Regularized Constant Modulus Algorithm: An improvement on Convergence rate and Steady-State Statistics for blind adaptive eqalization

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Abstract— This paper addresses the problems of poor convergence and high value of mean square error, embedded with various blind adaptive equalization techniques in mobile communication and proposes an improved design providing enhancement in performance and ease of operation. The effectiveness of proposed approach is evaluated against different algorithms and simulations prove that design facilitate in reducing convergence time and low MSE. Proposed algorithm also ensures that equalizer does not converge to any wrong solutions under time varying and noisy conditions.

Keywords— CMA, MMA, LMS, RLS, PSK, QAM, MSE

INTRODUCTION

Digital filters of fixed frequency response are widely used in mobile communication to attenuate noise and other unwanted signals. However filtering is done by adaptive digital filters in situation for which noise containing frequencies erratically place over the signal bandwidth. The coefficients of adaptive filter are updated at each transaction in assistance of a training signal with the target of minimizing the MSE between reference sequence and the solution of adaptive filter thus converges the filter to a global minimum. Adaptive algorithms like RLS, LMS, etc then updates the equalizer filter coefficients accordingly. But the power expense and utilization of bandwidth by this overhead sequence need to be avoided in ever growing mobile communication by means of substitution by smart equalizers, which use the statistical properties of signal to find the instantaneous error. Performance function of these equalizers is defined by a weight vector whose value optimizing process reduces the value of cost function and thus proceeds towards advancement in adaptive equalization process. Problem lies in choosing perfect values for the weight vector and preventing equalizer from wrong solutions.

Channel of propagation contains non-deterministic phenomena which causes distortion and other non-linear effects which should be taken into consideration to validate the effectiveness of proposed algorithm. The constant modulus algorithm (CMA), a well-known technique broadly employed in blind equalization whose standing origins from easiness in designing processes as well as robustness in operation. However, it undergoes slow convergence and possesses high probability for improper solutions, if not treated properly, as compared with other blind techniques. The error surface examination for CMA reveals that most designs are based on merged models of channel and equalizer, thus treating solution set in global space. To develop independent operational algorithm, as needed in many scenarios, error surface in terms of error equation needs to be reviewed for tractable analysis of channel. Requirement of an independent nature CMA equalizer arises by considering the fact that computations becomes restricted to complexity and decreases in efficiency with channel-equalizer space. This causes convergence ambiguities in transient behavior and optimization for parameters effecting convergence and MSE which should be addressed for systems demanding high speed data rates promising integrity of information as VDSL and Fiber Curb networks. Focusing the transient and level of error in convergence of the integral part of communication systems, this paper explains the related issues in next section. Discussion on generalizing CMA is organized in section IV with description of proposed approach is presented in section V. Next section expresses the effectiveness of proposed algorithm and proves the speedy convergence and least MSE among other members of CMA family. can put the page in this format as it is and do not change any of this properties.

PROBLEM STATEMENT

CMA and MMA both can lead to a set of wrong solutions at convergence acting as a phase splitter filter used for high data rates. Embedding phase splitting function in coefficients reduces the need of Hilbert Transform or a cross-coupled structure. But this may guide the set of filters. There is no need of carrier phase recovery according to the Godard algorithm eliminates the carrier phase recovery need and thus applicable to fading conditions such as dispersive channels, resulting in considerable equalization results including an open up eye pattern, indicating very low ISI and phase distortion. Constant Modulus Algorithm belongs to the Godard's extensive work where whose primary reduction in algorithm produces a minimum optimal value, a non-convex cost function stated as

$$J(n) = E\left[\left(|y(n)|^{p} - R_{p}\right)^{2}\right] \quad \text{with} \quad R_{p} = E\left[|u(n)|^{2p}\right] / E\left[|u(n)|^{p}\right] \tag{1}$$

With p being a positive integer and represents dispersion constant. It has a major negative aspect that it undergoes slow convergence as compared to training base algorithms. Moreover, cost function of CMA exhibits a non-linear behavior making the implementation complex. CMA with simple assumptions leads to wrong solutions and way out is presented in [3],[4] and [5] with LMS due to less sensitive nature of LMS towards eigenvalue spread in contrast to quadratic error surface of CMA. But LMS filters exhibits slow convergence as the contained information in information signal is insufficient to properly construct I/O mapping and additional noise makes it more worst. Solution of LMS always leads to global minimum convergence as it's a training based algorithm. LMS integrated CMA, any sequence having constant phase offset will be acknowledges and hence cost surface of CMA results in multiple minima. Second problem to be highlight is the amount of MSE generated during the blind adaptive process which is not acceptable for reliable communication. Increase in update taps minimized the MSE but at the cost of increase in convergence time. So this trade-off between transient behavior (convergence rate) and steady state response (amount of MSE) should be treated in an optimized manner and this approach is presented in this paper.

MATHEMATICAL FRAMEWORK OF CMA & MMA

The work of Sato provides the basic idea for blind equalization whose algorithm is BUSSGANG type in which cost function reduces to a minimum optimized solution as mathematically given in the eq.3 in the form of difference between non-memory non-linear estimate of transmitted data and the output of transversal filter as

$$J(n) = E[\left(\alpha\, sgn(y(n)) - y(n)\right)^{2}]$$

$$\alpha = E[(\alpha\, sgn(y(n))]^{2} \Big/ E[(\alpha\, sgn(y(n))] \tag{2}$$

Where the *sgn* represents signum function which roots non-linearity and constant α sets the gain of equalizer. The Tap weight vector *w(n)* for tap input vector *u(n)* is updated with step size *μ* in accord with the stochastic gradient algorithm generally expressed as

$$w(n+1) = w(n) + \mu u(n) e^{*}(n) \tag{3}$$

Expression for error signal is given by

$$e(n) = y(n)|y(n)|^{p-2}(R_{p} - |y(n)|^{p}) \tag{4}$$

The second case of Godard algorithm by taking p=2 reveals that the cost function of Eq.1 minimizes the value to mathematical framework of Constant Modulus Algorithm CMA [2].

$$J(n) = E\left[\left(|y(n)|^{2} - R_{2}\right)^{2}\right] \text{Where } R_{p} = E\left[|u(n)|^{4}\right] / E\left[|u(n)|^{2}\right] \tag{5}$$

The above term provides initial convergence. In terms of carrier phase offset, CMA has distinction over other Bussgang algorithms due to the property of using the cost function *J(n)* for its solution with lesser MSE. The DCT for general training sequence {d(k),. . . . ,d(k-N+1)} is given by

$$d_{DCT,n}(k) = \sum c_{kn} d(k-n) \tag{6}$$

Where c_{kn} are the DCT coefficients. Signed-Error CMA proposes that computations for multiplication process during the update process of tap weight vector can be minimized with replacement of equalizer tap equation coefficients with differential sign, positive or negative depending on the nature of result of summation. The real-valued solution is given by the following equation

$$w(n+1) = w(n) + \mu r(n)\,\mathrm{sgn}(y_n(\gamma - y_n^2)) \quad (7)$$

Signed error methods generally result in rough convergence with high steady state MSE which is encountered by Dithered Sign-Error CMA in which SE-CMA was implemented in a short manner with only few features as design was based on thoughtful merging of standard CMA generated noise [8]. The DSE-CMA algorithm is given by the update equation

$$w(n+1) = w(n) + \mu r(n)\alpha\,\mathrm{sgn}(y_n(\gamma - y_n^2) + \alpha d_n) \quad (8)$$

Where d_n is a uniformly distributed i.i.d, independently indentical distribution, over the interval of (-1, 1]. Multi Modulus Algorithm MMA defines cost function as complex function of real and imaginary components of vector output of equalizer.

$$J_{MMA} = E\{[y_R^2 - R_{2,R}]^2\} + E\{[y_I^2 - R_{2,I}]^2\} \quad (9)$$

Real and imaginary parts consider both modulus and phase of the output to achieve carrier phase recovery. The update for tap-weight vector for MMA is given by

$$w(n+1) = w(n) - \mu.e^*(n)u(n) \quad (10)$$

Where *e(n)* also composes real and parts in MMA.

GENERALIZATION OF CMA

Regularization of a divergent expression drives the expressions to a finite and meaningful value with the aid of a regulating factor. Regularization causes increase in complexity but prevents the process from overfitting and the result is always obtained in the direction of regularization factor. The problems embedded with blind adaptation techniques are mainly their poor convergence property when judged against other conventional techniques deployed with abet of training sequences. Generally speaking, a gradient algorithm integrated in a blind adaptation schemes will give a better solution, an idea that got the shape of well-known Constant Modulus Algorithm (CMA) utilizing the information signal unique property of constant modularity for adapting process [7].

For the schemes using CMA, there exists a trivial solution of phase shift that each minima will experience. Suppose, for an equalizer of length L, constant modulus cost function will have L^2 number of minima, each having a unique phase shift and representing a valid solution. Taking practical conditions in to discussion, all L^2 solutions could not be accepted equally. For M-array PSK signal of differentially encoded source sequence, only M different solutions or M number of sequences are considerable for which solution converges to global minima. Other solutions represent local minima even though all minima have same cost. The method of initialization the equalizer is an important factor to obtain the information of minimum point on which algorithm will force the equalizer to converge to either local or global minimum. So as a depended of initialization, CMA based equalizers may converge to a local or a global minimum with the problem of slow convergence as mentioned earlier. Another important consideration here is that adaptive algorithms are also very much dependent on their cost function for their convergence characteristics and core task during the adaptation process is minimization of cost function. Derivation of expression for cost function includes estimation of error signal between the filter output and desired signal and this derivation can be altered, to achieve any desired response by either changing the whole function or as an alternative, varying the error equation. Varying of error equation involves understanding of basics involved in defining the cost function which is normally defined as MS of error signal but this holds for conditions where noise distribution is Gaussian. To bring the systems in to account having no-Gaussian interfering noise distribution, a Non-MS Error criterion would be advantageous. So in process of varying the definition of cost function, start with changing the error equation and analyzing each obtained expression for convergence and steady state response with some obviously provide improved convergence and other offers elimination of probability to local minima convergence. Few points must be consider to ensure global minima convergence during the adaptation process related to initialization which is discussed below.

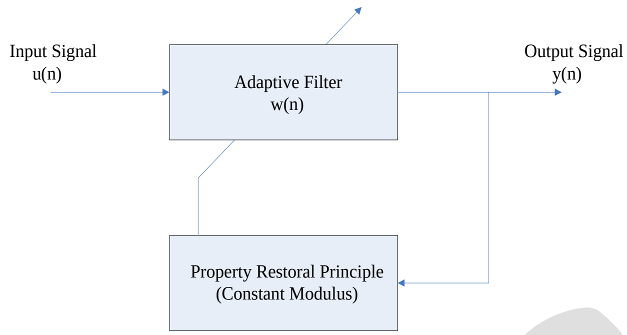

Figure 1. Blind Adaptive Filter Property Restoral Principle

Since, calculated output signal for filter is given as

$$y(n) = w^H(n)u(n) \tag{11}$$

Eq.4 and above equation both depicts explains clearly that basic adaptive filter structure and proposed method have same update rule for coefficients and generation of output samples. Only difference lies in the estimation of the error signal. The evaluation and computation of error signal is primary in the filter coefficient adaptation process from an initially random value to an optimal value. Error equation for proposed design for of constant modulus algorithm is given as

$$e(n) = y(n)(R_p - |y(n)|^2) \tag{12}$$

Putting both error equations side by side with same amount of error we have

$$d(n) = y(n)(R_2 - |y(n)|^2) + y(n) \tag{13}$$

Where *d(n)* represents the generated reference signal for the proposed case. In the contrast of the Eq.12, the proposed equalizer structure is shown in fig.2.

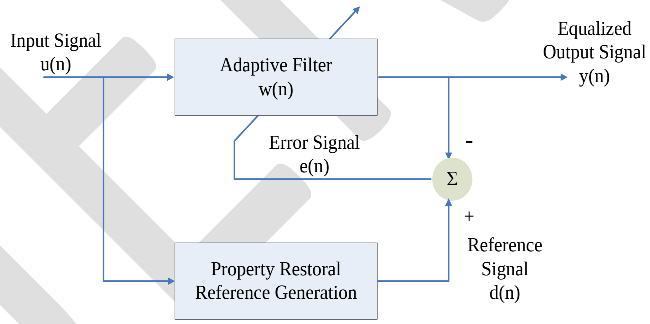

Figure 2. Proposed Equalizer Structure

It can be observed that performance for both techniques is same since reference signal is generated to produce and equivalent error signal as from a blind adaptive process. Another enhancement of selecting this approach includes generation of reference signal at the receiving side instead of transmitter; hence overheads in the transmission are avoided providing increased capacity for bandwidth and gateway to the attached losses with transmission of reference signal. Regularization helped the process of receiver training to cope up the distortion and non-Gaussian noise by comparatively analyzing both generated and known sequence. Second proposed structure comprise multiplexing of Regularized CMA and MMA algorithm in which former algorithm brings the cost to an initial threshold value, then MMA decomposes the constellation into regions and modulus is calculated for each. This approach further reduces the cost with the help of moduli of regions. MMA makes the 2nd adjustment through second coefficient generation.

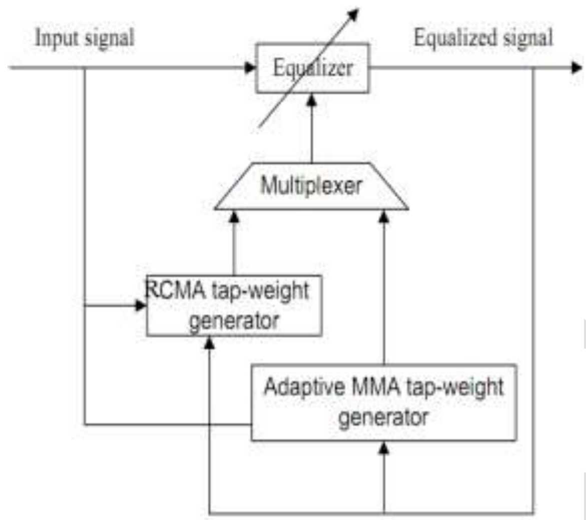

Figure 3. Combining Regularized Constant Modulus Algorithm and Multi Modulus Algorithm

PROPOSED WEIGHT UPDATE TECHNIQUE

Cost function to be minimized in a Recursive least-Square algorithm is termed as $\varepsilon(n)$, where n is the variable length of observable data, given as

$$\varepsilon(n)=\sum_{i=1}^{n}\beta(n,i)\left|e(i)\right| \tag{14}$$

The $\beta(n,i)$ represents weighting factor and is given by

$$\beta(n,i)=\lambda^{n-i}, \quad i=1,2,3,\ldots,n.$$

With λ<1 a positive constant, which makes the weighting factor $\beta(n,i)$ zero with the passage of time and due to this property it is termed as "forgotten factor". Regularization is explained as under. To solve the issues caused by LMS, the cost function is given as:

$$\varepsilon(n)=\sum_{i=1}^{n}\lambda^{n-i}\left|e(i)\right|^{2}+\delta\lambda^{n}\left\|w(n)\right\|^{2} \tag{15}$$

The sum of weighted errors squares is given by:

$$\sum_{i=1}^{n}\lambda^{n-i}\left|e(i)\right|=\sum_{i=1}^{n}\lambda^{n-i}\left|d(i)-w^{H}(n)u(i)\right|^{2} \tag{16}$$

The regularization term is given by

$$\delta\lambda^{n}\left\|w(n)\right\|^{2}=\delta\lambda^{n}w^{H}(n)w(n) \tag{17}$$

δ denotes the regularization parameter. Now normal equation are formulated again making the time average N-by-N correlation matrix of the tap input vector $u(i)$ as

$$\Phi(n)=\sum_{i=1}^{n}\lambda^{n-i}u(i)u^{H}(i)+\delta\lambda^{n}I \tag{18}$$

Moreover cross-correlation N-by-1 matrix between the tap input and the desired response of transversal filter is given by

$$z(n)=\sum_{i=1}^{n}\lambda^{n-i}u(i)d^{*}(i) \tag{19}$$

The expression for relation among correlation matrix *Φ(n)* and tap weight vector $\hat{w}(n)$ along with cross correlation vector for recursive least square is given by:

$$\Phi(n)\hat{w}(n)=z(n) \tag{20}$$

Computations of *Φ(n)* and *z(n)* are derived by taking *i=n* which make the corresponding term to be separate from the equation, now it can be written as

$$\Phi(n)=\lambda\left[\sum_{i=1}^{n-1}\lambda^{n-1-i}u(i)u^{H}(i)+\delta\lambda^{n}I\right]+u(n)u^{H}(n)$$
$$=\lambda\Phi(n-1)+u(n)u^{H}(n) \tag{21}$$

Similarly

$$z(n)=\lambda z(n-1)+u(n)d^{*}(n) \tag{22}$$

Using matrix inversion lemma

$$\Phi^{-1}(n)=\lambda^{-1}\Phi^{-1}(n-1)-\frac{\lambda^{-2}\Phi^{-1}(n-1)u(n)u^{H}(n)\Phi^{-1}(n-1)}{1+\lambda^{-1}u^{H}(n)\Phi^{-1}(n-1)u(n)} \tag{23}$$

For convenience let $P(n)=\Phi^{-1}(n)$, thus Eq.23 becomes

$$k(n)=\frac{\lambda^{-1}P(n-1)u(n)}{1+\lambda^{-1}u^{H}P(n-1)u(n)} \tag{24}$$

Substituting *P(n)* and *k(n)* in the above equation

$$P(n)=\lambda^{-1}P(n-1)-\lambda^{-1}k(n)u^{H}(n)P(n-1)$$

Eq.24 implies that

$$k(n)=\lambda^{-1}P(n-1)u(n)-\lambda^{-1}k(n)u^{H}(n)P(n-1)u(n)$$
$$=\left\lfloor\lambda^{-1}P(n-1)-\lambda^{-1}k(n)u^{H}(n)P(n-1)\right\rfloor u(n) \tag{25}$$

$$k(n)=P(n)u(n)=\Phi^{-1}(n)u(n) \tag{26}$$

Time updating of tap weight vector

$$\hat{w}(n)=\Phi^{-1}(n)z(n)=P(n)z(n)$$
$$=\lambda P(n)z(n-1)+P(n)u(n)d^{*}(n) \tag{27}$$

Putting values of *P(n)* only in first term of Eq.5 gives desired recursive updating tap weight vector $\hat{w}(n)$ as

$$\begin{aligned}\hat{w}(n) &= P(n-1)z(n-1) - k(n)u^H(n)P(n-1)z(n-1) \\ &+ P(n)u(n)d^*(n) \\ &= \Phi^{-1}(n-1)z(n-1) - k(n)u^H(n)\Phi^{-1}(n-1)z(n-1) \\ &+ P(n)u(n)d^*(n) \\ &= \hat{w}(n-1) - k(n)u^H(n)\hat{w}(n-1) + P(n)u(n)d^*(n) \\ &= \hat{w}(n-1) - k(n)\left[d^*(n) - u^H(n)\hat{w}(n-1)\right] \\ &= \hat{w}(n-1) + k(n)\xi^*(n)\end{aligned} \tag{28}$$

$$\begin{aligned}\xi(n) &= d(n-1) - u^T(n)\hat{w}^*(n-1) \\ &= d(n) - \hat{w}^H(n-1)u(n)\end{aligned} \tag{29}$$

Where $\xi(n)$ is the priory estimation error. Thus we have

$$\begin{aligned}\gamma(n) &= P(n-1)u(n) \\ k(n) &= \frac{\gamma(n)}{\lambda + u^H(n)\gamma(n)}\end{aligned} \tag{30}$$

$$P(n) = \lambda^{-1}P(n-1) - \lambda^{-1}k(n)u^H(n)P(n-1)$$

So $\Phi^{-1}(n)$ update equation becomes

$$\begin{aligned}\Phi^{-1}(\mathrm{n}) &= \lambda^{-1}\Phi^{-1}(\mathrm{n}-1) \\ &-\lambda^{-1}\mathrm{k}(\mathrm{n})\mathrm{u}^{\mathrm{H}}(\mathrm{n})\Phi^{-1}(\mathrm{n}-1)\end{aligned} \tag{31}$$

SIMULATION RESULTS

Results for the quantitative analysis for convergence rate and steady state MSE are presented in this section. The proposed methodology was simulated employing BPSK and 16-QAM modulated signals with parameter values of SNR equal to 25dB and 0.05 of step size. The generated reference signal through blind estimation is also test out to verify its credibility against basic requirements of any reference signal used in the process of equalization. Starting from the correlation between reference signal generated and the source symbols which are transmitted from the source, Fig.4 shows the correlation plot demonstrating clearly the strong relationship of correlation between the two. Hence initial condition is satisfied for the generated sequence to be used as a reference signal.

Generated reference signal must have to satisfy the basic requirements of a standard reference sequence which include correlation with source symbols, zero correlation with channel noise and an above zero DCT plot. Thus, the reference signal has been formed as to hold a constant modulus resulting in a constant modulus output signal. Subsequent requirement for the reference signal is to have zero correlaration with respect to noise. Fig. 4 shows that the channel symbols, which were made to be non-i.i.d, are strongly correlated at zero delay. Another point to be consider is if DCT value for generated sequence becomes zero for a bandgap of size M starting from m_1 to frequency m_2, this spectral gap M (M = m_1 - m_2) will freeze all filter weights occurring at these spectral position and finally the updating process of coefficients will discontinue. For this reason, the original tap value is forced to be kept at instances where spectrum value decreases to zero.

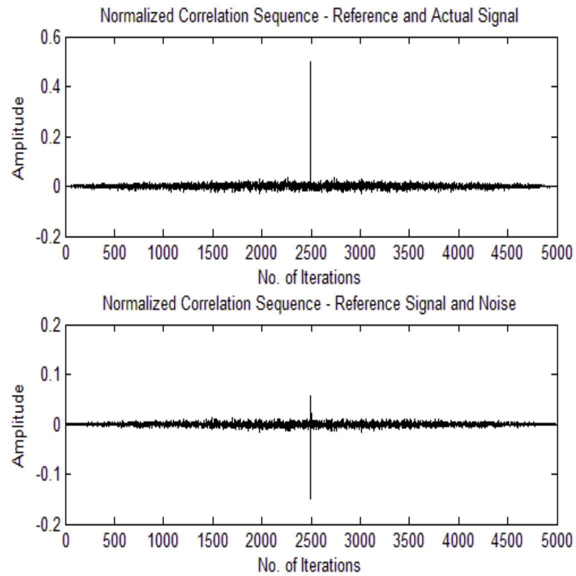

Figure 4.Normalized Correlation Sequence between the Generated Reference Signal and the Noise Sequence

Correlation of the training signal with noise sequence is shown in the below normalized correlation plot showing correctness of selection of reference signal as inverse proportional relation exists between the two. This relation guides much better in the adaption process in the noisy channels. In continuous with these two validations, fig.5 shows that the frequency contents of training sequence well maintains an-above zero DCT (Discrete Cosine Transform) and not hold the zero value for a considerable time even when sometime it touches that. Other generated sequences held zero value at some positions, thus causing problems in adaptation process and increase in convergence time.

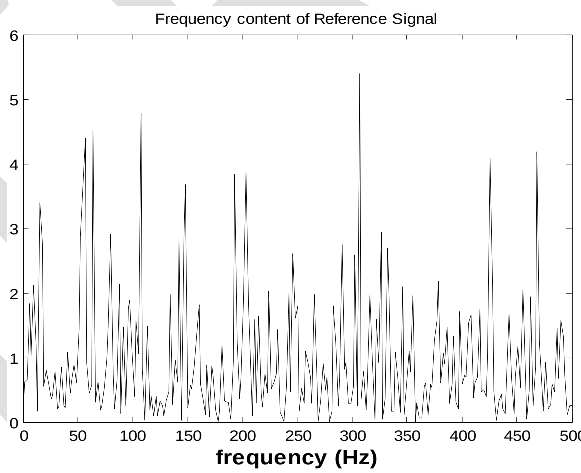

Figure 5. Discrete Cosines Transform of of the Reference Signal

The channel has a time varying impulse response and initialized by h = [-0.001 0.1 -0.45 0.9 -0.45 0.1 -0.001 and is modeled to have Rayleigh fading with complex AWGN noise addition as in [11]. The initial value for MSE is somehow analogous before steady state for all the algorithms, but in contrast with the other algorithms, proposed technique converges to its steady state very quickly.

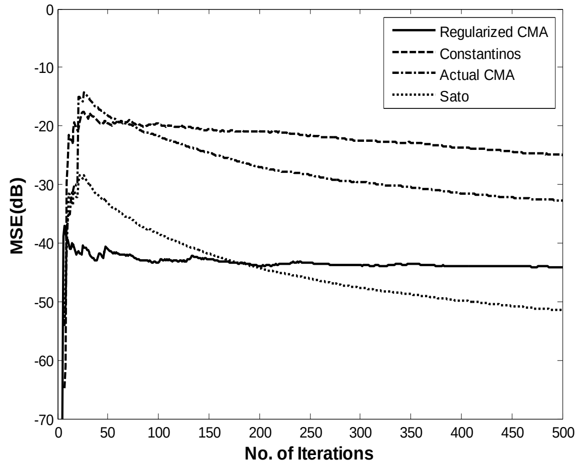

Figure 6. Learning Curves of Different Algorithms for BPSK

For QAM modulated signals, speed of convergence and accuracy of the proposed regularized CMA is shown in Fig.7.

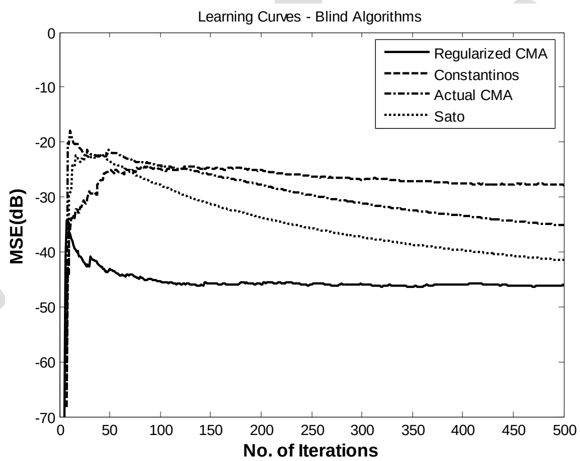

Figure 7. Learning Curves of Blind Algorithms for QAM

Regularized Constant Modulus Algorithm RCMA clearly converges prior to other competitor techniques for the 40dB acceptable value of MSE. QAM scheme is modeled as complex number being two dimensional in nature having real and imaginary parts. Improved performance was achieved by combining the properties of proposed technique with MMA. Multiplexing of RCMA and MMA performs even better for the reason of quick convergence property of RCMA and low value for final steady state mean square error of MMA. A comparison of second proposed architecture of equalizer, mixing sato with regulized CMA, with other algorithms is shown in Fig. 8 which reveals lowest MSE value convergence as compared to work presented in [6], [8] and [9]. This proves that weight coefficients trajectory of equalizer, over any finite time, converges to the ordinary differential equation solution in spite of constant step size and considerable noise in channel. Speedy convergence guarantees the implementation of the algorithm in high speed data rate mobile communication systems employing 4G modulation techniques OFDM and MC-CDMA [11,12].

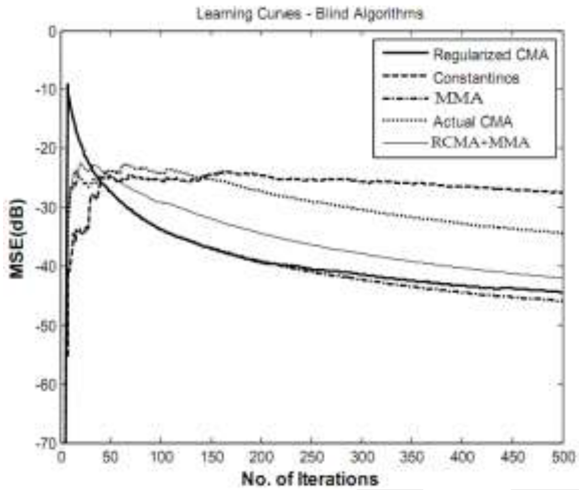

Figure 8. Comparision of Proposed technique with other algorithms

CONCLUSION

Performance of equalizers in modern communication systems is evaluated in terms of convergence rate and mean square error and existing LMS algorithms gives poor convergence where noise level is beyond critical values. The proposed algorithm diminishes the trade-off between the two properties by generating a smart training sequence and blindly using low mean square error property of CMA, thus providing improved performance at a much reduced complexity. The weight updating technique for the regularized CMA was of RLS producing considerable low value for mean error and quick response to optimal solution convergence, hence providing better results than traditional Constant Modulus Algorithm Family algorithms.

Although computer manufacturing industry is progressing toward ultra fast parallel processing day by day, the complexity matters are trivial in wireless communication designs, journey towards optimization and improvement never ends. Prospective applications for the proposed technique in adaptive filter applications are in echo cancellation and signal rejection/ extraction for radar systems etc. To a particular extent, goal for attaining optimal coefficient values through swift convergence rate has been successfully achieved though the effort presented in this research. Some potential future recommendations could be extension of the proposed algorithm to radius-directed equalization RDE, which uses multiple radii to give much smaller value of error than CMA at low orders [1].

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