

**UFA SCIENTIFIC GROUP BY E. A. MUKHACHEVA:  
APPLIED OPERATIONAL RESEARCH PROBLEMS**

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**Abstract.** The Ufa Scientific Group busy with the research of cutting-packing problems was founded more than 50 years ago in the Ufa Aviation Institute. The class of cutting-packing problems includes problems of various applied interpretation. They are regarded as NP-hard problems, and development and research of effective methods of their solution is actual nowadays. This paper is devoted to the review of the basic problems and the methods of their solution developed under the guidance of Prof. E. A. Mukhacheva.

**Keywords:** cutting-packing problems; Ufa Scientific Group; E. A. Mukhacheva; exact methods; approximating method; heuristic method

## 1. INTRODUCTION

The Ufa scientific group (USG), has been actively engaged in applied operational research problems during more than 50 years. The USG coming into being is connected with L.V.Kantorovich and one of his scientific work, that has become basic for all our activity [1]. L. V. Kantorovich was known to have a huge range of scientific interests, including cutting problems [2]. As it was mentioned the USG of cutting-packing (Cutting and Packing, C&P) took place in the beginning of 1960th. At that time the future head of the group Elita Alexandrovna Mukhacheva (1930–2011), became interested in problems of linear programming. In 1962 E. A. Mukhacheva entered the postgraduate course of Institute of mathematics of the USSR Academy of Science (Siberian Department) to the mathematic-economic department headed by the founder of linear programming L. V. Kantorovich. L. V. Kantorovich's priority in this area had already been recognized in the world by the time. In 1967 E. A. Mukhacheva defended the thesis "Numerical methods of solving some classes of problems of linear programming" and got the scientific degree of the candidate of physical and mathematical sciences. Since then in the Ufa aviation institute there started the successful study of C&P problems.

## 2. APPLIED CUTTING-PACKING PROBLEMS

Cutting stock problems represent the important technological problem and their optimum solution

allows one to minimize the available resources consumption. These problems include: a linear material cutting, longitudinal cutting of tapes and rolls, cutting of sheets into rectangular items, cutting materials of different sizes, cutting for serial and custom-made products, packing of subjects into limited area, figured item cutting, allocation of circles, of choice of the best sizes of a material for the subsequent cutting, and many others. Such problems arise in mechanical engineering, metallurgy, woodworking and clothing industry, pulp-and-paper industry, etc. Each industry adds some additional requirements to the statement of problems, and, therefore, needs different adaptation of known algorithms and demands to develop new ones.

Many problems not referring at first sight to the class of cutting-packing problems appear to come down to them in the final analysis. For example, the scheduling problem, the routing problem, the problem of decomposition of multiply connected orthogonal polygon and many other applied problems.

The problems of cutting, packing or covering may appear as intermediate in other problems or take turns in one and the same problem. For example, let us consider a logistic problem in which to every conveyance corresponds its routing towards goods consumer. Then the search of each conveyance routing [3, 4] and the finding of goods allocation plan in cargo bay turn out to be the problems of C&P class.

The sequence order of the vehicle routing is also taken into account so, that the goods for the last consumer are loaded first.

Each object domain brings additional requirements in the way of solving the above problems and, therefore, the known algorithms should be adapted.

The presence of two groups of objects is common to this class of problems. A correspondence between the elements of these groups is established and estimated. There exist linear cutting (one-dimensional) problems, rectangular (two-dimensional) and parallelepiped (three-dimensional) cutting-packing problems. Among these problems the guillotine cutting-packing problems stay aside. Problems of nesting, that is, of accommodations of details of the complex geometrical form in the set areas are especially allocated. The main things for them are the information problems of the figure specifying, the account and maintenance of their non-crossing, their coding, etc. The list of geometrical properties of the items and material can be supplemented and considered in mathematical model by some physical and-or economic parameters. The detailed classification of the basic models of C&P problems in Russia is shown in [5].

### 3. THE ALGORITHMS AND METHODS OF C&P PROBLEM SOLVING DEVELOPED IN THE USG

*The use of mathematical programming methods.* For solving guillotine cutting problems E. A. Mukhacheva developed the method directed toward the dynamic programming application [6]. She also developed the conditional optimization algorithms based on linear programming, they took into account the real industry specificity. At that time these and many other research papers were aimed at application of linear programming for solving real industrial problems. This purpose was to some extent reached in mass and batch production. E. A. Muhacheva's book [7] was written and published under the guidance of L. V. Kantorovich. However the C&P problems as a matter of fact are problems of discrete optimization and their solution by means of linear programming represents no more than a continuous relaxation. It is expedient to use various ways of a rounding off of a relaxation and construction of various remainder problems.

The further development of exact algorithms on the basis of linear integer programming that is approach of a continuous relaxation to an integer optimum was realized by German scientists with the participation of the Ufa scholar G. N. Belov [8]. They have offered to solve the problems of linear and guillotine cutting by the Gomori method of se-

cant planes. Researching in this field is being conducted by them to this day. Similar problems were considered earlier by some authors from Ufa, namely: E. A. Muhacheva and S. M. Ibatullina offered a method with construction of initial basis to solve a cutting problem for material of various prices in the set assortment [9]. However they did not go beyond the continuous relaxation, i.e. application in mass production.

#### *Exact methods of combinatory optimization.*

The methods based on linear integer programming are not always acceptable for solution of linear and guillotine cutting problems, therefore the sorting out methods have received their development. The known branch and bound method (MBB) developed due to introduction of the search reduction procedures in the papers of Ufa scholars [10]. V. M. Kartak offered the element grouping method, allowing considerably reduce dimensions of the problems under consideration. He also offered to reduce a 2D packing problem into sheets and a strip to two 1D packing problems with additional restrictions [11]. The method is generalized for a case of N-dimensional packing [12]. M. Mesjagutov offered the exact method of solving a problem of one-dimensional continued packing based on the dominance criterion and on the rules of cutting off for reducing the sorting-out process [13]. These authors pay profound attention to searching for the bottom bounds [14].

*Approximating and heuristic methods.* While studying C&P problems and developing methods of their solution one should pay special attention to the generalized problem of knapsack loading. The binary, limited, unlimited, two-dimensional knapsack problems find their application in algorithms for solution of C&P problems. Great attention is paid to the problem of searching for subset sums (Subset Sum Problem, SSP) which is also named an independent estimation of the knapsack problem. A. F. Valeeva and E. A. Muhacheva used the SSP problem as a part of the dynamic sorting-out algorithms for solving linear and rectangular packing problems [15].

Some simple heuristics for guillotine cutting problems of large dimensions have been developed in the USG. A number of such problems have been developed by A. I. Ermachenko and T. M. Sirasetdinov [16]. They used the recursive method with implicit sorting-out of card fragments where the full card is built only once. This method insures good results for solving problems with minor package but with a great number of different rectangles.

Among the simple algorithms of solving the problems of packing into a strip and sheets those

ones that serve as decoders in multipass algorithms stand separately. Their development became urgent when the relative methods of local optimum search appeared and were developed. They can calculate the objective function value and restore the packing sketch. The effective decoders with block representation of packing have been developed in the USG by A. V. Chiglintsev, A. S. Filippova [17]. On the basis of packing block coding two variants of the pair decoder scheme and reconstruction decoders were offered. The first is a simple one-pass algorithm that works sequentially with two, vertical and horizontal, block-structures of packing. The second is a more complex algorithm with returns, but it provides, as a rule, a better packing.

The effective method of sequentative value correction (Sequentative Value Correction, SVC) belongs to multipass algorithms. It originated from the idea of the objectively-conditioned values by L. V. Kantorovich [18]. The SVC method is realized according to the modified scheme “the first suitable with ordering” with priority and iteration procedures. The element ordering is based on economic sense of dual variables in LP. The method can be denoted as general for the C&P problems solving. It is applied in cases of linear and guillotine cutting, 2D and 3D packing, and it is also developed by the Ufa scientists for solving the nesting problems [19–21].

Simple and multipass heuristics well-known for solving 1D and 2D problems are mainly used to solve 3D packing problems [22]. The situation becomes complicated as one more coordinate appears and there are no exact algorithms to apply in practice, though there exist some exact algorithms for 2D packings that are generalized for 3D problems. For example V. Kartack suggested the method of connected matrix generalized for solving N-dimension packing [12]. Simple heuristics are widely applied for solving nesting problems. It had been the only way to construct admissible cutting cards for a long time. The approximation and decomposition method reducing the problem to consecutive solution of several simple problems was widely used. For example, for the case of objects close to rectangular, the following scheme is used: first, a rectangular of the minimal area covering the object is found; the rectangles are packed in the set area; the initial objects after “sorting out” the rectangular shells are condensed [23]. This method is quite simple in regard to defining the conditions of mutual non-crossing. Generally when nesting problems are being solved, the initial information representing, the movement of objects and the conditions of their non-crossing checking become important.

The discrete-logic and chain coding methods has shown to be quite promising [23].

***Probabilistic methods of an optimum local search.*** Elements of chance entering of the determined algorithm raises its productivity. So, for example, the efficiency of mentioned above the SVC and dynamic search algorithms has raised after stochastic elements entering [10, 15]. And non-determined simple heuristics used by A. R. Usmanova showed the unexpectedly good results [24]. The scholars of the USG pay much attention to studying and raising efficiency of the genetic algorithm applied for solving C&P problems [25]. Efficiency of the classical algorithm depends on the used decoder. Some original block decoder, its modifications and algorithms of decoding have been developed [26] for their application in general evolutionary schemes where genes are blocks, that is, rectangular fragments of packing. Special block technique of constructing algorithms of optimum local search takes special place among the evolutionary algorithms. U. I. Valiahmetova has developed a multi-method technology of constructing solution algorithms [27], it is based on the idea of enumerating simple heuristics as a code of packing. Other metaheuristics are also successfully developed in the USG. Methods and algorithms of solving some other applied problems of operational research have been investigated and developed, they include problems of routing, problems of production distribution, problem of geometrical covering, etc [28–30].

#### 4. THE UFA SCIENTIFIC SGROUP COMMUNICATION WITH THE SCIENTIFIC WORLD

The changes that began in Russia in the 1980th, made it possible for Russian scientists, including those from Ufa, to actively publish their papers abroad in the editions, devoted to operational research, and to participate in the international conferences. Besides there appeared an opportunity to cooperate with German experts in the field of C&P, namely with the professors J. Terno, G. Scheithauer from Institute of Numerical Mathematics Dresden University of Technology. Students, post-graduate students, and teachers of USATU have been actively cooperating with the Dresden colleagues. The cooperation includes scientific training, participation in joint seminars and conferences. The exchange of experience, undoubtedly, allows developing and investigating of new methods of solving NP-hard C&P problems.

The Ufa scientists actively participate in the community named SICUP (Special group on inter-

ests in the field of packing). SICUP unites many researchers interested in the given problem worldwide. And foreign researchers take part in the Russian special editions and joint researches. Thus the collection of papers of Russian and foreign scientists devoted to C&P problems was issued under E. A. Mukhacheva editorship [31].

#### 4. CONCLUSION

A wide range of problems connected with combinatorial methods of solving these difficult and urgent C&P problems are still to be investigated despite of great achievements made by scientists from Ufa under direction of Elita Alexandrovna Mukhacheva. Many scientists from the collective headed by E. A. Mukhacheva have moved to other countries and continue working in this area. Contribution to science made by E. A. Mukhacheva and her pupils renders appreciable influence on the major results in the field of solving C&P problems in the world.

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