PUNCTE FIXE COMUNE PENTRU APLICATIILE M IN SPATIILE METRICE DIFUZE

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***REZUMAT**: In aceasta lucrare, obtinem doua teoreme ale punctelor comune fixe, una pentru doua perechi de aplicatii simple si cu valoare precisa si alta pentru patru aplicatii puncte fixe care satisfac conditiile de tip implicit, contractiv. 2000 Clasificarea matematica a subiectului: 47H10 , 54H25.*

***Cuvinte cheie**:* Aplicatie punct fix, Aplicatie M, Aplicatie Subcompatibil, Relatie implicita.

1. Introducere si preliminarii

Teoria punctelor difuze a fost introdusa de L.Zadeh [9] in 1965.George si Veeramani [1] au modificat conceptul de spatiu metric, difuz introdus de Kramosil si Michalek [7].Grabiec[10] a demonstrat principiul de contractie in fixarea spatiilor metrice difuze introdus in [1]. Pentru teoremele punctelor fixe in spatii metrice difuze, unele dintre referintele interesante sunt [1,3,4,5,10-18].

In consecinta, avem nevoie de urmatoarele

Definitia 1.1; ([2]: O operatie binara $*$: $[0,1] \times [0,1] \to [0,1]$ este o norma-t daca indeplineste urmatoarele conditii:

(i) $*$ este asociativa si comutativa,
(ii) $*$ este continua,
(iii) $a*1= a$ pentru toate $a \in[0,1]$,
(iv) $a*b \leq c*d$ oricand $a \leq c$ si $b \leq d$, pentru fiecare $a,b,c,d \in [0,1]$.

Doua exemple tipice ale normei-t continue sunt $a* b = ab$ si $a*b =\min \{a,b\}$.

COMMON FIXED POINTS FOR M-MAPS IN FUZZY METRIC SPACES

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***Abstract** : In this paper, we obtain two common fixed point theorems , one for two pairs of single and set-valued mappings and another for four set-valued mappings satisfying implicit contractive type conditions. 2000 Mathematics Subject Classification : 47H10 , 54H25*
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1. Introduction and preliminaries

The theory of fuzzy sets was introduced by L.Zadeh [9] in 1965.George and Veeramani [1] modified the concept of fuzzy metric space introduced by Kramosil and Michalek [7].Grabiec[10] proved the contraction principle in the setting of fuzzy metric spaces introduced in [1].For fixed point theorems in fuzzy metric spaces some of the interesting references are[1,3,4,5,10-18].

In the sequel, we need the following

Definition 1.1; ([2]: A binary operation $*$: $[0,1] \times [0,1] \to [0,1]$ is a continuous t-norm if it satisfies the following conditions

(i) $*$ is associative and commutative,
(ii) $*$ is continuous,
(iii) $a*1= a$ for all $a \in[0,1]$,
(iv) $a*b \leq c*d$ whenever $a \leq c$ and $b \leq d$, for each $a,b,c,d \in [0,1]$.

Two typical examples of continuous t-norm are $a* b = ab$ and $a*b =\min \{a,b\}$.

Definition 1.2: ([1]: A 3-tuple $(X,M, *)$ is

Definitie 1.2: ([1]: A 3-tuple (X,M, $*$) se numeste spatiu metric difuz daca X este un punct ne-arbitrar (care nu este gol), $*$ este o norma-t continua si M este un punct difuz pe $X^2 \times (0, \infty)$, care satisface urmatoarele conditii pentru fiecare x,y,z $\in$ X si fiecare t si s > 0,

(1) M(x,y,t) > 0 ,

(2) M(x,y,t) = 1 daca si numai daca x = y ,

(3) M(x,y,t) = M(y,x,t) ,

(4) M(x,y,t) $*$ M(y,z,s) $\leq$ M(x,z,t+s) ,

(5) M(x,y,.):(0, ∞) $\to$ [0,1] este continuu.

Lema 1.3 ([10]): Fie (X,M, $*$) un spatiu metric difuz. Atunci M(x,y,t) nu este descrescator in legatura cu t, pentru toate x,y in X .

Definitia 1.4: Fie (X,M, $*$) un spatiu metric difuz. M este continuu la $X^2 \times (0, \infty)$ daca $\lim_{n\to\infty} M(x_n, y_n, t_n) = M(x,y,t)$ de fiecare data cand o insiruire $\{(x_n, y_n, t_n)\}$ in $X^2 \times (0,\infty)$ se apropie de un punct (x,y,t) $\in X^2 \times (0,\infty)$, i.e., de fiecare data cand $\lim_{n\to\infty} M(x_n, x, t) = \lim_{n\to\infty} M(y_n, y, t) = 1$ si $\lim_{n\to\infty} M(x, y, t_n) = M(x,y,t)$.

Lema 1.5. ([8]): Fie (X,M, $*$) un spatiu metric difuz. Atunci M este o functie continua pe $X^2 \times (0,\infty)$.

In aceasta lucrare, sa presupunem ca $\lim_{t\to\infty} M(x, y, t) = 1$ si B(X) este valoarea tuturor subseturilor delimitate ale spatiilor metrice, difuze (X,M, $*$).

Pentru A,B $\in$B(X) si pentru fiecare t > 0 , notati $\delta_M(A,B,t) = \inf\{M(a,b,t): a\in A, b\in B\}$.

Daca A este format dintr-un singur punct a, notam $\delta_M(A,B,t) = \delta_M(a,B,t)$. Daca B este format, de asemenea, dintr-un singur punct b, notam $\delta_M(A,B,t) = M(a,b,t)$.

Din definitie se poate observa ca:

$\delta_M(A,B,t) = \delta_M(B,A,t) \geq 0$,

$\delta_M(A,B,t) = 1 \Leftrightarrow A = B = \{a\}$,

$\delta_M(A,B,t+s) \geq \delta_M(A,C,t) * \delta_M$ called a fuzzy metric space if X is an arbitrary (non-empty) set, $*$ is a continuous t-norm and M is a fuzzy set on $X^2 \times (0, \infty)$, satisfying the following conditions for each x,y,z $\in$ X and each t and s > 0,

(1) M(x,y,t) > 0 ,

(2) M(x,y,t) = 1 if and only if x = y ,

(3) M(x,y,t) = M(y,x,t) ,

(4) M(x,y,t) $*$ M(y,z,s) $\leq$ M(x,z,t+s) ,

(5) M(x,y,.):(0, ∞) $\to$ [0,1] is continuous.

Lemma 1.3 ([10]): Let (X,M, $*$) be a fuzzy metric space. Then M(x,y,t) is non-decreasing with respect to t , for all x,y in X .

Definition 1.4: Let (X,M, $*$) be a fuzzy metric space. M is said to be continuous on $X^2 \times (0, \infty)$ if $\lim_{n\to\infty} M(x_n, y_n, t_n) = M(x,y,t)$ whenever a sequence $\{(x_n, y_n, t_n)\}$ in $X^2 \times (0,\infty)$ converges to a point (x,y,t) $\in X^2 \times (0,\infty)$, i.e., whenever $\lim_{n\to\infty} M(x_n, x, t) = \lim_{n\to\infty} M(y_n, y, t) = 1$ and $\lim_{n\to\infty} M(x, y, t_n) = M(x,y,t)$.

Lemma 1.5. ([8]): Let (X,M, $*$) be a fuzzy metric space. Then M is continuous a function on $X^2 \times (0,\infty)$.

Throughout this paper, assume that $\lim_{t\to\infty} M(x, y, t) = 1$ and B(X) is the set of all non-empty bounded subsets of fuzzy metric space (X,M, $*$).

For A,B $\in$B(X) and for every t > 0 ,denote $\delta_M(A,B,t) = \inf\{M(a,b,t): a\in A, b\in B\}$.

If A consists of a single point a, we write $\delta_M(A,B,t) = \delta_M(a,B,t)$. If B also consists of a single point b, we write $\delta_M(A,B,t) = M(a,b,t)$

It follows immediately from the definition that

$\delta_M(A,B,t) = \delta_M(B,A,t) \geq 0$,

$\delta_M(A,B,t) = 1 \Leftrightarrow A = B = \{a\}$,

$\delta_M(A,B,t+s) \geq \delta_M(A,C,t) * \delta_M(C,B,s)$ for all A,B,C in B(X) and for all s,t > 0 .

Definition 1.6: A sequence $\{A_n\}$ in B(X) is said to converge to a set A $\in$ B(X) if $\delta_M(A_n, A, t) \to 1$ as $n \to \infty$ for all t > 0.

Lemma 1.7: Let $\{A_n\}$ and $\{B_n\}$ be sequences in B(X) converge to A and B in

(C,B,s) pentru toate A,B,C in B(X) si pentru toate s,t > 0 .

Definitia 1.6: Un sir $\{A_n\}$ in B(X) se apropie de un set $A \in B(X)$ daca $\delta_M(A_n,A,t) \to 1$ ca $n \to \infty$ pentru toate t > 0.

Lema 1.7: $\{A_n\}$ SI $\{B_n\}$ sa fie siruri in B(X) care se apropie de A si B in B(X), respectiv . Atunci $\delta_M(A_n, B_n, t) \to \delta_M(A, B, t)$ ca $n \to\infty$ pentru toate t > 0 .

Dovada. Avem $\delta_M(A_n, A, t) \to 1$ si $\delta_M(B_n, B, t) \to 1$ ca $n\to\infty$ pentru toate t > 0.

Pentru r,s,t > 0 , avem $\delta_M(A_n, B_n, r + t + s) \geq \delta_M(A_n, A,r) * \delta_M(A,B,t) * \delta_M(B,B_n,s)$.

Fie $n \to\infty$,

$\lim_{n\to\infty} \delta_M(A_n, B_n, r + t + s) \geq 1 * \delta_M(A,B,t) * 1 = \delta_M(A,B,t)$.

Daca $r \to 0$, $s \to 0$, obtinem $\lim_{n\to\infty} \delta_M(A_n, B_n, t) \geq \delta_M(A,B,t)$ ….(i)

In mod asemanator, putem demonstra ca $\lim_{n\to\infty} \delta_M(A_n, B_n, t) \leq \delta_M(A,B,t)$ ….(ii)

Din (i),(ii), avem $\lim_{n\to\infty} \delta_M(A_n, B_n, t) = \delta_M(A,B,t)$.

Lema 1.8: Daca pentru doua puncte A si B in B(X) si pentru un numar pozitiv q < 1,

$\delta_M(A,B,qt) \geq \delta_M(A,B,t)$ pentru toate t > 0 atunci A = B = multime cu un singur element.

Definitie 1.9: ([6]): Aplicatiile $f : X\to X$ si $F : X\to B(X)$ sunt subcompatibile daca sunt commutate la puncte de coincidenta, i.e. , pentru fiecare punct u in X astfel incat $Fu = \{fu\}$, avem $Ffu = f Fu$.

Definitia 1.10: Aplicatiile f, F : $X\to B(X)$ se muta intamplator, daca se muta la punct de legatura, ex, pentru fiecare punct u in X astfel incat Fu = fu , avem Ff u = f Fu.

Definitia 1.11: Aplicatiile $f : X\to X$ si $F : X\to B(X)$ sunt o pereche de aplicatii M daca exista un sir$\{x_n\}$ in X astfel incat pentru fiecare t > 0 , sa avem $M(fx_n ,z,t) \to 1$ si $\delta_M(Fx_n,\{z\},t) \to 1$ ca $n\to \infty$ pentru cateva $z \in X$.

B(X) respectively . Then $\delta_M(A_n, B_n, t) \to \delta_M(A, B, t)$ as $n \to\infty$ for all t > 0 .

Proof. We have $\delta_M(A_n, A, t) \to 1$ and $\delta_M(B_n, B, t) \to 1$ as $n\to\infty$ for all t > 0.

For r,s,t > 0 , we have

$\delta_M(A_n, B_n, r + t + s) \geq \delta_M(A_n, A,r) * \delta_M(A,B,t) * \delta_M(B,B_n,s)$.

Letting $n \to\infty$,

$\lim_{n\to\infty} \delta_M(A_n, B_n, r + t + s) \geq 1 * \delta_M(A,B,t) * 1 = \delta_M(A,B,t)$.

Now letting $r \to 0$, $s \to 0$, we get $\lim_{n\to\infty} \delta_M(A_n, B_n, t) \geq \delta_M(A,B,t)$ ….(i)

Similarly, we can show that $\lim_{n\to\infty} \delta_M(A_n, B_n, t) \leq \delta_M(A,B,t)$ …….(ii)

From (i),(ii), we have $\lim_{n\to\infty} \delta_M(A_n, B_n, t) = \delta_M(A,B,t)$.

Lemma 1.8: If for two sets A and B in B(X) and for a positive number q < 1,

$\delta_M(A,B,qt) \geq \delta_M(A,B,t)$ for all t > 0 then A = B = singleton.

Definition 1.9: ([6]): The mappings $f : X\to X$ and $F : X\to B(X)$ are subcompatible if they commute at coincidence points, i.e. , for each point u in X such that $Fu = \{fu\}$, we have $Ffu = f Fu$.

Definition 1.10: The mappings f, F : $X\to B(X)$ are said to be coincidentally commuting if they commute at coincidence points, i.e., for each point u in X such that Fu = fu , we have Ff u = f Fu.

Definition 1.11: The mappings $f : X\to X$ and $F : X\to B(X)$ are said to be a pair of M-maps if there exists a sequence $\{x_n\}$ in X such that for every t > 0 , we have

$M(fx_n ,z,t) \to 1$ and $\delta_M(Fx_n,\{z\},t) \to 1$ as $n\to \infty$ for some $z \in X$.

2. Implicit Relation

Let Φ denote the class of all continuous functions $\phi : [0,1]^6 \to R_+$ satisfying

$\phi(u,1,1,v,v,1) \geq 0$ or $\phi(u,1,v,1,1,v) \geq 0$ or $\phi(u,v,1,1,v,v) \geq 0$ implies $u \geq v$.

Example 2.1: $\phi(t_1, t_2, t_3, t_4, t_5, t_6) = t_1 –$

2. Relatii implicite

Sa numim Φ ca fiind clasa pentru toate functiile continue ϕ : $[0,1]^6 \to R_+$ indeplineste $\phi(u,1,1,v,v,1) \geq 0$ sau $\phi(u,1,v,1,1,v) \geq 0$ sau $\phi(u,v,1,1,v,v) \geq 0$ implica $u \geq v$.

Exemplul 2.1: $\phi(t_1, t_2, t_3, t_4, t_5, t_6) = t_1 - \min\{t_2, t_3, t_4, t_5, t_6\}$.

Exemplul 2.2 : $\phi(t_1, t_2, t_3, t_4, t_5, t_6) = t_1 - \min\{t_2, t_3, t_3 t_5, t_4 t_6\}$.

Exemplul 2.3 : $\phi(t_1, t_2, t_3, t_4, t_5, t_6) = t_1 - \dfrac{t_3 t_4 + t_5 t_6}{1 + t_2}$

3. REZULTATE PRINCIPALE

Teorema 3.1. Fie ca f,g sa fie aplicatii pe un spatiu metric difuz (X,M,*) si F,G: X→B(X) sa fie doua aplicatii cu puncte fixe, astfel incat

(3.1.1)

$$\phi\begin{pmatrix}\delta_M(Fx,Gy,qt), M(fxgy,t), \delta_M(fx,Fx,t)\\ \delta_M(gy,Gy,t), \delta_M(fx,Gy,t), \delta_M(gy,Fx,t)\end{pmatrix} \geq 0$$

Pentru toate $x,y \in X$, $t > 0$ si $q \in (0,1)$, unde $\phi \in \Phi$,

(3.1.2) (f, F) si (g, G) sunt perechi subcompatibile,

(3.1.3) (a) (f, F) este o pereche de aplicatii M , $Fx \subseteq g(X)$ $\forall x \in X$ si f (X) este inchis

(sau)

(3.1.3) (b) (g,G) este o pereche de aplicatii M , $Gx \subseteq f(X)$ $\forall x \in X$ si g (X) este inchis.

Atunci f,g, F si G au un unic punct fix in X .

Dovada. Sa presupunem ca (3.1.3) (a) detine.

$\min\{t_2, t_3, t_4, t_5, t_6\}$.

Example 2.2 : $\phi(t_1, t_2, t_3, t_4, t_5, t_6) = t_1 - \min\{t_2, t_3, t_3 t_5, t_4 t_6\}$.

Example 2.3 : $\phi(t_1, t_2, t_3, t_4, t_5, t_6) = t_1 - \dfrac{t_3 t_4 + t_5 t_6}{1 + t_2}$

3. MAIN RESULTS

Theorem 3.1. Let f,g be self maps of on a fuzzy metric space (X,M,*) and let F,G: X→B(X) be two set- valued maps such that

(3.1.1)

$$\phi\begin{pmatrix}\delta_M(Fx,Gy,qt), M(fx,gy,t),\\ \delta_M(fx,Fx,t)\\ \delta_M(gy,Gy,t), \delta_M(fx,Gy,t),\\ \delta_M(gy,Fx,t)\end{pmatrix} \geq 0$$

for all $x,y \in X$, $t > 0$ and $q \in (0,1)$, where $\phi \in \Phi$,

(3.1.2) (f, F) and (g, G) are subcomtible pairs,

(3.1.3) (a) (f, F) is a pair of M-maps , $Fx \subseteq g(X)$ $\forall x \in X$ and f (X) is closed

(or)

(3.1.3) (b) (g,G) is a pair of M-maps , $Gx \subseteq f(X)$ $\forall x \in X$ and g (X) is closed.

Then f,g, F and G have a unique common fixed point in X .

Proof. Suppose (3.1.3) (a) holds.

Since(f,F) is a pair of M-maps , there exist a sequence{xn} in X and some $z \in X$ such that $\lim_{n\to\infty} M(fxn,z,t)=1$ and $\lim_{n\to\infty} \delta M(Fxn,\{z\},t)=1$ for all $t > 0$.

Since $Fx \subseteq g(X)$ $\forall x \in X$, there exist $\alpha n \in Fxn$ and $yn \in X$ such that $\alpha n=gyn$ $\forall$ n.

Also $M(gy_n,z,t) = M(\alpha_n,z,t) \geq \delta_M(Fx_n,\{z\},t) \to 1$ as $n\to\infty$.

Suppose $\lim_{n\to\infty} \delta_M(Gy_n,A,t)=1$ for some $A \in B(X)$. Now

Din moment ce (f,F) este o pereche de aplicatii M, exista un sir $\{x_n\}$ in X si cateva $z \in X$

Astfel incat $\lim_{n\to\infty} M(fx_n,z,t)=1$ si $\lim_{n\to\infty} \delta_M(Fx_n,\{z\},t)=1$ pentru toate $t > 0$.

Din moment ce $Fx \subseteq g(X)\ \forall\ x \in X$, exista $\alpha_n \in Fx_n$ si $y_n \in X$ astfel incat $\alpha_n = gy_n\ \forall\ n$.

De asemenea $M(gy_n,z,t) = M(\alpha_n,z,t) \geq \delta_M(Fx_n,\{z\},t) \to 1$ deoarece $n\to\infty$.

Sa presupunem $\lim_{n\to\infty} \delta_M(Gy_n,A,t)=1$ pentru cateva $A \in B(X)$. Acum

$$\phi\begin{pmatrix}\delta_M(Fx_n,Gy_n,qt), M(fx_n,gy_n,t), \delta_M(fx_n,Fx_n,t) \\ \delta_M(gy_n,Gy_n,t), \delta_M(fx_n,Gy_n,t), \delta_M(gy_n,Fx_n,t\end{pmatrix} \geq 0$$

Sa presupunem $n\to\infty$, obtinem

ϕ ($\delta_M(z,A,qt)$, 1,1, $\delta_M(z,A,t)$, $\delta_M(z,A,t)$,1) ≥ 0

$\delta_M(z,A,qt) \geq \delta_M(z,A,t)$ din valoarea ϕ. Ca urmare a lemei, avem $A=\{z\}$. Astfel

$\lim_{n\to\infty} Gy_n=\{z\}$.

Din moment ce f(X) este inchis, exista $u \in X$ astfel incat $z = fu$. Acum,

$$\phi\begin{pmatrix}\delta_M(Fu,Gy_n,qt), M(fu,gy_n,t), \delta_M(fu,Fu,t) \\ \delta_M(gy_n,Gy_n,t), \delta_M(fu,Gy_n,t), \delta_M(gy_n,Fu,t\end{pmatrix} \geq 0$$

Sa presupunem $n\to\infty$, obtinem

ϕ($\delta_M(Fu,z,qt)$,1, $\delta_M(Fu,z,t)$,1,1 $\delta_M(Fu,z,t)$) ≥ 0

$\delta_M(Fu,z,qt) \geq \delta_M(Fu,z,t)$ care implica faptul ca $Fu=\{z\}$. Astfel $Fu=\{z\}=\{fu\}$.

Din moment ce $\{z\}=Fu \subseteq g(X)$, exista $w \in X$ astfel incat $z = gw$. Acum,

$$\phi\begin{pmatrix}\delta_M(Fx_n,Gw,qt), M(fx_n,gw,t), \delta_M(fx_n,Fx_n,t) \\ \delta_M(gw,Gw,t), \delta_M(fx_n,Gw,t), \delta_M(gw,Fx_n,t\end{pmatrix} \geq 0$$

$$\phi\begin{pmatrix}\delta_M(Fx_n,Gy_n,qt), M(fx_n,gy_n,t), \\ \delta_M(fx_n,Fx_n,t) \\ \delta_M(gy_n,Gy_n,t), \delta_M(fx_n,Gy_n,t), \\ \delta_M(gy_n,Fx_n,t\end{pmatrix} \geq 0$$

Letting $n\to\infty$, we get ϕ ($\delta_M(z,A,qt)$, 1,1, $\delta_M(z,A,t)$, $\delta_M(z,A,t)$,1) ≥ 0

$\delta_M(z,A,qt) \geq \delta_M(z,A,t)$ from property of ϕ. By lemma, we have $A=\{z\}$.Thus

$\lim_{n\to\infty} Gy_n=\{z\}$.

Since f(X) is closed , there exists $u \in X$ such that $z = fu$. Now,

$$\phi\begin{pmatrix}\delta_M(Fu, Gy_n, qt), M(fu, gy_n, t), \\ \delta_M(fu, Fu, t) \\ \delta_M(gy_n, Gy_n, t), \delta_M(fu, Gy_n, t), \\ \delta_M(gy_n, Fu, t\end{pmatrix} \geq 0$$

Letting $n\to\infty$, we get ϕ($\delta_M(Fu,z,qt)$,1, $\delta_M(Fu,z,t)$,1,1 $\delta_M(Fu,z,t)$) ≥ 0

$\delta_M(Fu,z,qt) \geq \delta_M(Fu,z,t)$ which implies that $Fu=\{z\}$.Thus $Fu=\{z\}=\{fu\}$.

Since $\{z\}=Fu \subseteq g(X)$, there exists $w \in X$ such that $z = gw$. Now,

$$\phi\begin{pmatrix}\delta_M(Fx_n, Gw, qt), M(fx_n, gw, t), \\ \delta_M(fx_n, Fx_n, t) \\ \delta_M(gw, Gw, t), \delta_M(fx_n, Gw, t), \\ \delta_M(gw, Fx_n, t\end{pmatrix} \geq 0$$

Letting $n\to\infty$, we get ϕ ($\delta_M(z,Gw,qt)$,1,1, $\delta_M(z,Gw,t)$, $\delta_M(z,Gw,t)$ $\delta_M(z,Gw,t)$) ≥ 0

$\delta_M(z,Gw,qt) \geq \delta_M(z,Gw,t)$ which implies that $Gw=\{z\}$. Thus $Gw=\{z\}=\{gw\}$.

Since (f,F) is sub compatible,we have $Fz=Ffu=f\,Fu=\{fz\}$. Now,

$$\phi\begin{pmatrix}\delta_M(Fz, Gw, qt), M(fz, gw, t), \\ \delta_M(fz, Fz, t) \\ \delta_M(gw, Gw, t), \delta_M(fz, Gw, t), \\ \delta_M(gw, Fz, t\end{pmatrix} \geq 0$$

which implies ϕ ($\delta_M(Fz,z,qt)$, $\delta_M(Fz,z,t)$, 1,1, $\delta_M(Fz,z,t)$, $\delta_M(Fz,z,t)$) ≥ 0

$\delta_M(Fz,z,qt) \geq \delta_M(Fz,z,t)$ so that $Fz = \{z\}$. Thus $Fz=\{z\}=\{fz\}$.

Since (g,G) is sub compatible, we have $Gz = Ggw = gGw = \{gz\}$. Now, Sa presupunem $n\to\infty$, obtinem

ϕ ($\delta_M(z,Gw,qt),1,1, \delta_M(z,Gw,t), \delta_M(z,Gw,t)\ \delta_M(z,Gw,t)$) ≥ 0

$\delta_M(z,Gw,qt) \geq \delta_M(z,Gw,t)$ ceea ce implica faptul ca Gw={z}. Astfel, Gw={z}={gw}.

Din moment ce (f,F) este subcompatibil, avem Fz=Ffu=f Fu={fz}. Acum,

$$\phi\begin{pmatrix} \delta_M(Fz,Gw,qt), M(fz,gw,t), \delta_M(fz,Fz,t) \\ \delta_M(gw,Gw,t), \delta_M(fz,Gw,t), \delta_M(gw,Fz,t \end{pmatrix} \geq 0$$

Care implica

ϕ ($\delta_M(Fz,z,qt)$, $\delta_M(Fz,z,t)$, 1,1, $\delta_M(Fz,z,t)$, $\delta_M(Fz,z,t)$) ≥ 0

$\delta_M(Fz,z,qt) \geq \delta_M(Fz,z,t)$ astfel incat Fz = {z}. Astfel Fz={z}={fz}.

Din moment ce (g,G) este subcompatibil, avem Gz = Ggw = gGw = {gz}. Acum,

$$\phi\begin{pmatrix} \delta_M(Fu,Gz,qt), M(fu,gz,t), \delta_M(fu,Fu,t) \\ \delta_M(gz,Gz,t), \delta_M(fu,Gz,t), \delta_M(gz,Fu,t \end{pmatrix} \geq 0$$

ceea ce implica

ϕ ($\delta_M(z,Gz,qt)$, M(z,gz,t), 1,1, $\delta_M(z,Gz,t)$, $\delta_M(z,Gz,t)$) ≥ 0

$\delta_M(z,Gz,qt) \geq \delta_M(z,Gz,t)$ astfel incat Gz = {z}. Astfel Gz = {z} = {gz}.

Astfel z un punct comun fix al F,G,f si g.

Caracterul unic al punctelor comune, fixe apar usor din (3.1.1). Asemanator, putem demonstra teorema, daca (3.1.3)(b)detine. □

Acum, demonstram teorema unui punct comun, fix pentru patru aplicatii ale punctelor fixe.

Theorema 3.2. Fie F,G,f si g:X→ B(X) aplicatii puncte fixe ce respecta (3.2.1)

$$\phi\begin{pmatrix} \delta_M(Fx,Gy,qt), \delta_M(fx,gy,t), \delta_M(fx,Fx,t) \\ \delta_M(gy,Gy,t), \delta_M(fx,Gy,t), \delta_M(gy,Fx,t \end{pmatrix} \geq 0$$

pentru toate $x,y\in X$, $t > 0$ si $q\in(0,1)$, unde $\phi \in \Phi$,

(3.2.2)(a) Sa presupunem ca exista un sit

$$\phi\begin{pmatrix} \delta_M(Fu, Gz, qt), M(fu, gz, t), \\ \delta_M(fu, Fu, t) \\ \delta_M(gz, Gz, t), \delta_M(fu, Gz, t), \\ \delta_M(gz, Fu, t \end{pmatrix} \geq 0$$

which implies ϕ ($\delta_M(z,Gz,qt)$, M(z,gz,t), 1,1, $\delta_M(z,Gz,t)$, $\delta_M(z,Gz,t)$) ≥ 0

$\delta_M(z,Gz,qt) \geq \delta_M(z,Gz,t)$ so that Gz = {z}. Thus Gz = {z} = {gz}.

Thus z is a common fixed point of F,G,f and g.

Uniqueness of common fixed point follows easily from (3.1.1). Similarly, we can prove the theorem if (3.1.3)(b) holds.

Now, we prove a common fixed point theorem for four set –valued mappings.

Theorem 3.2. Let F,G,f and g:X→ B(X) be set- valued mappings satisfying (3.2.1)

$$\phi\begin{pmatrix} \delta_M(Fx, Gy, qt), \delta_M(fx, gy, t), \\ \delta_M(fx, Fx, t) \\ \delta_M(gy, Gy, t), \delta_M(fx, Gy, t), \\ \delta_M(gy, Fx, t \end{pmatrix} \geq 0$$

for all $x,y\in X$, $t > 0$ and $q\in(0,1)$, where $\phi \in \Phi$,

(3.2.2)(a) Suppose that there exists a sequence $\{x_n\}$ in X such that $\{Fx_n\}$ and $\{fx_n\}$ converge to the same limit {z} for some $z\in X$. (or)

(3.2.2)(b) Suppose that there exists a sequence $\{y_n\}$ in X such that $\{Gy_n\}$ and $\{gy_n\}$ converge to the same limit {z} for some $z\in X$.

(3.2.3) Suppose that the pairs (f,F) and (g,G) are coincidentally commuting,

(3.2.4) Suppose fu={z}=gv for some $u,v\in X$.

(3.2.5) Suppose that Fz or fz is a singleton and Gz or gz is a singleton.

Then z is the unique common fixed point of F,G,f and g, Also z is the unique common fixed point of F and f as well as of G and g.

Proof. Suppose (3.2.2) (a) holds.

$\{x_n\}$ in X astfel incat $\{Fx_n\}$ si $\{fx_n\}$ sa se apropie de aceeasi limita {z} pentru anumite $z \in X$. (or)

(3.2.2)(b) Sa presupunem ca exista un sir $\{y_n\}$ in X astfel incat $\{Gy_n\}$ si $\{gy_n\}$ sa se apropie de aceeasi limita {z} pentru cateva $z \in X$.

(3.2.3) Sa presupunem ca perechile (f,F) si (g,G) se muta in mod intamplator,

(3.2.4) Sa presupunem ca fu={z}=gv pentru anumite $u,v \in X$.

(3.2.5) Sa presupunem ca Fz sau fz este o multime cu un singur element Gz sau gz este o multime cu un singur element.

Atunci z este un punct fix unic, comun F,G,f si g, De asemenea, z este unicul punct comun, fix al F si f la fel ca la G si g.

Dovada. Sa presupunem ca (3.2.2) (a) are.

$$\phi\begin{pmatrix} \delta_M(Fx_n, Gv, qt), \delta_M(fx_n, gv, t), \delta_M(fx_n, Fx_n, t) \\ \delta_M(gv, Gv, t), \delta_M(fx_n, Gv, t), \delta_M(gv, Fx_n, t) \end{pmatrix} \geq 0$$

Fie $n \to \infty$, obtinem

ϕ(δ_M(z,Gv,qt),1, 1 ,δ_M(z,Gv,t), δ_M(z,Gv,t),1)) ≥ 0

δ_M(z,Gv,qt) $\geq$ δ_M(z,Gv,t) astfel incat Gv = {z}.Astfel Gv = {z} = gv.

Din moment ce (g,G) se muta intamplator, avem Gz = Ggv = gGv = gz = multime cu un singur element din(3.2.5). Acum,

$$\phi\begin{pmatrix} \delta_M(Fx_n, Gz, qt), \delta_M(fx_n, gz, t), \delta_M(fx_n, Fx_n, t) \\ \delta_M(gz, Gz, t), \delta_M(fx_n, Gz, t), \delta_M(gz, Fx_n, t) \end{pmatrix} \geq 0$$

Fie $n \to \infty$, obtinem

ϕ(δ_M(z,Gz,qt), δ_M(z,Gz,t),1,1, δ_M(z,Gz,t), δ_M(z,Gz,t)) ≥ 0

δ_M(z,Gz,qt) $\geq$ δ_M(z,Gz,t) astfel incat Gz={z}.Astfel Gz = {z} = gz.

$$\phi\begin{pmatrix} \delta_M(Fu, Gz, qt), \delta_M(fu, gz, t), \delta_M(fu, Fu, t) \\ \delta_M(gz, Gz, t), \delta_M(fu, Gz, t), \delta_M(gz, Fu, t) \end{pmatrix} \geq 0$$

care implica

$$\phi\begin{pmatrix} \delta_M(Fx_n, Gv, qt), \delta_M(fx_n, gv, t), \\ \delta_M(fx_n, Fx_n, t) \\ \delta_M(gv, Gv, t), \delta_M(fx_n, Gv, t), \\ \delta_M(gv, Fx_n, t) \end{pmatrix} \geq 0$$

Letting $n \to \infty$, we get

ϕ(δ_M(z,Gv,qt),1, 1 ,δ_M(z,Gv,t), δ_M(z,Gv,t),1)) ≥ 0

δ_M(z,Gv,qt) $\geq$ δ_M(z,Gv,t) so that Gv = {z}.Thus Gv = {z} = gv.

Since (g,G) is coincidentally commuting, we have Gz = Ggv = gGv = gz = singleton from(3.2.5). Now,

$$\phi\begin{pmatrix} \delta_M(Fx_n, Gz, qt), \delta_M(fx_n, gz, t), \\ \delta_M(fx_n, Fx_n, t) \\ \delta_M(gz, Gz, t), \delta_M(fx_n, Gz, t), \\ \delta_M(gz, Fx_n, t) \end{pmatrix} \geq 0$$

Letting $n \to \infty$, we get ϕ(δ_M(z,Gz,qt), δ_M(z,Gz,t),1,1, δ_M(z,Gz,t), δ_M(z,Gz,t)) ≥ 0

δ_M(z,Gz,qt) $\geq$ δ_M(z,Gz,t) so that Gz={z}.Thus Gz = {z} = gz.

$$\phi\begin{pmatrix} \delta_M(Fu, Gz, qt), \delta_M(fu, gz, t), \\ \delta_M(fu, Fu, t) \\ \delta_M(gz, Gz, t), \delta_M(fu, Gz, t), \\ \delta_M(gz, Fu, t) \end{pmatrix} \geq 0$$

which implies

ϕ(δ_M(Fu,z,qt), 1, δ_M (Fu,z,t), 1,1, δ_M (Fu,z,t), δ_M(Fu,z,t)) ≥ 0

δ_M(Fu,z,qt) $\geq$ δ_M(Fu,z,t) so that Fu={z}.Thus Fu={z}=fu.

Since (f,F) is coincidentally commuting, we have Fz=Ffu=f Fu=fz=singleton from(3.2.5). Now,

$$\phi\begin{pmatrix} \delta_M(Fz, Gz, qt), \delta_M(fz, gz, t), \\ \delta_M(fz, Fz, t) \\ \delta_M(gz, Gz, t), \delta_M(fz, Gz, t), \\ \delta_M(gz, Fz, t) \end{pmatrix} \geq 0$$

which implies

ϕ(δ_M(Fz,z,qt), δ_M(Fz,z,t), 1,1, δ_M (Fz,z,t), δ_M(Fz,z,t)) ≥ 0

δ_M(Fz,z,qt) $\geq$ δ_M(Fz,z,t) so that Fz ={z}. Thus Fz = fz. Thus z is a common fixed point of F,G,f and g.

$\phi(\delta_M(Fu,z,qt), 1, \delta_M(Fu,z,t), 1,1, \delta_M(Fu,z,t), \delta_M(Fu,z,t)) \geq 0$

$\delta_M(Fu,z,qt) \geq \delta_M(Fu,z,t)$ astfel incat Fu={z}.Astfel Fu={z}=fu.

Din moment ce (f,F) se muta intamplator, avem Fz=Ffu=f Fu=fz=multime cu un singur element din (3.2.5). Acum,

$$\phi\begin{pmatrix} \delta_M(Fz,Gz,qt), \delta_M(fz,gz,t), \delta_M(fz,Fz,t) \\ \delta_M(gz,Gz,t), \delta_M(fz,Gz,t), \delta_M(gz,Fz,t) \end{pmatrix} \geq 0$$

care implica

$\phi(\delta_M(Fz,z,qt), \delta_M(Fz,z,t), 1,1, \delta_M(Fz,z,t), \delta_M(Fz,z,t)) \geq 0$

$\delta_M(Fz,z,qt) \geq \delta_M(Fz,z,t)$ astfel incat Fz ={z}. Astfel Fz = fz. Astfel z este un punct fix, comun al F,G,f si g.

Caracterul unic al punctelor fixe, commune, deriva din (3.2.1).

Sa presupunem ca fw={w}=Fw pentru anumite $w \in X$. Acum,

$$\phi\begin{pmatrix} \delta_M(Fw,Gz,qt), \delta_M(fw,gz,t), \delta_M(fw,Fw,t) \\ \delta_M(gz,Gz,t), \delta_M(fw,Gz,t), \delta_M(gz,Fw,t) \end{pmatrix} \geq 0$$

care implica

$\phi(M(w,z,qt), M(w,z,t), 1,1, M(w,z,t), M(w,z,t)) \geq 0$

$M(w,z,qt) \geq M(w,z,t)$ astfel incat w =z. Astfel z este unicul punct comun al f si F. In mod asemanator, putem demonstra ca z este unicul punct comun al g si G. In mod asemanator, putem demonstra teorema cand (2.3.2) (b) are. □

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Uniqueness of common fixed point follows easily from (3.2.1).

Suppose fw={w}=Fw for some $w \in X$. Now,

$$\phi\begin{pmatrix} \delta_M(Fw, Gz, qt), \delta_M(fw, gz, t), \\ \delta_M(fw, Fw, t) \\ \delta_M(gz, Gz, t), \delta_M(fw, Gz, t), \\ \delta_M(gz, Fw, t) \end{pmatrix} \geq 0$$

which implies

$\phi(M(w,z,qt), M(w,z,t), 1,1, M(w,z,t), M(w,z,t)) \geq 0$

$M(w,z,qt) \geq M(w,z,t)$ so that w =z. Thus z is the unique common fixed point of f and F. Similarly we can show that z is the unique common fixed point of g and G. Similarly, we can prove the theorem when (2.3.2) (b) holds.

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