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STRUCTURAL PATTERNS AND REPRESENTATION FORMS OF UNIVERSITY PHYSICS TEACHERS: BIOT-SAVART LAW AND AMPÈRE'S LAW

Abstract. *The way teachers organize and transfer their subject matter knowledge has played a crucial role in science education. However, it seems that more attention should be paid on the forms of representation and knowledge organization of teachers that they apply for transferring their knowledge. Therefore, teachers' knowledge organization and representation forms were investigated with an emphasis on the topics of Biot-Savart law and Ampère's law. Four physics teachers of introductory-level University who used, concept cards and concept maps were interviewed, and finally an interpretative analysis was performed. The results showed teachers' knowledge organization was strongly connected only for the topic of Biot-Savart law. Moreover, descriptive and explanatory mathematical models were the most applied forms of representation that teachers used for representing their knowledge. The possibility to recognize the differences in knowledge organization and representation forms of different teachers is the first step towards developing more effective teaching and learning solution.*

Key words: *Ampère's law, Biot-Savart law, concept maps, knowledge organization, representation forms.*

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Introduction

In science teacher education teachers' subject matter knowledge (SMK) is a key factor, on which pedagogically successful approaches are based. Of course, there are many equally important aspects, like paying attention on students' qualitative understanding, pre-conceptions, motivation and context of teaching (Duit et al., 2007) but it is very difficult to imagine successful and well-planned teaching, which does not pay attention to the organization of the subject content. However, taking into account the important role of SMK as starting point for didactical and pedagogical solutions, relatively little attention has been paid on how teachers organize and conceive the structure of the subject content.

First category of teachers' disciplinary knowledge, which is of interest, is teachers' SMK. In 1986, Shulman described teachers' subject matter knowledge (SMK) in terms of substantive knowledge and syntactic knowledge. Substantive knowledge concerns teachers' knowledge of concepts, principles, and facts in the disciplines as well as different ways of relating and organizing these concepts and facts. On the other hand, syntactic knowledge focuses on a set of rules that determine the knowledge of scientific inquiry e.g. recognizing a problem, and knowledge of science process e.g. control of variables. Of the studies focusing on SMK, several are in physics. It seems that most of the studies about physics teachers' SMK have been accumulated by the concepts of light and shadows, electricity, sound, force and motion, heat and temperature, energy, thermal properties of materials, gravity, and air pressure (Abell, 2007). As a result, there are few studies about substantive and syntactic structure of teachers' SMK that concern the concepts of magnetism or magnetostatics. Although SMK is a central theme in science education research, few studies, have concentrated on the structure and organization of physics teachers' knowledge for the purposes of teaching. Studies on teachers' understanding



of organization and relation between concepts in physics are “a largely unmapped field of study in the domain of SMK of teachers” (Abell, 2007, p. 1117). Understanding the structure of subject content is especially important in subject matter areas where knowledge is supposedly complex, basic concepts and laws are interconnected and students (and teachers) are known to struggle to form a picture of subject content. One area where all these aspects are realized is magnetism. Therefore, it is of interest to investigate how teachers organize and conceive their knowledge of the subject of magnetostatics. For these reasons, we have here selected two basic topics – the Biot-Savart's law and Ampère's law - of magnetism as a context, and we studied the SMK of expert teachers.

Second category of teacher's disciplinary knowledge, which is of interest in this study, is PCK. PCK concerns the most important part of teaching: representation of knowledge. Shulman (1986, 1987) stated that teachers should be able to represent their SMK that is pedagogically powerful and comprehensible for students. Therefore, a teacher with high ability of teaching possesses the knowledge of content and knowledge of pedagogy, which is called PCK. According to Shulman (1986), PCK includes (a) teacher knowledge of representation such as analogies, examples, and explanations; (b) knowledge of students' learning difficulties and also strategies to conquer those difficulties. Thus far, the focus of researchers in case of PCK has been on students' learning and their difficulties for learning rather than teachers' representation forms and strategies.

Here the knowledge representation of subject matter for teaching purposes of university physics teachers for the topics of Biot-Savart law and Ampère's law was examined from the viewpoints of 1) teachers' forms of representation that is pointed by Shulman (1986, 1987) as an important element of PCK, 2) teachers' knowledge organizations and structures, which according to Shulman belongs to the substantive structure of SMK 3) possible relation between representation forms and knowledge organization. In this regard, the theoretical background closely follows Shulman's framework for discussing the aspects of the teachers' SMK with a focus on their knowledge organization and teacher's PCK with an emphasis on their representation forms. It is worthy to mention that PCK of teachers is strongly influenced by SMK; though sometimes SMK is assumed as a category of their PCK (e.g. Grossman, 1990). Nevertheless, the relations between SMK and PCK and their components are discussed in the next section in theoretical background.

Since one of the purposes of this study is to investigate the structure and organization of teachers' knowledge, suitable portrayal tool for representing the structure is needed. Concept maps have been successfully used in some previous similarly oriented studies. For example, Ferry (1996) found concept maps as a useful tool for examining teachers SMK. Consequently, in this study, concept maps were used as a tool for visualizing and evaluating the organization of the SMK of teachers.

The empirical approach of the research is based on teachers' interviews and the interpretative analysis of the interviews. There are different kinds of interviews such as structured, non-structures, non-directive, and focus interviews. The interview that performed in this study was somewhat non-directive interview. So, the interviewer had minimal control and teachers were free to express the representation forms of their SMK and to show their knowledge organization. In this, the interviewer asked few questions either to clarify the answers when they were ambiguities or to check the confirmation of answers (Moser & Kalton, 1977). Since this article concentrates on structural patterns, knowledge organization of teachers, and their representation forms, using non-directive interviews appeared to be more applicable than other qualitative analysis such as questionnaires or videotaping the lessons. So, teachers could sketch their concept maps, represent their SMK, and present their knowledge organization in the designed interviews. The interviewed teachers here were experienced physics teachers who teach introductory first year of university level.

In carrying out the analysis, first, through interpretative analysis different forms that teachers use for representing and connecting their SMK were identified as a part of their PCK (see also Majidi & Mäntylä, 2011). These categorizations were emerged from the data analysis and they were motivated by the notions that models and experiments are two key components of knowledge construction (Mäntylä, 2011): models and modelling have a prominent role to construct and justify the knowledge in science education (Koponen, 2007); experiments are important in physical knowledge construction and hold a generative role on teaching physics (Koponen & Mäntylä, 2006). The analysis showed that the identi-



fied categories regarding teachers' representation forms include: mathematical representation which is a category of physics knowledge that helps students and teachers to construct representation of a physical process and reason about the process (see also Van Heuvelen, 1991); Analogies which play a role in learning and teaching as mapping tools (see also Glynn and Takahashi, 1998); visual models in the perspective of science education where figures and photos (two-dimensional structure) are very capable of to developing the visualization (see also Gilbert, 2005); Reasoning that concerns successful knowledge that encompasses comprehensive representation (see also Brachman & Levesque, 2004).

Second, the knowledge organization of interviewed teachers were pictured as the main part of their SMK. The results showed that university teachers' organization of knowledge in case of Biot-Savart law was richer and more connected than teachers' SMK in the case of Ampère's law. Also teachers' representation forms contain interesting and relevant differences for both cases of Biot-Savart law and Ampère's law. Third and finally, the results show how PCK and SMK of teachers including their components can be related together. It is encouraging to find how the present method of analysis is capable of revealing differences in teachers' ways of organizing SMK. The possibility to reveal such differences is a first step towards finding out whether or not such aspects of SMK and representation forms have any consequences to students' learning.

The aim of this research was to study the organization of SMK of university physics teachers and representation forms of their PCK for teaching purposes with the focus on two topics of Biot-Savart law and Ampère's law. In order to reach the goal, following research questions were formulated:

1. What is the representation forms (as a component of PCK) that teachers' use in representing and connecting their SMK of Biot-Savart law and Ampère's law?
2. What are the characteristics of the relational structures or organizations of the content elements (as a part of SMK) which construct the topics of Biot-Savart law and Ampère's law?
3. What is the relation between representation forms (as a part of PCK) to their knowledge organization (as a part of SMK) for the topics of Biot-Savart law and Ampère's law?

These questions were investigated and answered for the subject of magnetostatics with two specific topics of Biot-Savart law and Ampère's law. First question examined teachers' representation forms such as models and reasoning that they use to formulate and connect their content knowledge. Second question evaluated teachers' knowledge organization, which somewhat describes substantive knowledge of teachers and shows how different parts of teachers' knowledge relate together. Third question reveals the answer to the question of how components of SMK and PCK, which are knowledge organization and representation forms, are correlated to each other. Finally, the answers of these questions reveal the characteristics of teachers' SMK and their representation as a whole.

Methodology of Research

As mentioned earlier, knowledge organization is very important part of teachers' SMK but it an unmapped domain of study (Abell, 2007); on the other hand, representation forms is a very crucial notion concerning PCK, nevertheless a great deal of interest has been focused on students' misconceptions and their learning difficulties rather than teachers' representation forms (Abell, 2007). Therefore, it is quite motivating to examine different forms of representation that teachers apply to transfer their SMK with an emphasis on the organization of SMK. Further, the design of the study and research methods are developed and discussed.

Topic of Study: Biot-Savart Law and Ampère's Law

The Biot-Savart's law is described in terms of either moving electric charges or current elements which are assumed as origin of magnetic fields. According to some experiments and referring to international textbooks, Biot-Savart law and thus magnetic fields obey the superposition principle. So it is quite feasible to calculate magnetic fields of any current distribution using superposition principle and Biot-Savart law. The most popular examples of magnetic fields that have been calculated from Biot-Savart law are magnetic fields of long wire, current loop, and coil (Knight, 2008; Walker, et al. 2008).



Ampère's law describes the relation between electric current enclosed by a closed loop and corresponding total magnetic flux through the loop. Ampère's law, which is driven from Biot-Savart law for the magnetic field of long wire, can be employed for calculating current distributions which are highly symmetrical. The most common examples that their current distribution are symmetrical and can be calculated from Ampère's law are magnetic field of solenoid, inside wire, and toroid (Knight, 2008; Walker, et al. 2008). In summary, selected topics in this study are quite interesting because they can be organized in terms of many well motivated ways for knowledge organization.

Concept Cards and Concept Maps

The most important concepts relating to the studied topics differed along several dimensions: concepts relating to sources of magnetic field (Current length element, Electric current, Charge); typical concepts relating to Biot-Savart law and Ampère's law (Superposition principle, Magnetic field of long wire, Magnetic field of current in arc of wire, Magnetic field of solenoid, Magnetic field inside wire, Magnetic field outside wire, Ampèrian loop, Magnetic field of toroid, Magnetic dipole, Magnetic field line, Magnetic field of current loop); electrostatic concepts that could be used as analogies to studied topics (Electric field, Gauss's law, Coulomb's law); advanced concepts and laws (Vector potential, Stokes theorem, Maxwell equations) relating to studied topics. So, these concepts were chosen from the content of three introductory university physics textbooks chapters (Knight, 2008; Walker et al. 2008; Feynman et al. 2006) and they were written on concept cards (post-it notes). The ways to choose these concepts were studied and validated in another study (see Majidi & Mäntylä, 2011). There were also empty cards, where the interviewed teacher could write any concept that was missing from the given set of concepts during the interview.

In the interview room, there was a whiteboard, on which the concept maps were drawn. Teachers selected the concept cards and drew the lines between the concepts piece by piece. At the same time teachers explained and described the construction of the concept maps. In other words, teachers' concept maps consisted of concept cards as well as the connections (lines) between the concept cards. Teachers' explanations and reasons for connecting the concept cards were videotaped. The concept maps were redrawn in electronic format using CmapTools and the numbers reflecting the order of teachers' presentations were added to the concept boxes. The concept maps made by teachers enabled us to identify the key features of teachers' ways to organize the SMK and, consequently, the key features of substantive structure of SMK. Moreover, the accompanying explanations and justification that teachers utilized to represent and connect their SMK enabled us to recognize the characteristics of representation forms as a part their PCK.

Interviews and Interviewees

The teacher interviews of four male physics teachers were conducted in autumn 2010. Each teacher was interviewed separately and the interviews were videotaped. As Table 1 shows, the duration of interviews varied from 40 to 45 minutes. All teachers had PhD in physics and they have taught the subject of magnetostatics for at least 5 semesters (Table 1).

Table 1. Information of interviewed teachers.

	David	John	Nigel	Chris
Degree of education	PhD in Physics	PhD in Physics	PhD in Physics	PhD in Physics
Graduation year	2000	1986	1998	1992
Teaching magnetostatics	8 semesters	12 semesters	12 semesters	5 semesters
Duration of interview	45'	40'	40'	45'



In the interviews, teachers were asked to make concept maps depicting the way and order how they present the selected topics in their teaching. In the beginning of the interview, the aims of this study were discussed and then the purpose and way of using concept maps was explained to the interviewees. During interviews, teachers simultaneously presented their SMK by means of selecting concept cards; connecting those concepts; describing forms of representation as a part of their PCK; and finally explaining how they constructed their concept maps. Thus in the end of the interview there was a concept map on the white board with its construction and explanations that were videotaped.

During interviews, the task for the teachers was to present their forms of representations and knowledge organization for the topics of Biot-Savart law and Ampère's law. The way of interviewing and posing questions were practised in advance in order to test the interview method. Teachers were asked to "think out loud" and verbalize their thinking during the interview. When necessary, the interviewer asked more detailed questions, otherwise the structure of the interview was quite open, though the interviewer attended to keep the interview in focus. The interviews were transcribed verbatim and the concepts in the concept maps were coded to the interview transcripts.

Procedures

In this study, the data consist of four videotaped interviews and four concept maps. First the videotaped interviews were transcribed and the chronological orders of the concepts were added into related concept maps.

In order to give structure to the analysis, the transcripts were classified into four different domains, which are: 1) introduction of Biot-Savart law, 2) applications of Biot-Savart law, 3) introduction of Ampère's law, and 4) applications of Ampère's law (see also Oser & Baeriswyl, 2001).

As the first task, teachers described their representation forms and reasoned on what basis they connect content elements. These forms showed how they represent and formulate their SMK and revealed the properties and characteristics of connections between content elements. In order to recognize the representation forms teachers' statements were identified through the transcripts. Next, the results of analysis were compared to the results of another researcher. Then attention was paid to the emerged categories of representation forms in each statement, and finally, the results were triangulated. So the categories concerning representation forms of teachers were driven from the content analysis. However, the categories were inspired by representation forms that Shulman (1987) pointed out as the components of transformation of SMK into PCK. He argued that representation repertoire includes analogies, metaphors, examples, explanation, and so forth. In this study, the categories included experiments, different models (including visual, analogy, mathematical), reasoning, and statement of fact. However, the choice of models and experiments were motivated by the notion that they were important in physical knowledge construction (Mäntylä, 2011, Koponen & Mäntylä, 2006; Koponen, 2007). In this study, the identified models contained visual models such as figures and diagrams that teachers utilize to represent the SMK (Gilbert, 2005), mathematical models such as equations and formulas that physics teachers apply during their teaching (van Heuvelen, 1999), and finally analogy as a mapping tool between relevant contexts (Glynn and Takahashi, 1998). In addition to models and experiments, teachers employed a wide range of inductive and deductive reasoning in order to justify the relation between the magnetostatics concepts (Brachman & Levesque, 2004).

Then, each statement was sorted and categorized as a specific category in a parallel manner (Miles & Huberman, 1996). In this stage of comparison, consistency values of identified categories ranged from 75% to 80% for interviewed teachers; the measurements of consistency values were 75% for the case of David, 78% for John, 80% for Nigel, and 79% for Chris. Although measured values suggested good reliability, in order to improve the reliability, the identified categories were compared until authors reached to a consensus (Kvale, 1996). Finally, triangulation was achieved through the interpretations and categorization. As the second task, teachers' SMK with the focus on their knowledge organization have been investigated where concept maps were utilized to picture their knowledge arrangements.



Results of Research

Teachers' SMK was studied with an emphasis on its knowledge organization. Moreover, teachers' representation forms were analyzed as an important element of teachers' PCK. First, representation forms of teachers were identified and compared for different teachers. Next, the characteristics of teacher's SMK were analyzed and compared on the basis of organization of selected concepts in order to see how they correlate and differences exist between them.

Representation Forms

It is of interest to investigate what are different forms of representation that teachers use as a part of their PCK to formulate their SMK. The results of analysis of the content of interviews showed that teachers used representation forms such as experiment, analogy, statement of fact, reasoning, and models including descriptive- and explanatory mathematical model and visual model. Teachers used these forms in order to represent and formulate their SMK. A short description of each form is given as follow:

Experiment: The information of observations, evidences, and discoveries.

"The first thing we look is the magnetic interaction outside of conducting wire. In a way, it is the old experiment by Ørsted." (Chris)

Analogy: A similar role, when it is seen from the perspective of mapping between similar concepts.

"Ampere's law is connected to magnetic field in a similar way as Gauss's law is connected to electric field." (Chris)

Descriptive mathematical model: A mathematical model that connects concepts with mathematical format to describe new laws.

"Magnetic field of wire and magnetic dipole interact to each other for calculations to derive this field [Ampere's law]." (Nigel)

Explanatory mathematical model: A mathematical model that explains the applications or examples of those described laws.

"Ampere's law is then used as a basis to again calculate magnetic field inside wire and magnetic field of toroid." (John)

Statement of fact: The declarative knowledge that describes the facts.

"We have a moving charge as the source of magnetic field. Magnetic dipoles come from moving charges." (John)

Visual model: About illustrations and visual perceptions.

"A magnetic field line... is very closely related to magnetic field is used as visualization of magnetic field." (Nigel)

Reasoning: Modes of presentation that gives reasons or explanations or interpretations to justify the connections between concepts.



"The shape of magnetic fields lines is interpreted so that the magnetic field for itself must originate from magnetic dipole." (David)

Teachers applied different forms in order to represent and connect the concept elements of Biot-Savart law and Ampère's law while they were describing the introduction and application to these topics. For this reason, here the representation forms are presented in terms of four domains that introduced before (Oser & Baeriswyl, 2001). These forms were used with different frequencies. The frequencies of using representation forms in four domains are presented in Table 2.

Table 2. The different domains and frequencies of using representation forms in each domain (the numbers of cases is in the parentheses).

Category	Introduction of Biot-Savart law (N = 20)	Applications of Biot-Savart law (N= 11)	Introduction of Ampère's law (N= 20)	Application of Ampère's law (N= 17)
Experiment	(3)	-	(1)	(2)
Analogy	(1)	(3)	(1)	(4)
Des- math model*	(6)	-	(4)	(4)
Exp- math model*	(3)	(7)	(1)	(6)
Statement of fact	(3)	-	(4)	(1)
Visual model	-	(1)	(3)	-
Reasoning	(4)	-	(6)	-

*Des: descriptive; Exp: explanatory

The analysis of representation forms in the first domain indicated that these forms were homogeneously distributed (Table 2). The most applied form for introducing Biot-Savart law was descriptive mathematical model (30%); but visual model was not used in this domain.

The representation forms in second domain were not homogeneously distributed (Table 2) but they were rather accumulated by explanatory mathematical model (63%). Experiment, descriptive mathematical model, statement of fact, and reasoning were not applied in this domain.

The representation forms that teachers used to introduce Ampère's law (third domain) were somehow uniformly distributed (Table 2). The leading form in this domain was reasoning (30%) but experiment, analogy, and explanatory math models were rarely used in this domain.

The overall view of last domain in Table 2 shows that representation forms were not uniformly distributed. The most crucial form was explanatory mathematical model (35%). Visual model and reasoning are not employed in this domain.

Organization of Subject Matter Knowledge

The results of this section reveal how teachers organize their SMK. They applied different approaches in order to arrange their SMK. As illustrated in the previous section, teachers' representation forms were recognized and analyzed in respect to four domains. The results of this section were also presented and evaluated using those four domains.

Teachers' concept maps revealed the structural and relational features of their SMK. Teachers' concept maps differed in terms of connectedness of knowledge. The number of loops and cycles somewhat indicate the level of connectedness of knowledge. Because when concepts tie together, they construct interwoven structure. By assuming A, B, and C as three concepts, the notation of a loop or cycle would be written as $A \rightarrow B \rightarrow C \rightarrow A$, which means these three concepts are bound together (Koponen & Pehkonen, 2010). The loop starts from A and after its connection to B and C it again returns to A. Thus, concepts of A, B, and C produce a connected and meaningful structure. Loops could be larger and include more



concepts but the minimum number of concepts to make a loop is three. Therefore, teachers' concept maps were classified in respect to the connectedness of their knowledge organizations:

Strongly connected: There are many loops and cycles in teachers' concept maps and no dead-ended concepts.

Moderately connected: In contrast to the strongly connected organization, there are fewer loops in teachers' concept maps. Moreover, there are few dead-ended concepts.

Loosely connected: There are limited numbers of loops in teachers' concept maps. Besides, there are many dead-ended concepts.

First, David's knowledge organization is illustrated; next, the knowledge organization of John and Nigel is introduced, and finally, the knowledge organization of Chris is presented. The number of concepts refers to the relating figures. In analyzing the concept maps directions are not considered; instead the focus is on existence of the links between concepts. In other words, concepts maps in this study represent undirected graphs.

Knowledge organization of David: Figure 1 shows how David organized his SMK concerning two studied topics. In the first step, he described Biot-Savart law (first and second domains). In the next step, he expressed Ampère's law (third and fourth domains).

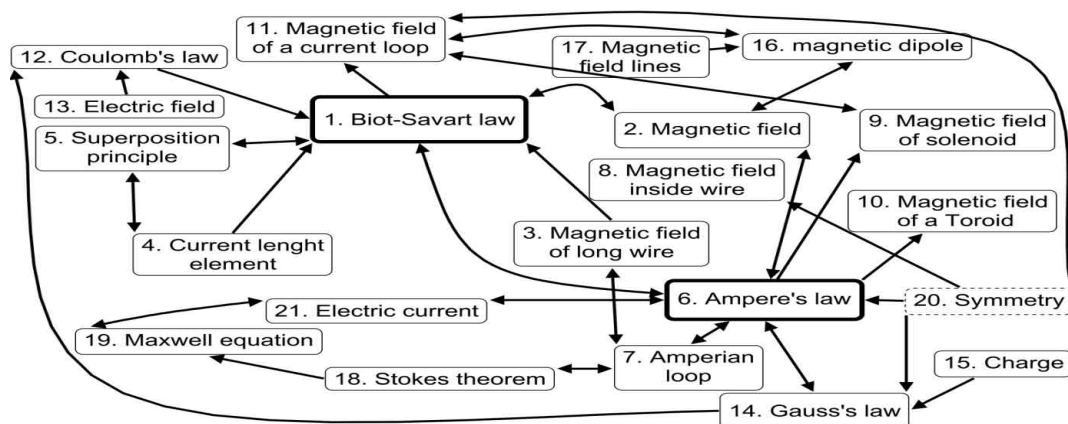


Figure 1: Concept map of David (numbers show the chronological order of the concepts). Dashed concepts were added by teacher.

An overall view of David's concept maps reveals that concepts C8, C10, C13, C15, and C17 in Figure 1 are dead-ended, which are not connected to other parts of the structure. However, his concept map has clear loops in his arrangement, which indicate the interconnectedness and integration between the concepts. In order to deepen the evaluation of David's knowledge organization, the structure of each domain is sketched individually in Table 3.

Table 3. Structural patterns of each domain in the concept map of David with an emphasis on loops and dead-ended concepts. Numbers refer to concept map of John in Figure 1.

Domains	Dead-ended concepts	Loops and cycles
First domain	-	(1→5→4→1)
Second domain	13, 17	(11→16→2→1→11)
Third domain	-	(2→1→3→6→7→2)
Last domain	8, 10, 15	(21→6→7→18→19→21), (20→6→14→20), (6→9→11→20→6)



As Table 3 shows, the structure of first domain includes one loop but no dead-ended concepts, so its structure is strongly connected. The organization of second domain includes one loop, but there are two dead-ended concepts that decrease the integration and coherency of the structure, consequently it is moderately connected. On one hand, third domain includes one loop; on the other hand it excludes dead-ended concepts, so its structure is strongly connected. Last domain includes three loop and it contains three dead-ended concepts, so its structure is moderately connected. Roughly speaking, David's overall knowledge organization is moderately connected. He utilized different but not mutual concepts to arrange his knowledge.

Knowledge organization of John and Nigel: The knowledge organizations of John and Nigel are somehow similar, because they followed the same steps to organize their SMK (Figures 2-3). They introduced and explained the introduction and applications of Biot-Savart law and Ampère's law, respectively. Although John and Nigel organized their SMK in a similar way, they selected different concepts so they made different loops and interconnections in their knowledge arrangements. Concepts of magnetic field (2), Amperian loop (6), and symmetry (20) were emphasized in David's map.

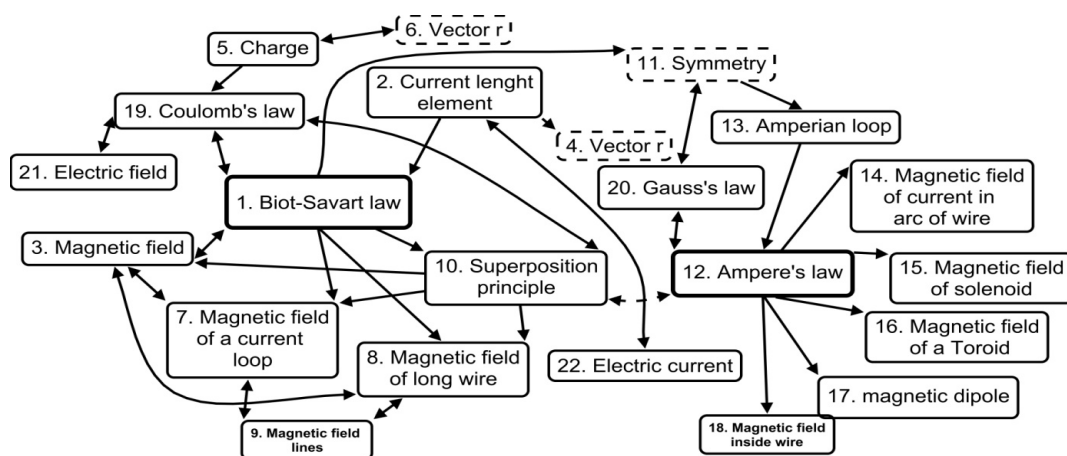


Figure 2: Concept map of John (numbers show the chronological order of concepts). Dashed concepts are added by teacher.

The concept map of John reveals that the concepts C4/6, C14, C15, C16, C17, C18, C21, and C22 in Figure 2 are dead-ended, which are not connected to other parts of the structure. However, his concept map has clear loops and cycles, which are strongly connected. In order to extent the analysis, the knowledge structure of John is drawn on the basis of four domains (Figure 2).

Table 4. Structural patterns of each domain in the concept map of John with an emphasis on loops and dead-ended concepts. Numbers refer to concept map of John in Figure 2.

Domains	Dead-ended concepts	Loops and cycles
First domain	-	-
Second domain	21, 22	(1→8→10→1), (1→7→10→1), (1→7→3→1), (1→8→3→1), (1→3→10→1), (3→7→10→3), (3→8→10→3), (1→10→7→3→1), (1→10→8→3→1), (1→7→8→9→1), (1→7→3→10→1), (1→10→8→9→7→1), (1→10→8→9→7→3→1)
Third domain	-	11→13→20→14→11
Last domain	14, 15, 16, 17, 18	-



As Table 4 shows, the structure of the first domain includes no loops and is has one dead-ended concepts, so its structure is moderately connected. Although, the organization of second domain includes two dead-ended concepts, its structure contains many strongly connected loops, and therefore its structure is interpreted as strongly connected. The structure of the third domain has one loop and no dead-ended concepts; hence its structure is strongly connected. There are no loops or cycles within the last domain and nearly all concepts are dead-ended, thus its structure is with no doubt loosely connected. In conclusion, John's overall knowledge organization is more or less moderately connected. As Table 4 shows, most of the loops and cycles in his knowledge organization contained the common concepts of magnetic field (3), magnetic field of long wire (8), magnetic field of current loop (7), and superposition principle (10).

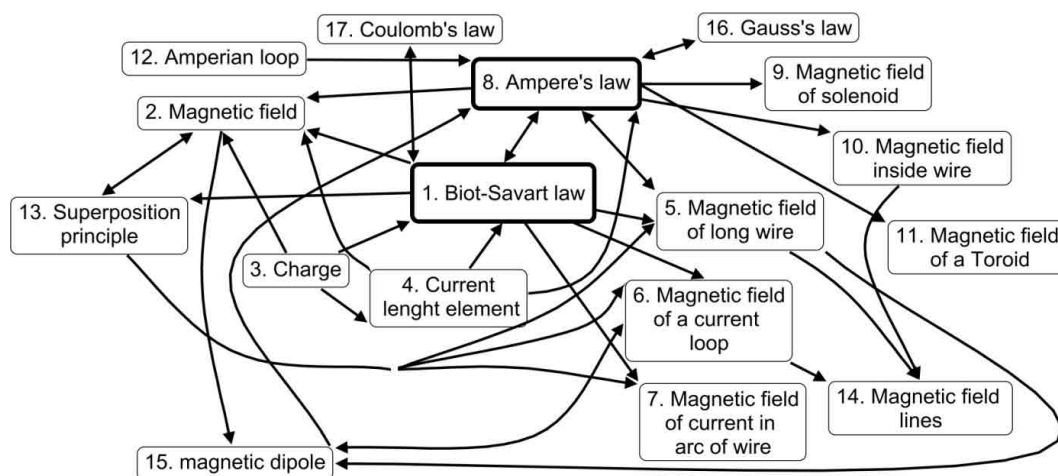


Figure 3. Concept map of Nigel (numbers show the chronological order of the concepts). Dashed concepts were added by teacher.

Nigel's concept map reveals that the concepts C9, C11, C12, C16, and C17 in Figure 3 are disjointed. These concepts are not connected to other parts of the structure (Figure 3). In order to deepen the analysis of knowledge organization of Nigel, Table 5 is given.

Table 5. Structural patterns of each domain in the concept map of Nigel with an emphasis on loops and dead-ended concepts. Numbers refer to concept map of John in Figure 3.

Domain	Dead-ended concepts	Loops and cycles
First domain	-	1→2→3→1, 1→3→4→1, 1→2→3→4→1
Second domain	17	1→2→13→1, 1→5→13→1, 1→7→13→1, 1→2→6→13→1, 1→2→7→13→1, 1→2→5→13→1, 1→2→6→13→1, 1→2→5→15→1, 1→2→15→6→1, 1→13→6→1
Third domain	12	-
Last domain	9, 11, 16	8→2→15→8, 5→8→10→14→5

As Table 5 shows, the structure of the first domain includes three loops and no dead-ended concepts, so its structure is strongly connected. The organization of second domain consists of many loops, even though the structure contains one dead-ended concept, its organization is interpreted as strongly connected. The structure of third domain is loosely connected because it has no loop and



it contains one dead-ended concept. Finally, the last domain includes one loop and there are some dead-ended concepts, thus its structure is moderately connected.

According to Table 5, Nigel mainly utilized the concepts of magnetic field (2), superposition position (13), magnetic field of long wire (5), and charge (3) to create loops and cycles in his knowledge organization. In conclusion, Nigel's overall knowledge organization is approximately moderately connected.

Knowledge organization of Chris: In contrast to other teachers, Chris started to organize his knowledge with the subject of electrostatics rather than magnetostatics (Figure 4). He introduced and explained the application of Biot-Savart and Ampère's law, respectively.

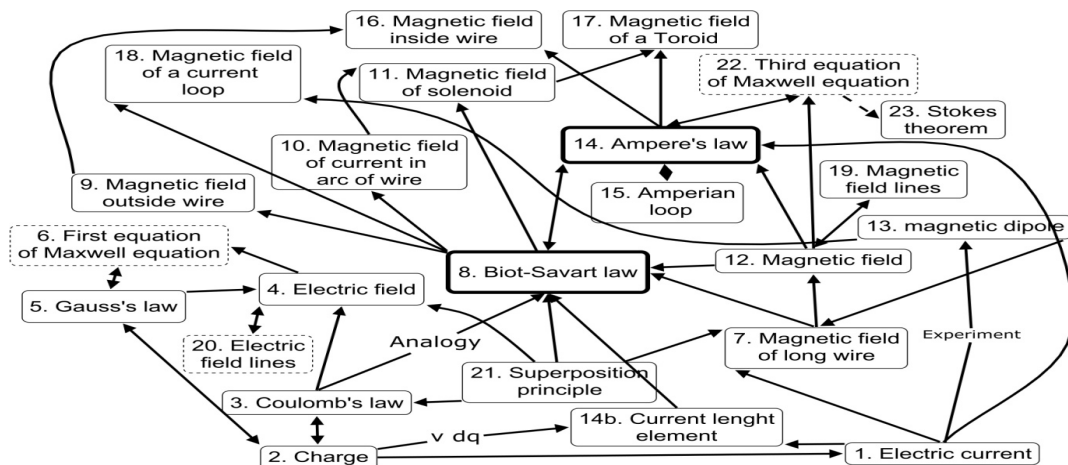


Figure 4: Concept map of Chris (numbers show the chronological order of the concepts). Dashed concepts were added by teacher.

Unlike other teachers who started their knowledge organization with the concept of Biot-Savart law, Chris utilized the concept of electric current (1 in Figure 4) as a starting point. First, he completed the description of the subject of electrostatics. He continued to introduce Biot-Savart law and then explained the examples and applications of that law. In the same way, he presented Ampère's law. Meanwhile, he enlightened the interconnections between other concepts. It appears that the concept map of Chris includes many cycles and excludes disjointed concepts. There are only two disjointed concepts (19 and 23) in his concept map (Figure 4). Again, the structure of each domain is shown individually in Table 6.

Table 6. Structural patterns of each domain in the concept map of Chris with an emphasis on loops and dead-ended concepts. Numbers refer to concept map of Chris in Figure 4.

Domains	Dead-ended Concept	Loops and cycles
First domain	-	3→8→21→3, 1→2→14b→2, 1→7→8→14b→1, 1→2→3→8→14b→2, 1→2→3→7→8→12→1, 1→2→3→8→14b→1, 1→2→3→7→8→1
Second domain	-	1→7→13→1, 12→7→8→12, 8→10→11→8, 8→9→16→11→10→8, 8→18→13→7→8
Third domain	23	8→12→14→8
Last domain	19	12→22→14→12



As Table 6 shows, first domain of the knowledge organization of Chris includes many loops and no dead-ended concepts, so its knowledge structure is undeniably strongly connected. In a similar way, the organization of a second domain is also strongly connected. The structure of third domain is moderately connected because it includes one loop and one dead-ended concept. Lastly, the organization of last domain is much similar to the third domain.

Consequently, the overall knowledge organization of Chris in term of its structure is moderately connected. He emphasized the concepts of charge (2), coulomb's law (3), magnetic field of long wire (7), magnetic field (12), current-length element (14b), and superposition principle (21) in his knowledge arrangement.

Relation of Representation Forms to Knowledge Organization

The result of this section show how representation forms, which is an important part of PCK, is related to knowledge organization as a part of teachers' SMK. The differences between different teachers in their use of representation forms and in their organization of their SMK are summarized in Table 7.

Table 7. Relation of representation forms to teachers' knowledge organization for each domain (numbers represent the frequency of using forms).

Domains	Representation forms	Knowledge organization
1. Introduction of Biot-Savart law	Descriptive math model 30 %	David, Nigel, Chris: strongly connected John: moderately connected
2. Application of Biot-Savart law	Explanatory math model 63 %	Chris, John, Nigel: strongly connected David: moderately connected
3. Introduction of Ampère's law	Reasoning 30 %	John, David: strongly connected Chris: moderately connected Nigel: loosely connected
4. Application of Biot-Savart law	Explanatory math model 35 %	David, Nigel, Chris: moderately connected John: loosely connected

SC: Strongly Connected, MC: Moderately Connected, LC: Loosely Connected

Table 7 is quite revealing in several ways such as representation forms and knowledge organization of interviewed teachers as a whole. According to Table 7, teachers can construct well organized SMK by employing mathematical models (explanatory and descriptive) for expressing the introduction and application of Biot-Savart law.

As third and last domains of Table 7 indicate, using representation form of reasoning does not lead to well-organized SMK. Moreover, the knowledge arrangement of teacher for either introduction or applications of Ampère's law were not as well-connected as the case of Biot-Savart law.

In summary, the overall SMK of teachers regarding both introduction and application of Ampère's law were not as well structured as the case of Biot-Savart law. In contrast to the context of Biot-Savart law where mathematical models were the dominant representation forms, reasoning is considered as a governing representation form in the context of Ampère's law.

Discussion

This research put an effort to visualize the SMK of teachers with an emphasis on their knowledge organization. So far only few researchers have been studied the SMK of teachers, concerning magnetostatics with the focus on two sub-topics of Biot-Savart law and Ampère's law. On the other hand, scholars have been focused on understanding physics concepts rather than investigating the organization of knowledge (Abell, 2007). In this study, we have investigated and compared the SMK of four physics university teachers from the viewpoint of 1) representation forms as the main component of their PCK for formulating the SMK (Shulman, 1986; 1987); 2) the relational and structural differences



in teachers' knowledge organizations that belongs to teachers' substantive structure of their SMK that also influences teachers' PCK (Shulman, 1986); and 3) the relation of representation forms to knowledge organization of teachers' SMK. The relational connections between the notions of SMK and PCK as well as their components were illustrated within the theoretical framework of this study.

The results of the analysis indicate that teachers used seven forms to connect and represent their SMK. These forms were emerged from the content analysis of teaching interviews, and then validated by doing parallel analysis where the results were triangulated (Kvale, 1996; Miles & Huberman, 1996). The identified representation forms that emerged from the analysis include: experiments, models (including descriptive- and explanatory mathematical model, visual model, and analogies), reasoning, and statement of fact. Nevertheless, the choice of representation forms including models and experiments were motivated by other studies (Koponen, 2007; Koponen & Pehkonen, 2010; Majidi & Mäntylä, 2011; Mäntylä, 2011).

The results concerning the frequencies of using representation forms indicate that teachers mostly used descriptive and explanatory mathematical model for describing Biot-Savart law (Table 2, Table 7). Since teachers SMK of Biot-Savart law featured a broad coverage of concepts and well-organized structure, it can be suggested that using mathematical model somewhat leads to an integrated and well-connected SMK for the physics university teachers. Also, findings of this research indicate that concepts of magnetic field, magnetic fields of long wire and current loop, symmetry, Amperian loop, superposition principle, current length element, Coulomb's law, and charge are the most dominant elements to describe and explain the studied topics of Biot-Savart law and Ampère's law in the context of magnetostatics.

As seen in Table 2 and Table 7, teachers applied reasoning and mathematical models for describing Ampère's law. The reason might be because Ampère's law is deduced from theory (Knight, 2008; Walker et al. 2008). Results showed, instead of mathematical models, reasoning played a prominent role in representing teachers' SMK of introducing the topic of Ampère's law. In the domain of applications of Ampère's law, teachers' concept maps were moderately connected where there were some dead-ended concepts which disconnect teachers' knowledge organization into some untied parts. In brief, one might expect that using more mathematical models might have been better worked here.

This study visualized the knowledge organization of teachers' SMK, which was somewhat an unmapped field of study specifically for the case of physics (see Abell, 2007). The results revealed the similarities and differences in teachers' knowledge organization. Here concept maps were utilized as a tool to contrast and evaluate the differences between teachers' knowledge organizations. Here, loops and cycles indicate the connectedness of teachers' knowledge. Similarly, cyclical paths between concepts were studied where the structural analysis of the concept maps of physics teacher students were based on the operationalisation of important structural features (Koponen & Pehkonen, 2010). According to what Shulman (1986) stated, teachers should be able to organize their content elements, properly. However, the results of this study showed that in some cases the knowledge organization of teachers were not well-connected, while in some other cases the structure of their SMK was highly connected.

The results, when taken together, show that there are many similarities in the university teachers' way to organize their SMK, but there are also clear differences. Nevertheless, the study shows that such differences can be detected and provides some new tools to represent these differences.

Furthermore, studying the representation forms of teachers clarifies one of the most important elements of their PCK (Shulman, 1986; 1987). Consequently, this study identifies and describes experienced teachers' SMK and makes explicit the representational components of experienced teachers.

It appears that more attention should be devoted to investigate the SMK of teachers on the basis of describing and explaining Ampère's law, with the focus on teachers' knowledge organizations and representation forms. On the other hand, it seems more studies should be conducted in order to investigate the possible impacts of mathematical models, as a dominant representation forms, on teachers' PCK. Moreover, the impact of mathematical models on teachers' SMK and their knowledge organization must be further investigated. There might be some obstacles that teachers experience while they describe and organize their SMK of Ampère's law. As a result, further studies are needed to be done in order to suggest appropriate approaches to overcome such obstacles.



Conclusions

In this study, teachers' representation forms are analysed, which reveal how they apply to formulate and connect their SMK as the main category of their PCK. Their representation forms consist of seven different forms including: descriptive- explanatory mathematical models, visual models, analogies, statement of fact, and experiment. Descriptive- explanatory mathematical models were the most applied representation forms for the case of Biot-Savart law. However, explanatory mathematical model and reasoning found an essential role in teachers' representation forms for the case of Ampère's law.

Also, the SMK of university physics teachers is examined with the focus on their knowledge organization concerning two topics of Biot-Savart law and Ampère's law that influence PCK. In contrast to knowledge organization of teachers for the topic of Biot-Savart law which was strongly connected, their knowledge was moderately connected concerning Ampère's law.

The results concerning the relation of representation forms and knowledge organization, for the topic of Biot-Savart law, revealed that employing mathematical models generates strongly connected knowledge. In conclusion, the possibility to recognize the differences in teachers' knowledge organization and representation forms appears to develop more effective teaching and learning solutions and curriculum plans.

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