, the salt crystals are pushed to the stationary nodes [16].

c.   Example 3 – Polarization of Transverse Electromagnetic (EM) Waves

Almost all textbooks introduce this topic by the list of possible polarizations; linear, circular and elliptical. Later, readers are informed by variations of circular and elliptical polarizations to be clockwise (CW) and counter clockwise (CCW) directions. They are forced to imagine looking towards the incoming wave to decide whether this rotation is CW or CCW. The mathematical derivations further complicates the problem. For a transverse EM wave propagating in an arbitrary direction defined by the spherical coordinates

$$\boldsymbol{E}(r) = (\boldsymbol{u}_1 E_1 \cos(\omega t + kr + \theta_2) \\ + \boldsymbol{u}_2 E_2 \cos(\omega t + kr + \theta_2) \tag{6}$$

where$r$ is the distance from the origin, $\boldsymbol{u}_1$ and $\boldsymbol{u}_2$ are the perpendicular unit vectors lying on the transverse plane[14]. Eq. (6) can be rewritten using phasors [19–25].

$$\boldsymbol{E}(r,t) = (\boldsymbol{u}_1 E_1 e^{j\theta_x} + \boldsymbol{u}_2 E_2 e^{j\theta_y})e^{j\boldsymbol{k}\bullet\boldsymbol{r}}. \tag{7}$$

The rigorous analysis of the polarization using (6 and 7) is outside the scope of this work. Let us assumethat the wave propagation is along the +z-axis. Then, the electric field can be expressed as

$$\boldsymbol{E}(z,t) = \boldsymbol{u}_x E_x \cos(\omega t + kz + \theta_x) \\ + \boldsymbol{u}_y E_y \cos(\omega t + kz + \theta_y) \tag{8}$$

and similarly using the phasors,

$$\boldsymbol{E}(z) = (\boldsymbol{u}_x E_x e^{j\theta_x} + \boldsymbol{u}_y E_y e^{j\theta_y})e^{-jkz} \tag{9}$$

The polarization can be determined using the relative phase between the x and y components. Polarization islinear when;one of $E_x$and$E_y$components are
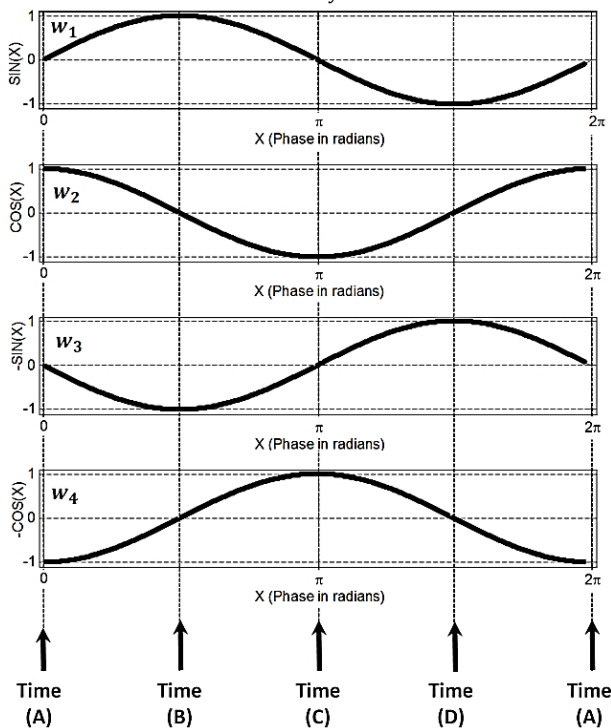
zero, or when they are in phase or out of phase by 180°.

$$\Delta\theta = \theta_y - \theta_x = n\pi. \qquad (10)$$

Circular polarization is much harder to visualize. It is the case when $E_x = E_y$, and when

$$\Delta\theta = \theta_y - \theta_x = (2n+1)\pi/2. \qquad (11)$$

It is defined to be clockwise and counter-clockwise when $\Delta\theta = 90^0$ and -90^0, respectively. Elliptical polarization is the case when $E_x \neq E_y$, and when

$$\Delta\theta = \theta_y - \theta_x \neq n\pi/2. \qquad (12)$$

It is often very hard for the students to understand the physics behind the mathematics, especially when lecture lacks any visual support.

Polarization of an EM wave is defined by the motion of the tip of the electric wave vector, projected to the transverse field. This movement is a function of time and place, and the tip could travel with the speed of light in a helix trajectory. Unfortunately, the time and concentration allocated for this complex topic aregenerally very limited.

The polarization example can be analyzed by using a similar mathematical problem in mechanical engineering. Initially, the elliptical polarization can
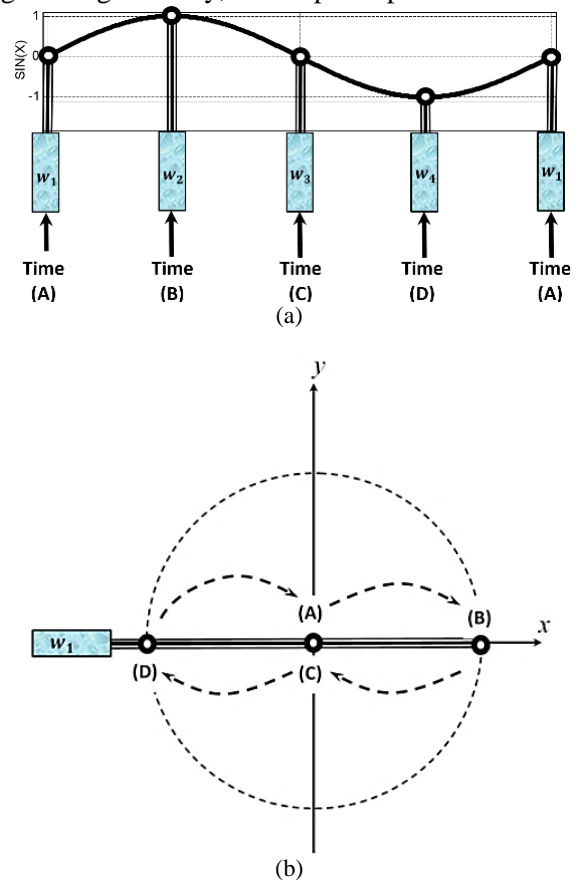
be eliminated, and the phase of the phasor ($\Delta\theta$) can be limited to four alternatives, and waveforms; w_1 thru w_4 are shown to oscillate out of phase according to Fig. 6. The phase differences are clear when they are evaluated at time instants; A, B, C and D. Now let us assume that each waveform represents the length of a mechanical piston as illustrated in Fig. 6 (a). The first piston's motion is given in time (Fig. 6 (b)).

The piston modeling of the phase function is helpful for visualization the EM wave's linear x-polarization. Linear y-polarization follows similarly. Combination of two pistons illustrated in Fig. 7 is much more visual when compared to the electric field formulated in (6–9). It is shown that at time instant (A) $w_1 = 0$ along x-axis and $w_2 = 1$ as illustrated in Fig. 7 (a). Comparing also with Fig. 5, it is now obvious that w_1 will increase from 0 to 1 while w_2 will decrease from 1 to 0 along the circular path to reach the point (B). This visualization is expected to be clear for 100% of the classroom to beclockwise. It should also be clear that polarization becomes counter-clockwise if w_4 is used instead of

w_2 for the y-axis as shown in Fig. 7 (b). Eqns. (6-12) state the same mechanism using mathematics.
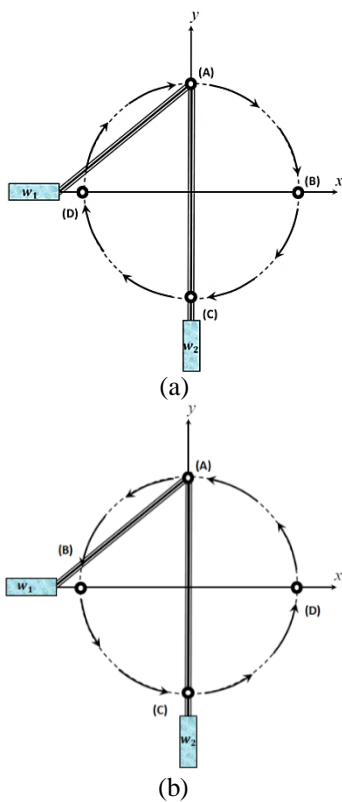

(a)

(b)

Fig. 7.Motion of two mechanical pistons pushing in x and y axis, using waveforms w_1 and w_2 respectively, (a) clockwise rotation, (b) counter-clockwise rotation.

d. Example 4 – Optical Filtering Using Polarizers

Polarizing filters are analyzed both in undergraduate and graduate level optics courses. A common example is studied here.

A total transverse EM wave propagating along the z-axis can be written as the sum of the two linearly polarized waves as given in (13).

$$E_{in}(z) = E_x(z) + E_y(z) \qquad (13)$$

where

$$E_x(z) = u_x E_x e^{-jkz} \qquad (14)$$

$$E_y(z) = u_y E_y e^{-jkz} \qquad (15)$$

Let us place two polarizing filters in front of the propagating wave E_{in} as shown in Fig. 8 (a). The optical transfer functionsfor the polarizing filters are illustrated in Fig. 9.It is almost obvious that the output of Fig. 8 (a) is zero. First filter stops the x-polarized component given in (14) and the remaining y-polarized wave of (15) is stopped by the second filter. Students use this clear information to falsely deduce that it is also true for the case shown in Fig. 8

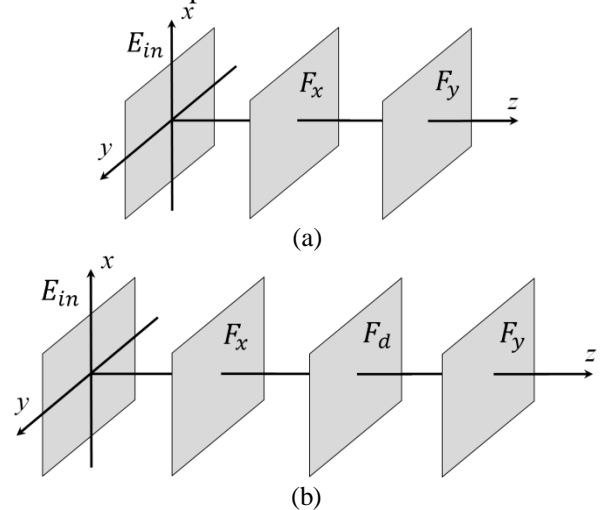(b). Even rigorous derivationsare observed not to convince even top students in the class.

(a)

(b)

Fig. 8. Multiple stage optical filtering, E_{in} is the input and (a) two filtering stages, F_x is the x-polarized and F_y is the y-polarized filters, (b) similar to (a) but a third diagonally polarized filter F_d is placed in between the two filters.
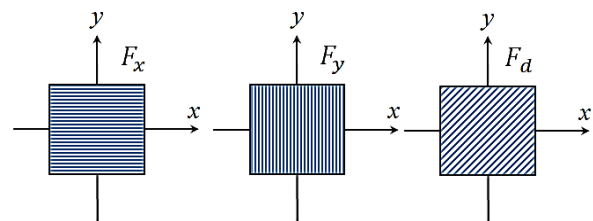

Fig. 9.Three optical polarizing filters drawn on the transverse plane.

This example is a good validation of the visualization problem analyzed in this work. It is really hard to convince students that the output could be different from zero due to the presence of F_x and F_y. It seems almost true that; the x and y polarized components of E_{in} are stopped by F_x and F_y, and this must yield a zero output. The function of the second optical filter,F_d, isnot very clear and is the source of this confusion. The related vector analysis is summarized in Fig. 10 (a). The input field E_{in} can be decomposed into x and y polarized waves (Fig. 2). F_x stops the x-polarized component of E_{in} as illustrated in Fig. 10. (b) so that, the input to the filterF_d is only the y-polarized E_1. It is interesting to observe that the output of F_d, E_2, does have an x-polarized component. This is the missing link that even if F_x stops all of the x-polarized wave initially, F_d rotates its output so that x-polarization reappears. The successive filtering results given in Fig. 10 (b) can be equivalently used to in a different domain. The mathematics suggest that the output should not be zero and successive filtering has an overall effect of rotating the input polarization by 135 degrees.This discussion could be helpful for some students. Unfortunately, it is found that most of the students

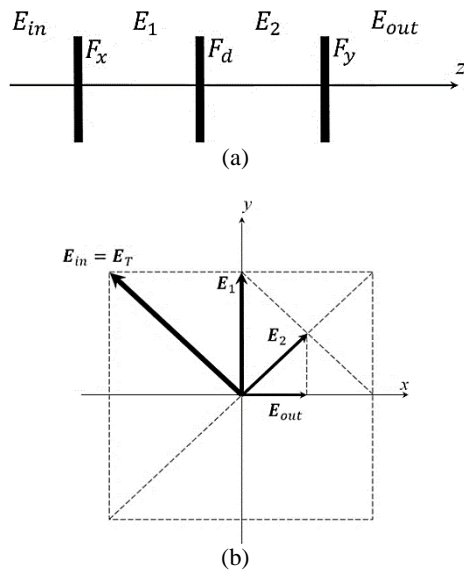fail to answer this exam question. This is possibly because complex mathematicalinformation

(a)

(b)

Fig. 10. Three stage optical filtering using F_x, F_d and F_y, (a) simple two dimensional drawing, (b) summary of vector projections of $\mathbf{E}_{in}$ through $\mathbf{E}_{out}$.
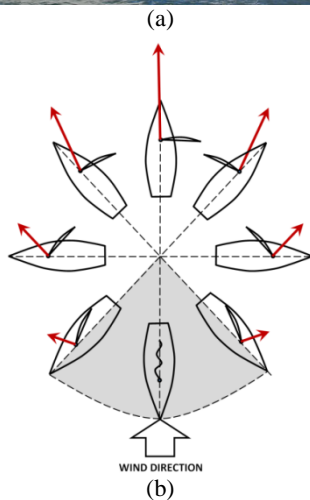

(a)

(b)

Fig.11. Sail boats moving with the force of wind, (a) two sail boats moving towards the wind flow (photograph), http://news.mulletboatracing.co.nz/ (b) wind force components on sail boats for different angles (boats in gray directions cannot profit from the wind).

is stored in the short-time memory if not supported visually.

It is necessary to move on to another domain to examine what is really going on. Following the steps proposed in Fig. 1, the problem can be simplified and moved into a more visual domain that has equivalent
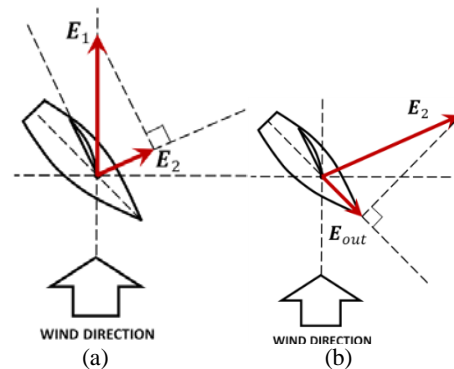

(a)   (b)

Fig.12. Sail boats moving with the force of wind, (a) sail transfers wind forces component as E_1, (b) the keel of the boat projects sail's force to the boat rudder.

Fig.13. The place and shape of the keel of the boat, http://en.wikipedia.org/wiki/Keel

mathematics as summarized in Fig. 10 (a). The new domain should illustrate the unpredictable nonzero outcome physically. For this example, the physics of sailing is examined. Sails travel towards the wind direction with the power of wind itself as shown in Fig. 11. Please notice the wind direction is aligned with the flags on the bridge. Both boats travel almost towards the wind, and it would not be wrong to say that it is unbelievable. Projections of wind power can be analyzed in three steps similar to the original problem. First, wind force is partially transferred to the boat by the sail as illustrated in Fig. 12 (a). Then, this force is projected once again as a forward force by the keel of the boat Fig. 12 (b). The keel is a long plate underneath the boat's body as shown in Fig. 13. It is interesting that the wind force E_1, and its projected force E_{out} clearly have opposing vectoral force components along the axis of the boat.

4. Statistical Analysis of Exam Results

The exam scores are expected to be normally distributed when the number of students converges infinity. It is a common mistake to assume the class distribution to be normal in practical applications. Thus, the mean and the standard deviation values are

often not very helpful. Statistical methods valid for finite setof observations should be used since the
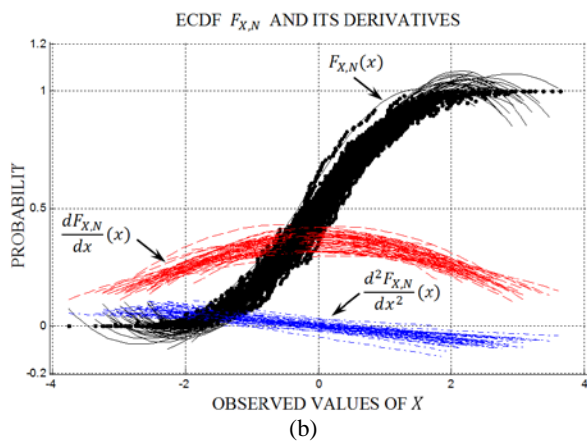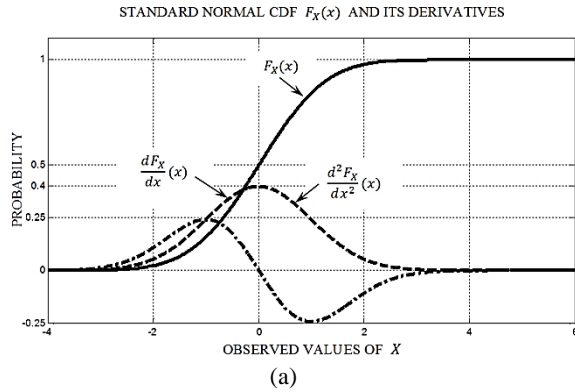

(a)

(b)

Fig.14. (a) The CDF and its first and second derivatives for the standard normal distribution, (b) the ECDF and its first and second derivatives for 100 observed samples of (a), 50 independent sets are used.

number of students could be as small as 10. In this work, the exam scores are examined using the empirical cumulative distribution function (ECDF) [26–28].

For an ordered set of N elements, $X_n = \{x_n : x_n \in \Re, n = 1, 2, \cdots, N\}$ where $x_p < x_r$ for $1 \leq p < r \leq N$, the ECDF is defined as

$$
F_{X,N}^e(x) = \begin{cases} 0 & x < x_1 \\ n/N & x_n \leq x \leq x_n \quad (1 \leq n \leq N-1) \\ 1 & x_N \leq x \end{cases} \quad (16)
$$

wherethe ECDF, $F_{X,N}^e$, converges to the CDF, F_X for N $\rightarrow \infty$. Note also that the probability density function (pdf) and the CDF is related by

$$
Prob\{x < x_n\} = F_X(x_n) = \int_{-\infty}^{x_n} f_X(x)\, dx \quad (17)
$$

$$
f_X(x_n) = \left[\frac{d}{dx} F_X(x)\right]_{x=x_n}. \quad (18)
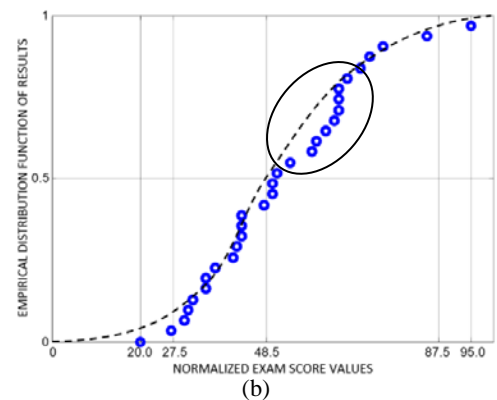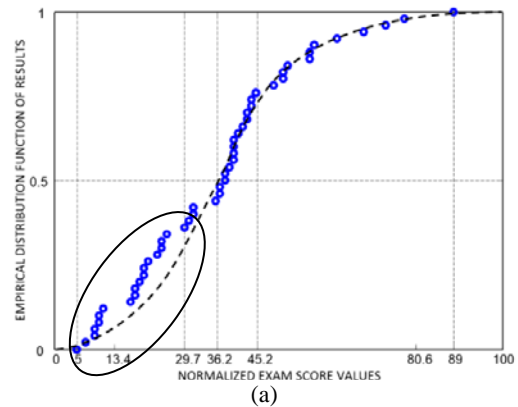$$

(a)

(b)

Fig. 15. The ECDF results for the EEM-323 for semesters; (a) Fall 2013, (b) Fall 2014.

The CDF and the pdf are the asymptotical functions valid only when $N \rightarrow \infty$. For a finite set of exam results, pdf can be estimated numerically. Least squares estimation for the pdf can be used as shown in Fig 14. Note that, the first derivative of the least squared estimation of ECDF (LSE-ECDF) is the least squared estimate of the empirical pdf (LSE–epdf). One further derivative of LSE–epdf illustrated in Fig. 14 (b) shows that the score domain can be divided into two regions where the zero crossing (middle) point is the observed value for the mean.

For small number of observations (exam scores), assumption of any type of distribution, including normal distribution is not accurate. Thus, mean and standard deviation values are not accurate. However, the EPDF and its derivatives provide statistical information for any type of observed distribution even for a classroom of just ten students. Note also that, it is possible to compare the EPDF results even if the number of students are not equal.

5. Numerical Results

The transformation method summarized in Sect. 2 is used in the development of third year undergraduate exams for the EEM-323

Electromagnetic Wave Theory II (Fall2013 and Fall 2014 semesters). Both set of exams are developed to be equivalent. In the first year, conventional teaching method is used. In the following year, fundamental topics are taught using the transformation to visual dual problem domain analyzed in this work. The canonical questions given in Sect. 3 are used in preparing of the quizzes, midterm and final exams.Individual scores for those preselected questions are added to form a single reference test score. It is then normalized to 100 for each student. Those combined scores are statistically analyzed. The ECDF results are given in Fig. 15. Note that the students below the average form a statistically lagging group (shown inside the ellipse) as illustrated in Fig. 15 (a). On the following year, it is observed that visual transformation of the abstract topics helps those students to catch up with the rest of the class. It is interesting to observe that similar improvement is observed for the above the average students as shown inside the ellipse (Fig. 15 (b)).

6. Conclusion

It is observed that classical method of teaching could divide the classroom into two groups with different mean and standard deviation values. On the other hand, visualization of abstract topics helpsthe classroom's observed scoresto form a population with the same distribution. The visualization method is found helpful to students, especially the below average students. They are observed to catch up with the rest of the class, forming a population of the same distribution.

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