Fatigue Life Prediction of Composite Under Two Block Loading

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Abstract— **The damage evolution mechanism is one of the important focuses of fatigue behaviour investigation of composite materials and also the foundation to predict fatigue life of composite structures for engineering applications. This paper is dedicated to damage investigation of composite materials under two block loading cycle fatigue conditions. The loading sequence effect and the influence of the cycle ratio of the first stage on the cumulative fatigue life are studied. Two loading sequences, i.e., high-to-low and low-to-high cases are considered. The proposed damage indicator is connected cycle by cycle to the S-N curve and the experimental results are in agreement with model expectations. Previous experimental research is employed for validation.**

Keywords- fatigue; damage acumulation; composite

I. INTRODUCTION

Composite materials were first used in aircraft engine rotor blades in the 1960s [1] and their use became more and more important in the construction of several framework in various domain. Fatigue behavior of these materials was a subject of thorough and extensive studies, due to the large utilization of these materials in different applications. Fatigue life assessment has been described with more than 70 cumulative damage hypotheses [2], the best known is the Miner rule [3]. Several researchers have investigated the fatigue phenomenon in composite materials [4-9]

Howe and Owen [10] studied the accumulation of damage during cyclic loading with the objective of obtaining useful working relationships of the Miner-rule that might be used in design. With the aid of optical microscopy they studied the development of debonding sites and resin cracks in chopped-strand-mat/polyester composites and they suggested that, although debonding did not itself cause reductions in strength, it served to initiate resin cracks which did weaken the material.

Mandell, et al [11] demonstrated that the data from various fiberglass composite materials in the data base may be characterized by a power law curve fit when they are normalized to the ultimate tensile or compressive strength of the composite. Wohler curve for different loading ratios (R), require a correction using the Goodman diagram

There are many studies of the behavior of composite materials under cyclic loading, and reviews are given in [12-14]. Approaches in the fatigue problems of composites can be divided into two classes: the Wöhler curve method and the damage accumulation theory. The Wöhler curve method [15–19] has been widely employed in engineering to deal with the fatigue issue of composites. However, only under the conditions of low stress and simple stress state, is the method suitable. The damage accumulation theory, which can be applied under complex loading conditions, is a hotspot in the research of the fatigue of composites. Several approaches have been proposed, such as the residual strength model presented in [20-24].

II. SOME DAMAGE MODELS FOR COMPOSITE MATERIALS

The mechanism of damage in composites is one of the important topics in the study of the fatigue behavior of composite materials and also the basis for predicting the fatigue life of composite structures for engineering applications [25].

The fatigue damage of composites is more complex than those of metals. Failure of composite materials under cyclic loading can occur following four scenarios:

- Cracking of the matrix

- Interfacial debonding

- Delamination

- Breaking Fibers

A. The Dzenis model

The Dzenis model [26] treats the process of fatigue damage in composite materials as a process related to the load. That is to say, the accumulation of damage in the load cycles is time dependent. For this study the effects of variable amplitude, frequency and shape of the cycle on the fatigue behavior of composite materials are considered. The Dzenis model is given by the following formula:

$$D_{\varepsilon_i} = \overline{s_{i,j}^2} K_{\sigma,j} + \overline{\sigma_j^2}(t) D_{s_{ij}} \qquad (1)$$

where $\overline{s_{i,j}}$ is the laminate compliances, $K_{\sigma,j}$ is the correlation functions, $\overline{\sigma_j}(t)$ is the applied stresses and $D_{s_{ij}}$ is the dispersion.

B. The Kang-Kim model

Kang and Kim [27] presented the fatigue behaviour of laminated carbon/epoxy with an impact-induced damage under two blocks tensile loading. To describe this behaviour, the concept of reduction in the strength of the material is introduced.

The model is given by the equation:

$$D_R = \frac{\sigma_0 - \sigma_{Re}}{\sigma_0 - \sigma_2} + \left\{\frac{\sigma_{Re} - \sigma_1}{\sigma_0 - \sigma_2}\right\} \cdot \frac{n_{imp,1}}{N_{imp,1R}} + \left\{\frac{\sigma_{Re} - \sigma_2}{\sigma_0 - \sigma_2}\right\} \cdot \frac{n_{imp,2R}}{N_{imp,2R}} \qquad (2)$$

where σ_0 is the ultimate tensile strength, σ_1, σ_2 are the applied stresses, σ_{Re} is the residual tensile strength, $n_{imp,1}$ is the number of cycles at σ_1, $n_{imp,2R}$ is the number of cycles at σ_2, $N_{imp,1R}$ is the residual life in the first loading and $N_{imp,2R}$ is the residual life in the second loading.

C. The Rognin et al model

The authors [28] used experimental data and numerical methods to characterized the composite material. The effects of fatigue are often evaluated by conducting experiments with two blocks loading (high-low/low-high). The purpose of the study was to predict, using probability methods, the fatigue resistance of the coupon and show the relationship between fractions accumulation of damage during the experiment. The authors propose a formula for the accumulation of damage as follows:

$$D_R = \sum_{i=1}^{m-1}\left(\frac{n_i}{N_i}\right)' + \frac{n_m}{N_m} \qquad (3)$$

where D_R is the fatigue damage variable, n_i and N_i are respectively the actual applied number of cycles and the number of cycles to failure

D. The Jen-Yang model

Jen and Yang [29] studied experimentally the cumulative damage of carbon nanotubes in composite material under two blocks loading. The content of the chemically modified carbon multiwall nanotubes used for the sample is 0.5% by weight. The effect of loading and the influence of the cycle rate of the first block on the cumulative damage were studied. The authors make their proposal as following:

$$D_s = \frac{S_n - S_f}{S_i - S_f} \qquad (4)$$

where S_i, S_n, and S_f are the magnitudes of stiffness corresponding to the initial cycle, the nth cycle, and the final stable cycle, respectively.

E. The proposed model

Under cyclic stress, structural loading will occur in the field of micro cracks in composite materials and these loads lead to fatigue damage. With an increase in the number of charging cycles, the amount of the loading increases and the damage to the material will accumulate in phase that leads to a change in the microscopic and macroscopic mechanical properties of materials. Based on experimental studies [4, 11, 18, 26, 29] we can conclude that the damage evolution of composite material is not linear. During the initial period of loading cracks appear in the matrix and the matrix cracks when it reaches saturation, fiber breakage occurs, and the damage is growing rapidly in this material as we can as shown in Figure 1.
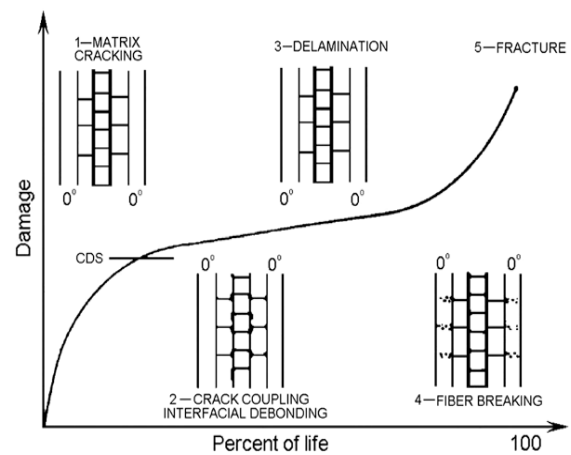

Fig. 1.   The evolution of fatigue damage in a unidirectional composite material.

According to the mechanisms of fatigue damage of composite materials, findings and observations from previous models, a new comprehensive model of fatigue damage is presented to describe the rule and stiffness degradation of composite materials for two blocks loading, as follows:

$$\left(\frac{n_1}{N_{f1}}\right)^{\left[\left(\frac{\sigma_u}{\sigma_1}\right)-1\right]} + \left(\frac{n_2}{N_{f2}}\right) = 1 \qquad (5)$$

where n_1 is the cycle number corresponding to σ_1, n_2 is the cycle number corresponding to σ_2, N_{f1} is the number of cycles to failure corresponding to σ_1, N_{f2} is the number of cycles to failure corresponding to σ_2 and σ_u is the ultimate tensile strength.

III.   APPLICATION AND VALIDATION OF THE PROPOSED MODEL

The proposed model is verified using experimental results from the literature. These results are consisted of two-block loading sequences with transitions from low to high (L–H) and high to low (H–L) load levels. Plumtree et al [30] conducted tests for fatigue in cyclic tests on $[\pm45]_{2S}$ angle ply carbon–epoxy specimens using stress ratios with an R (minimum/maximum stress) of 0.1 and -1.0. After a given number of cycles under known loading conditions, the cyclic stresses were changed and the test continued to failure under the new conditions. The loading conditions, the test results reported in [30], the theoretical predictions of the proposed model and Miner's rule are given in Table I for increasing and decreasing block types of loading respectively.

TABLE I.   EXPERIMENTAL RESULTS AND THE PREDICTIONS OF THE PROPOSED MODEL.

σ_1 (MPa)	σ_2 (MPa)	n_1	N_{f1}	N_{f1}	n_2 (Exp)	n_2 (predicted)	n_2 (Miner)
105	64	675	1406	196807	108244	158733	102340
96	58	511	4258	410417	578688	408543	361167
101	61	511	2222	285291	407966	276482	219674
106	64	511	1246	199721	131816	171820	117836
110	66	511	763	155130	141168	87982	51193
64	110	85960	199907	764	1153	744	435
64	108	85960	199907	972	846	947	554
64	109	85960	199907	866	745	844	494

According to the experimental data [30], the results show that the difference between the predicted residual fatigue life and the experimental data are acceptable because of the big scatter of fatigue life and most points are within 1.5 times range as shown in the Figure 2, on the other side predicted life calculated by the Miner's rule are 2.5 greater than the experimental results in two cases. The predicted residual fatigue life by the proposed algorithm is in good agreement with the experiment, considering the bigger scatter of composites.
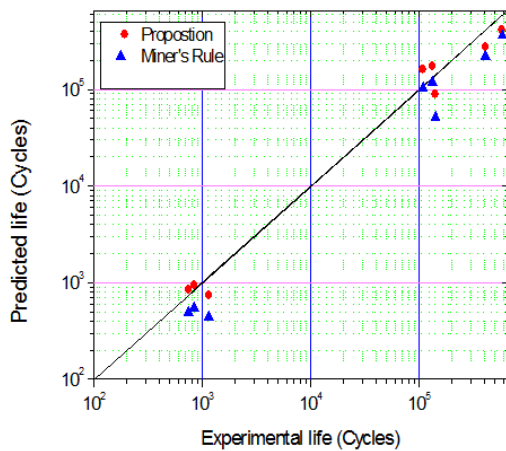

Fig. 2.   Experimental and predicted residual fatigue lives of laminates

Table II compares the experimental results reported in [31] and the predictions of model proposed in this paper. As shown in Figure 3, the majority of the results calculated by the proposed prediction model is conservative, as they are in the neighborhood of the experimental results.

TABLE II.   EXPERIMENTAL RESULTS AND THE PREDICTIONS OF THE PROPOSED MODEL.

| σ_1 (MPa)	σ_2 (MPa)	n_1	N_{f1}	N_{f1}	n_2 (Exp)	n_2 (predicted)	n_2 (Miner)
315	340	87200	115150	8800	520	343	2136
315	340	87000	115150	8800	150	345	2151
315	340	86300	115150	8800	1408	355	2205
315	340	57700	115150	8800	1750	827	4390
315	340	57550	115150	8800	2280	830	4402
315	340	40300	115150	8800	2027	1226	5720
315	340	28700	115150	8800	3320	1584	6607
315	340	26500	115150	8800	2640	1666	6775
315	340	25300	115150	8800	2464	1713	6867
315	340	17650	115150	8800	6170	2068	7451
315	340	17000	115150	8800	38140	2104	7501
315	340	13000	115150	8800	14300	2356	7807
315	340	12500	115150	8800	24030	2392	7845
340	315	8500	8800	115150	15250	235	3926
340	315	7480	8800	115150	17060	1096	17273
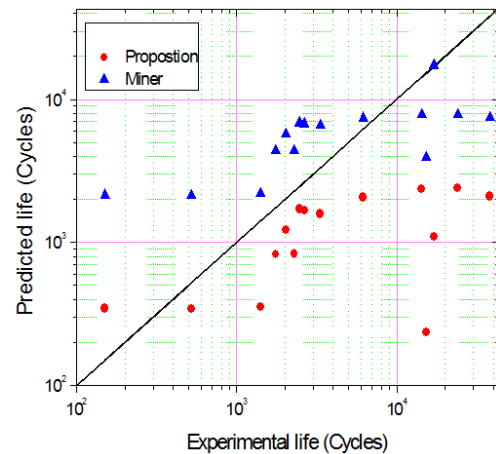
Fig. 3.   Experimental and predicted residual fatigue lives of laminates

In this investigation, the relative error of prediction represents the relative difference between the experimental and calculated lines using the proposed model and the Miner's rule. The REP is defined by :

$$REP(\%) = \frac{N_{exprimental} - N_{calculated}}{N_{exprimental}} x\,100 \qquad (6)$$

The corresponding predictions of the proposed model and those calculate by Miner's rule are gathered and presented in Figure 4. It is clear in this figure that the predictions are very good. All the relative errors in the proposed model are less than 10% except for one load condition, which leads to an error of

28.42% (Decreasing blocks). It should also be noticed that the REP in the absolute value calculated by the proposed model are lower than the REP calculated by Miner's rule.
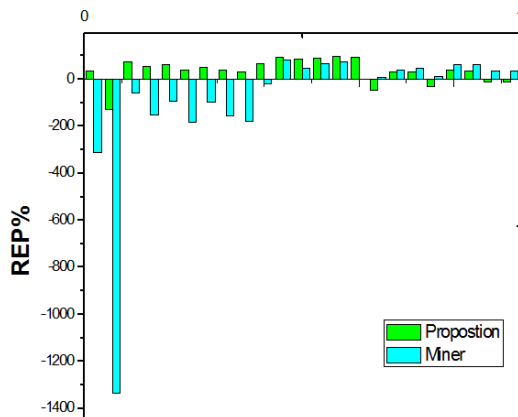

Fig. 4.   Relative errors of prediction for calculate lives using the proposed model and Miner's rule

IV.   CONCUSION

The paper presents a non-linear damage accumulation model to predict the remaining fatigue life of the second stage. The use of this model is simple, it has no parameters to be determined, and requires only the knowledge of the *S-N* curve. A comparison between our proposition and the Miner's rule was made and some deviation is evident. The two-level loading examples show that the model can predict residual fatigue life of composite materials quite well. The theoretical analyses are well in conformance and are in good agreement with the experimental data for all materials tested in this investigation for the residual life as well as for the cumulative damage. From this viewpoint, we hope that our model may eventually find broad use. The proposed model may be extended to complex random loading.

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