# MATHEMATICAL MODELING AND SIMULATION OF THE ASYNCHRONOUS MACHINE IN THE U,V,0 AXIS SYSTEM 

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#### Abstract

The paper presents the differential equations which represent the mathematical model of the asynchronous machine in the u,v,0 axis system, and the simulation in the Simulink - Matlab, implemented basis of these equations. The asynchronous machine been very used in industry, the mathematical modeling is extremely helpful because allow solving many problems existing in this moment.


### 4.1. Determination of the differential equations

When we conclude the differential equations and when we analyse the transient process of the asynchronous machine, we use the ipotesis and the usual restrictions, referable to "ideal machine":
> The machine is unsaturated;
> Core loss isn't present;
> The phase's coilings are symetrics and the alteration of phase are with 90 electrical degrees for biphase machines and with 120 electrical degrees for thriphase machine;
> The coiling' s magnetomotive voltage and the magnetic fields are sinusoidal distributed along the air gap;
$>$ The air gap is uniform.
> In emergency situations we can taking into consideration the saturation of the magnetic circuit, core loss, rotor's asymetri etc, which make hardly solving the equations because they become very complicated.
The configuration of the voltage equations for the three-phase of the stator's and rotor's motor are:

$$
\left\{\begin{array}{l}
u_{A}=\frac{d \psi_{A}}{d t}+R_{s} i_{A}  \tag{1}\\
u_{B}=\frac{d \psi_{B}}{d t}+R_{s} i_{B} \\
u_{C}=\frac{d \psi_{C}}{d t}+R_{s} i_{C}
\end{array}\right.
$$

$$
\left\{\begin{array}{l}
u_{\mathrm{a}}=\frac{d \psi_{a}}{d t}+R_{r} i_{a}  \tag{2}\\
u_{b}=\frac{d \psi_{b}}{d t}+R_{s} i_{b} \\
u_{c}=\frac{d \psi_{c}}{d t}+R_{s} i_{c}
\end{array}\right.
$$

where:

$$
\begin{aligned}
& \Psi_{A}\left(\Psi_{a}\right) \text { - total flux of the stator's (rotor's) phase; } \\
& i_{A}\left(i_{a}\right) \text { - the amperage of the stator's (rotor's) phase; } \\
& R_{s}\left(R_{r}\right) \text { - the resistance of the stator's (rotor's) phase coiling. }
\end{aligned}
$$

The expresion of the total flux of the stator's (rotor's) phase are:

$$
\begin{align*}
& \Psi_{A}=L_{A} i_{A}+M_{A B} i_{B}+M_{A C} i_{C}+i_{a} M \cos \gamma+i_{b} M \cos \left(\gamma+\frac{2 \pi}{3}\right)+i_{c} M \cos \left(\gamma-\frac{2 \pi}{3}\right)  \tag{3}\\
& \Psi_{a}=L_{a} i_{a}+M_{a b} i_{b}+M_{a c} i_{c}+i_{A} M \cos \gamma+i_{B} M \cos \left(\gamma+\frac{2 \pi}{3}\right)+i_{C} M \cos \left(\gamma-\frac{2 \pi}{3}\right)
\end{align*}
$$

where:

$$
\begin{aligned}
& L_{A}=L_{B}=L_{C}=L_{s} \text {-the inductivity of the stator's phase; } \\
& L_{a}=L_{b}=L_{c}=L_{r} \text {-the inductivity of the rotor's phase; } \\
& M_{A B}=M_{A C}=M_{B C}=M_{s} \text {-the mutual inductivity between stator's phases; } \\
& M_{a b}=M_{a c}=M_{b c}=M_{r} \text {-the mutual inductivity between rotor's phases; }
\end{aligned}
$$

The electromagnetic moment of the asynchronous machine can be write like as electromagnetic energy's partial differentiation with rotation angle of the rotor:

$$
\begin{equation*}
M_{e m}^{\prime}=\left(\frac{\partial W_{e m}}{\partial \gamma}\right)_{p} \tag{4}
\end{equation*}
$$

where:

$$
\begin{equation*}
W_{e m}=0,5\left(\Psi_{A} i_{A}+\Psi_{B} i_{B}+\Psi_{C} i_{C}+\Psi_{a} i_{a}+\Psi_{b} i_{b}+\Psi_{c} i_{c}\right) \tag{5}
\end{equation*}
$$

The equation of the circuit rotoris:

$$
\begin{equation*}
M_{e m}^{\prime}-M_{m e c}^{\prime}=\frac{1}{p} J \frac{d \omega_{r}}{d t} \tag{6}
\end{equation*}
$$

where:
$J$ - angular impulse of the rotor and actuator mechanism;
$M_{\text {mec }}^{\prime}$ - resistant couple.

The system of the asynchronous machine is represented to equations (1) $\div(6)$.
For simplify the modeling of the tree-phase asynchronous machine, we make the conversion from the real axis system into the $u, v, 0$ axis system. This system rotate from the stator
with synchronous velocity which mean it is immobile from the statoric field of the asynchronous machine, in stationary regime ( $\omega_{k}=\omega_{s}$ ). Because the voltages $u_{v s}$ and $u_{u s}$ are constants value, in the study of the transitory regime of the asynchronous machine it is preferable to use this system.

### 4.2. The equations of the asynchronous machine in the $\mathbf{u}, \mathbf{v}, \mathbf{0}$ axis system

 In the $\mathrm{u}, \mathrm{v}, 0$ axis system, the equation are:$$
\left\{\begin{array}{l}
\frac{d \psi_{s u}}{d t}=u_{s u}-\frac{R_{s}}{\sigma \cdot x_{s}} \psi_{s u}+\omega_{s} \psi_{s v}+\frac{x_{m} R_{s}}{\sigma \cdot x_{s} \cdot x_{r}} \Psi_{r u}  \tag{7}\\
\frac{d \psi_{s v}}{d t}=u_{s v}-\frac{R_{s}}{\sigma \cdot x_{s}} \psi_{s v}+\omega_{s} \psi_{s u}+\frac{x_{m} R_{s}}{\sigma \cdot x_{s} \cdot x_{r}} \Psi_{r u} \\
\frac{d \Psi_{r u}}{d t}=\frac{x_{m} R_{r}}{\sigma \cdot x_{s} \cdot x_{r}} \Psi_{s u}-\frac{R_{s}}{\sigma \cdot x_{r}} \psi_{r u}-s \omega_{s} \Psi_{r v} \\
\frac{d \Psi_{r v}}{d t}=\frac{x_{m} R_{r}}{\sigma \cdot x_{s} \cdot x_{r}} \Psi_{s v}-\frac{R_{r}}{\sigma \cdot x_{r}} \psi_{r v}-s \omega_{s} \Psi_{r u} \\
\frac{d \omega_{r}}{d t}=\frac{M_{e m}-M_{m e c}}{J} \\
M_{e m}=\frac{3}{2} \cdot \frac{x_{m}}{x_{s} x_{r}}\left(\Psi_{r u} \Psi_{s v}-\Psi_{r v} \Psi_{s u}\right) \\
s=\frac{\omega_{s}-\omega_{r}}{\omega_{s}}
\end{array}\right.
$$

This system allow determination of the transitory electromagnetic moment and the angular speed, in the study transitory process of the asynchronous machine. In this equations (7) the amperages are not presents, because for theirs determination we need to know supplementary relations. This way to write the equation of the system (7), allow us obtain the stable mathematical model of the asynchronous machine.

### 4.3. Simulation of the asynchronous machine in the $\mathbf{u}, \mathbf{v}, \mathbf{0}$ axis system

Using the equations system (7) the simulation diagram implemented with Simulink MATLAB is presented in figure 1. This diagram allow study the evolution of the three parameters ( $\left.\omega_{r}, M_{e m}, s\right)$.

### 4.4. Conclusions

Compared with the analogic simulation implemented with operational amplifiers, the numerical simulation presents the following advantages:
> allow easily modification of the many parameters of the asynchronous machine's mathematical model;
$>$ observation of the evolution of the many control variables;
$>$ drawing of the many characteristics;
In this way we can relieve the high flexibility of the numerical simulation.


Figure 1.

## BIBLIOGRAPHY

1. Sipalov G.A., Loos A.V., Mathematical modelation of the electrical machin, "The high school" 1980.
2. Constantin Ghiță, Electrical machines, "MATRIX ROM",2005
