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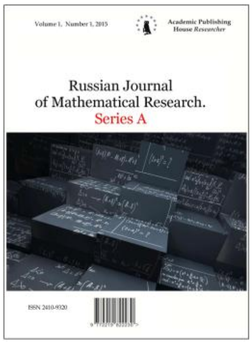

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Tolling Contracts With Two Driving Prices

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Abstract

The power plant tolling contract is one of most complicated derivative instruments among energy derivatives. The paper continues the investigations begun in [1]. Some papers introduce more complicated switching strategies. However, we deal with much simpler dispatch structures, to introduce a new method of evaluation.

Keywords: Tolling agreement; stopping time; switching; optimal strategy.

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Introduction

A tolling contract is an agreement between the buyer and the owner of a power plant. The latter lends the power plant to the buyer, who takes all the profit from selling the produced electricity. Running a power plant is not always profitable. Sometimes a fuel price (we take here only the power produced from fuel) is quite high, and this may lead to either having little (or no) profit or having costs. Here one can, of course, hold down the production by switching off the power generator. But this also incurs costs as you switch on the generator later. Each time the generator is being switched on from the off state, additional costs arise. These may include the amortization costs, costs of the additional energy used for switching on, or simply the lost time, as this plant needs time to start working again. We will assume that these costs occur instantaneously, even though they are not. Additionally there can be a restriction posed on the time of switching on. The main idea is that if the generator is switched off and on too fast, or just many times, the generator can break down or this can lead to mulfunction. To avoid having such problems, the plant owner may impose above restrictions.

From financial point of view, the tolling contract is interesting as it implies a starting cost, or just a price of a contract, which the buyer or holder pay, for having the right to regulate the activity of generator, and take the profit. In some sense this is an option derivative. One can find in some literature ([2] and [3]) that the price of tolling agreement coincides (matches) with a sum of spark spread options.

The value of the tolling contract is taken to as the maximum possible discounted profit the holder can get in a mathematical expectation sense. The maximum is taken over all strategies the holder can take. And at last, under the strategy one should understand the alternating between **on, off** states of the generator, and also the level of operating of generator. Here it is talked about different states of generator. Specifically, maximum output generating state, and minimum output. Of course, one can take much more intermediate states, but generally it is taken as said. For more details see [1], [4] - [6].

Some papers introduce more complicated switching strategies. In this paper we work with much simpler dispatch structures to introduce a new method of evaluation. The derivation made are just results of calculations. Here we suppose that there is no time lapse between switching, even more, as in reality, the cost of switching between maximum output level and minimum output level is either zero or very little, we take the first case. So we take that there are no costs, and no time lapse for switching between the two levels of on state. The mathematical structure of these states can be seen in several paper, specifically in [4].

The organization of this paper is as follows. In this section the tolling contract will be briefly described. In Section 2 the mathematical structure of tolling contract with two driving prices will be described. In the final Section 3 a key tool to valuation is given, using the techniques of stopping times.

The mathematical framework. Let's first introduce the following notations.

M - the maximal number of dispatch decision points. Generally decisions are made on everyday frequency. M coincides with the end point of the contract.

Time variable changes continuously from 0 to $2T$, $2T$- the duration of tolling contract. We take 2T instead of T, to signify the fact that each dispatch day consists of on-peak (day time) and off-peak (night time) periods.

$X_{1,t}$ ($X_{2,t}$) - the on-peak (off-peak) (or day (night)) time price of electricity at t. But here one should understand that when it is said at time t in real process it is not of the same index.

Y_t - the price of fuel at time t. In this case the index coincides with the point of the process.

$X_{1,t}$ ($X_{2,t}$) and Y_t are described by stochastic differential equations.

N - the maximal time you are allowed to switch on the station (i.e to take on position from off position.)

$\overline{Hr}$ ($\underline{Hr}$) - the heat rate when operating at maximum (minimum) output level.

$\overline{Q}$ ($\underline{Q}$)- the amount of generated electricity per unit time when operating at maximum (minimum) output level.

$$H_i(t) = \begin{cases} I(X_{i,t} - Y_t\overline{Hr} \geq 0)(X_{i,t} - Y_t\overline{Hr})\overline{Q} + \\ I(X_{i,t} - Y_t\overline{Hr} < 0)(X_{i,t} - Y_t\underline{Hr})\underline{Q} - \\ VOM_i(t), & if\ state\ on \\ \\ 0, & if\ state\ off \end{cases}$$

- the instantaneous dispatch income. It will be included in the integral over the period of operating ($VOM_i, i = 1,2$ represents variable operational costs). Here it can be seen that we have combined the maximum output and minimum output levels in one equation assuming automatically switching between the levels. Starting working at highest output level, it switches to the lowest only if the former is not profitable, and we have loses. It is obvious that if we have profit, in any case we use the maximum level of operating. However sometimes if we don't want to switch off the generator, we would operate under lower output level to mitigate our loses.

Remark 1. If the process is not profitable at maximum output level of operating, then it wouldn't be profitable in the minimum output level either (i.e. $\overline{\mathrm{Hr}} < \underline{\mathrm{Hr}}$) . The index $\mathrm{i} = 1{,}2$ here stand for the night or day time. Also it should be noted, that the time index of $X_{i,t}$ is not the same, or cannot be the same that of Y_t. Later we will specify the time index.

$C_{on}(t)$ - the cost of switching on. (i.e. **off** $\rightarrow$ **on)**. Below the t will be adjusted.

$s(t)$ - non-random costs (they can be time dependent or constant). These can include fixed costs, other associated costs which are not included in VOM.

$N(on)$ ($N(off)$) - the actual times of switching on (switching off), i.e the actual number of changes **off** $\rightarrow$ **on** (**on** $\rightarrow$ **off**). Obviously $N(on) \leq N$ and $N(off) \leq N$.

τ_i - a point of change from one state to another. We yet speak about it as i-th stopping time. However it is not that obvious that this is really a stopping time in its mathematical definition (it will be proved in the proposition).

τ_i takes values as follows: 0; $\frac{T}{2M}$; $\frac{T}{M}$; $3\frac{T}{2M}$; ... ; T.

Remark 2. Stopping times with odd indexes represents the switch **on** $\rightarrow$ **off**, and the stopping time with even indexes **off** $\rightarrow$ **on**. It is because we suppose that we start the contract with the **on** state. Hence when we have the first change it is switch off. The next change would be when we switch it on. And so on. If we denote the number of stopping times with **n**, it is obvious that we can have at maximum $2N - 1$ changes.

We come to the following statement.

Proposition 1. τ_i are stopping times.

It is obvious as they take only the values $\frac{T}{2M}$; $\frac{T}{M}$; $3\frac{T}{2M}$; ... ; T. They are independent of $t > 2\tau_i$, and the natural filtration flow associated with it. So if it is more convenient we can take τ-s to take double of these, without making any significant change. So we are interested in the price (or value) of tolling contracts. We aim to provide a simple tool for the calculations, and a better formula for further using stochastic optimal control tools, like for example Hamilton-Jacobi-Bellman equation. For the price we have the following formula:

$$V(0) = \sum_{j=0}^{N(off)} \int_{2\tau_{2j}}^{2\tau_{2j+1}} \sum e^{-rt} H_i(t) dt - \int_0^{2T} e^{-rt} s(t)\, dt - \sum_{j=1}^{N(on)} e^{-2r\tau_{2j}}\, C_{on}(\tau_{2j}),$$

where the first component represents the periods where the generator is in **on** state, the second component are fixed costs and the last are the costs of switching on.

Note that the sum under the first integral arises as the period between switching off and the next switching on can pass through several days and nights. Further the costs of switching on $C_{on}(t)$ have concrete formula depending on the prices on that period of time.

This initial price is a random variable, and hence we are interested in its expectation and the maximal value of its expectation with respect to the stopping times.

The tolling contract value

Firstly we consider the continuous time model. (However in general the prices are established at the beginning of a day (night), and are assumed not change during it.) Under these conditions we can have for the first summand

$$\sum_{j=0}^{N(off)} \int_{2\tau_{2j}}^{2\tau_{2j}+1} \sum e^{-rt} H_i(t)\, dt = \sum_{j=0}^{N(off)} \sum_{l \in N} \int_{2\tau_{2j}}^{2\tau_{2j}+1} (e^{-rt} I\left(2l-2 \leq t\frac{M}{T} < 2l-1\right) H_1(t) +$$
$$+ e^{-rt} I\left(2l-1 \leq t\frac{M}{T} < 2l\right) H_2(t))\, dt, \qquad (3.1)$$

where $I(A)$ is the indicator function of A.

Here we have assumed that the contracts start at day time. (One can just switch the indexes if interested in night time starting contracts.) Therefore, if t belongs to the first period (which is on-peak), $t \in [0, \frac{T}{M})$ and for the second period (which is off-peak) $t \in [\frac{T}{M}, 2\frac{T}{M})$.

We now specify the time indexes of electricity prices in $H_i(t)$ for the above formula.

$$V(0) = \sum_{j=0}^{N(off)} \sum_{l \in N} \int_{2\tau_{2j}}^{2\tau_{2j}+1} (e^{-rt} I(2l-2 \leq t\frac{M}{T} < 2l-1) \times$$
$$\times\ (I(X_{1,\gamma(t)} - Y_t\overline{Hr} \geq 0)(X_{1,\gamma(t)} - Y_t\overline{Hr})\overline{Q} +$$
$$+ I(X_{1,\gamma(t)} - Y_t\overline{Hr} < 0)\ (X_{1,\gamma(t)} - Y_t\underline{Hr})\underline{Q} - VOM_1(t)) +$$
$$+ e^{-rt} I(2l-1 < t\frac{M}{T} < 2l)(I\left(X_{2,\frac{T}{2M}\gamma(t)-\frac{T}{2M}} - Y_t\overline{Hr} \geq 0\right) \times \qquad (3.2)$$
$$\times (X_{2,\gamma(t)-\frac{T}{2M}} - Y_t\overline{Hr})\overline{Q} + I\left(X_{2,\gamma(t)-\frac{T}{2M}} - Y_t\underline{Hr} < 0\right) \times$$
$$\times (X_{2,\gamma(t)-\frac{T}{2M}} - Y_t\underline{Hr})\underline{Q} - VOM_2(t)))\, dt -$$
$$- \int_0^{2T} e^{-rt} s(t) dt - \sum_{j=1}^{N(on)} e^{-2r\tau_{2j}} C_{on}(2\tau_{2j})$$

where $\gamma(t) = \frac{T}{M}([t\frac{M}{T}] + 2\{t\frac{M}{T}\})$. Here and below $[x]$ is the integer part of x, while $\{x\}$ is the fractional part of x.

Consider the indexes of the processes. Suppose that we have a 2-rd night time. In the hole process it will the period from $[3\frac{T}{M}; 4\frac{T}{M}]$. In the index t should run through $[\frac{T}{M}; 2\frac{T}{M}]$. If we take the integer part of t time $\frac{M}{T}$ it will be 3. $\frac{3}{2}\frac{T}{M} - \frac{T}{2M} = \frac{T}{M}$ which was the desirable point.

Now we take that along one period the prices remain the same and rewriting (3.2) for the case $r = 0$, that is, assume there is no discounting. Thus in this case we get the following formula:

$$V(0) = \sum_{j=0}^{N(off)} \sum_{k=0}^{K_j-1} \sum_{l \in N} [I(2l - 2 \leq k + \frac{2M}{T}\tau_{2n} < 2l-1)$$

$$(I(X_{1;\left[\tau_{2n}\frac{M}{T}\right]+\left[\frac{k}{2}\right]} - Y_{2\tau_{2n}+k}\overline{Hr} \geq 0)(X_{1;\left[\tau_{2n}\frac{M}{T}\right]+\left[\frac{k}{2}\right]} - Y_{2\tau_{2n}+k}\overline{Hr})\overline{Q} +$$

$$I\left(X_{1;\left[\tau_{2n}\frac{M}{T}\right]+\left[\frac{k}{2}\right]} - Y_{2\tau_{2n}+k}\overline{Hr} < 0\right)(X_{1;\left[\tau_{2n}\frac{M}{T}\right]+\left[\frac{k}{2}\right]} - Y_{2\tau_{2n}+k}\underline{Hr})\underline{Q} - VOM_1(2\tau_{2n}+k))$$

$$+I(2l-1 \leq k + \frac{2M}{T}\tau_{2n} < 2l)(I(X_{2;\left[\tau_{2n}\frac{M}{T}\right]+\left[\frac{k}{2}\right]} - Y_{2\tau_{2n}+k}\overline{Hr} \geq 0)$$

$$(X_{2;\left[\tau_{2n}\frac{M}{T}\right]+\left[\frac{k}{2}\right]} - Y_{2\tau_{2n}+k}\overline{Hr}\})\overline{Q} + I\left(X_{2;\left[\tau_{2n}\frac{M}{T}\right]+\left[\frac{k}{2}\right]} - Y_{2\tau_{2n}+k}\overline{Hr} < 0\right) \tag{3.3}$$

$$(X_{2;\left[\tau_{2n}\frac{M}{T}\right]+\left[\frac{k}{2}\right]} - Y_{2\tau_{2n}+k}\underline{Hr})\underline{Q} - VOM_2(2\tau_{2n}+k))] - \int_0^{2T} s(t)dt - \sum_{j=1}^{N(on)} C_{on}(2\tau_{2j})$$

where $K_j = (\tau_{2j+1} - \tau_{2j})\frac{2M}{T}$ is the amount of periods between two stopping times. In the case $r \neq 0$, we get

$$V(0) = \frac{1}{r} \sum_{j=0}^{N(off)} \sum_{k=0}^{K_j-1} \sum_{l \in N} [I(2l - 2 \leq k + \frac{2M}{T}\tau_{2n} < 2l-1)$$

$$(e^{-r(2\tau_{2j}+k)} - e^{-r(2\tau_{2j}+k+1)})\,(I(X_{1;\left[\tau_{2n}\frac{M}{T}\right]+\left[\frac{k}{2}\right]} - Y_{2\tau_{2n}+k}\overline{Hr} \geq 0)$$

$$(X_{1;\left[\tau_{2n}\frac{M}{T}\right]+\left[\frac{k}{2}\right]} - Y_{2\tau_{2n}+k}\overline{Hr})\overline{Q} + I\left(X_{1;\left[\tau_{2n}\frac{M}{T}\right]+\left[\frac{k}{2}\right]} - Y_{2\tau_{2n}+k}\overline{Hr} < 0\right) \times$$

$$\times (X_{1;\left[\tau_{2n}\frac{M}{T}\right]+\left[\frac{k}{2}\right]} - Y_{2\tau_{2n}+k}\underline{Hr})\underline{Q} - VOM_1(2\tau_{2n}+k)) + I(2l-1 \leq k + \frac{2M}{T}\tau_{2n} < 2l)$$

$$(e^{-r(2\tau_{2j}+k)} - e^{-r(2\tau_{2j}+k+1)})(I(X_{2;\left[\tau_{2n}\frac{M}{T}\right]+\left[\frac{k}{2}\right]} - Y_{2\tau_{2n}+k}\overline{Hr} \geq 0)$$

$$(X_{2;\left[\tau_{2n}\frac{M}{T}\right]+\left[\frac{k}{2}\right]} - Y_{2\tau_{2n}+k}\overline{Hr}\})\overline{Q}$$

$$+I(X_{2;\left[\tau_{2n}\frac{M}{T}\right]+\left[\frac{k}{2}\right]} - Y_{2\tau_{2n}+k}\overline{Hr} < 0)(X_{2;\left[\tau_{2n}\frac{M}{T}\right]+\left[\frac{k}{2}\right]} - Y_{2\tau_{2n}+k}\underline{Hr})\underline{Q} - VOM_2(2\tau_{2n}+k))]$$

$$-\int_0^{2T} e^{-rt}s(t)dt - \sum_{j=1}^{N(on)} e^{-r[\tau_{2j}\frac{M}{T}]\frac{T}{M}}\, C_{on}(2\tau_{2j})$$

Thus formula (3.3) is a tool to analyze the process and to understand a pre-specified strategy.

Conclusion

In this paper, we have considered the tolling contracts where there are two driving prices (for power) - on-peak and off-peak. We have derived a convenient formula (3.3) for the further calculations, and derivation of the optimal strategy (strategies) for this stochastic optimal control problem. Specifically, we have shown that the price of tolling contract can be represented as a sum of stopped processes, which include the combination of prices (fuel and electricity), some fixed costs and the costs of switching on. Formula (3.3) provides and easier way to apply Hamilton-Jacobi-Bellman equation or just to calculate directly for a pre-specified dispatch rule(strategy). We have also calculated the initial price of tolling contract for certain strategy, using direct approach.

However this strategy cannot be said to be uniformly optimal. It shows that for concrete cases there is no need, to use other complicated techniques to obtain the initial price.

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