HOW TO INCLUDE RAINBOW FORMATION TO TEACHING OF PHYSICS AT HIGH SCHOOLS USING ICT?

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Abstract

The basic topics of physics include optics. In teaching of physics at high schools we explain how to assemble and use microscope or telescope but we don't explain to our students why we can see a rainbow in the sky. The article wants to point out that the rainbow formation is an interesting and didactically relevant theme of high school physics, also with strong ties to the mathematics and computer science. We can lead students to create a dynamic model of light beam motion in a drop of water using geometric sketchbook (i.e. Cabri Geometry or GeoGebra). Based on the understanding of the model then students determine so-called rainbow function, calculate and draw its graph using a spreadsheet (i.e. MS Excel or OpenOffice.org Calc). With the use of ICT the interpretation of rainbow formation is easy to understand for students, and it also adds an unusual, yet effective use of two kinds of application software.

Key words: *high school, physics, optics, rainbow formation, geometric sketchbook, spreadsheet.*

Introduction

Teachers of physics sometimes ask if it wasn't wiser to modulate the traditional contents of the high school physics when the information and communication technologies might support and enlarge the possibilities of the traditional lectures very significantly. One of the appropriate fields of physics for that is certainly optics. Most of the textbooks come very quickly from the basic principles, such as principles of reflection and refraction, to the mirror and lens display and finally to rather sophisticated optical devices such as various types of telescopes and microscopes. Even the founders of optics were fascinated by the natural phenomenon of light dispersion in a raindrop – i.e. a rainbow. Among these founders were such figures of the 17th century physics as Isaac Newton, Christiaan Huygens, or the first significant Czech physicist Jan Marcus Marci.

Why hasn't the high school physics dealt with the rainbow formation? The construction of the rainbow function is not difficult but the study of the function process and the determination of its extreme position of the individual colour spectrum demand knowledge of higher mathematics, in concrete terms the calculation of the derivation of a function. However, if the spreadsheet is used, there is no need for examining the function analytically. Instead we calculate the sufficient number of function amount with the help of the spreadsheet and we draw curves of the rainbow function for individual colours of the spectrum as the graphs of functions. We use proper dynamic geometry to facilitate the understanding of the phenomenon by our students and to demonstrate them that when a rainbow is formed by dual refraction on the air-water boundary there exists a certain boundary angle inside a small spherical raindrop and one or two reflections (formation of a primary and secondary rainbow).

Demonstration of a geometrical principle of the phenomenon and derivation of the rainbow function

Before starting the derivation, let's review the student's knowledge of mathematics and physics. It is sufficient for students to know the principle of reflection and refraction of light. In reflection and refraction the reflected (refracted) ray remains in the plane of impact. The plane of impact is determined by two straight lines – an impact ray and a perpendicular to the optical boundary in the place of impact. Moreover for the reflection is valid that angle of reflection is equal to the angle of impact – i.e. α = α′. For refraction is valid that the proportion of the sine of angle of impact to the sine of angle of refraction is equal to the relative index of refraction, i.e.

$$\frac{\sin\alpha}{\sin\beta} = n$$

This principle is often called Snell's law by its founder, Dutch mathematician and physician Willebrord Snell van Royen. It is also necessary to know the sine function and the inverse function (arcus sine). And furthermore to know that the sum of the inner angles of triangle is always 180º, in tetragon it is 360º, in pentagon it is 540º, and that the correspondent top angles are equal.

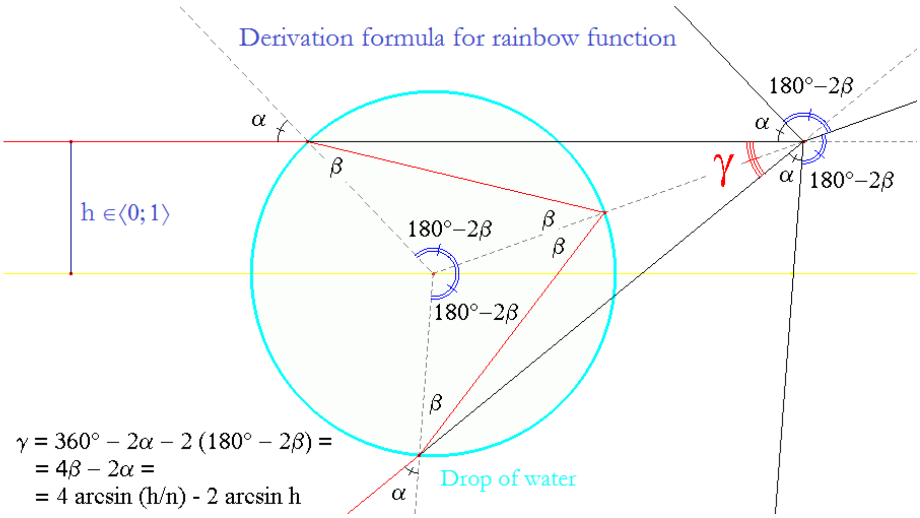


Figure 1: Derivation formula for rainbow function.

With the use of the above mentioned picture it is relatively easy to derive that for the resulting angle of refraction γ on a drop of water is valid this relation:

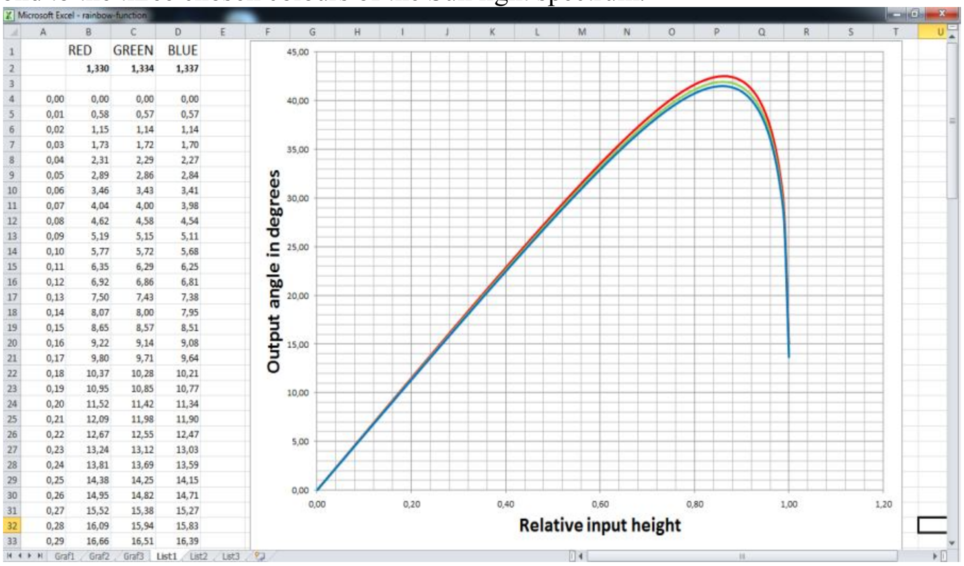
$$\gamma = 4\arcsin\left(\frac{h}{n}\right) - 2\arcsin(h),$$

where *h* is a relative distance of the dropping ray from the parallel ray crossing the centre of the drop (the real number from 0 to 1). The *n* is the relative index of refraction for the transition of the ray from air to water. Because the absolute index of air refraction is normally very close to 1 (1.00026), it is possible to substitute the absolute index of the water refraction for *n*.

Even more illustrative than a static picture is animation, which cannot be put into this article but it is possible to display it on the internet website in the Java applet image (Musílek, 2011). The picture is constructed in an appropriate environment such as Cabri Geometry II or GeoGebra. The most difficult part of the construction is application of Snell's law. It precisely models the refraction of a ray of light to perpendicular in the moment of entering water from air and vice versa, i.e. from the raindrop back to the air. However, the refraction principle is very easy to construct by axial symmetry. The geometrical model has dual usage from the didactic point of view. The completed model is shown to the students with a lower level of mathematic knowledge and physical intuition to examine its characteristics and to understand correctly the essence of the phenomenon. To more talented students it is given a task to construct the model so that they have to combine their knowledge of physics, geometry and skills of using special software – a geometry notebook.

Let's look at the rainbow function γ as to a real function of one real variable h, where index of refraction n that depends on a wave length (and also on the colour) of the dropping light is a parameter. The process of function for three different amounts of refraction index $n_R = 1.330$ (red light), $n_G = 1.334$ (green light), and $n_B = 1.337$ (blue light) can be calculated and depicted as a function type X-Y in the spreadsheet. In the spreadsheet MS Excel 2010 we prepare amounts of the independent variable h from 0 to 1 with the step of 0.01 in the column A and for this column we define a title h in the bookmark "Formula" by the function "Define title…". Columns B, C and D we name RED, GREEN and BLUE. In the second line of the chart we state the amounts of indexes of refraction and choose for them a title n. Then we can write the formula = (4*ARCSIN(*h/n*)-2*(ARCSIN(*h*))*180/PI() to the top left cell (B4). The formula figures out the amount of the function γ in radians and by multiplying it with $180/\pi$ it transfers it to the angle degrees. Then we copy the formula to the right and down to the whole area of the outputting amounts.

We use graph X-Y with the smoothed connectors and with the grid for more precise deduction of amounts. We re-colour the colours of individual lines so that they correspond to the three chosen colours of the Sun light spectrum.



	A	B	C	D
1		RED	GREEN	BLUE
2		1,330	1,334	1,337
3				
4	0,00	0,00	0,00	0,00
5	0,01	0,58	0,57	0,57
6	0,02	1,15	1,14	1,14
7	0,03	1,73	1,72	1,70
8	0,04	2,31	2,29	2,27
9	0,05	2,89	2,86	2,84
10	0,06	3,46	3,43	3,41
11	0,07	4,04	4,00	3,98
12	0,08	4,62	4,58	4,54
13	0,09	5,19	5,15	5,11
14	0,10	5,77	5,72	5,68
15	0,11	6,35	6,29	6,25
16	0,12	6,92	6,86	6,81
17	0,13	7,50	7,43	7,38
18	0,14	8,07	8,00	7,95
19	0,15	8,65	8,57	8,51
20	0,16	9,22	9,14	9,08
21	0,17	9,80	9,71	9,64
22	0,18	10,37	10,28	10,21
23	0,19	10,95	10,85	10,77
24	0,20	11,52	11,42	11,34
25	0,21	12,09	11,98	11,90
26	0,22	12,67	12,55	12,47
27	0,23	13,24	13,12	13,03
28	0,24	13,81	13,69	13,59
29	0,25	14,38	14,25	14,15
30	0,26	14,95	14,82	14,71
31	0,27	15,52	15,38	15,27
32	0,28	16,09	15,94	15,83
33	0,29	16,66	16,51	16,39

Figure 2: Calculation and display of rainbow function.

It is clear from the graph that the maximal angle of the light reflection on raindrops is 42.5º. It occurs in the red light and for the relative inputting height 0.86. In the same inputting height we get 41.9º for the green light and 41.5º for the blue light. And this is the explanation of dispersion of the solar light to the colour spectrum and the formation of the beautiful atmospheric phenomenon that is called rainbow. The places that we can see in the parallel view with the solar rays under the certain angle crate an imaginary circle. Because a rainbow can be seen only when looking at a rainy screen in the sky, it has most often shape of a circle arc. We can see the arc with the Sun behind us and the rainy screen in front of us. If it rains only in the part of the sky in front of us, we can see only a part of the arc and the rainbow is touching the horizon only in one side or it doesn't touch it anywhere. Because all the light is reflecting on drops of water under the smaller angles, or under the limit angle, the inside area under the rainbow arc is lighter than the area outside the arc.

We can examine the graph of the rainbow function with our students. The graph we created by the spreadsheet without any need of higher mathematics. The spreadsheet is a perfect device for modelling various phenomena at the level of high school physics. A very interesting article dealing with the problem of using of mechanics of a solid body is "Using the MS Excel application for calculation of the centre of gravity of a system of blocks" (Hubálovský, 2010).

Some of the students may have noticed that sometimes there can be seen two rainbow arcs in the sky. The second, which we haven't mentioned yet, can be occasionally seen outside the primary arc and it is a little thinner and its colours are in the opposite order than in the primary arc. It is called a secondary rainbow. When the secondary rainbow is formed, the light is not reflected only once but twice actually. Let's have a look at the picture:

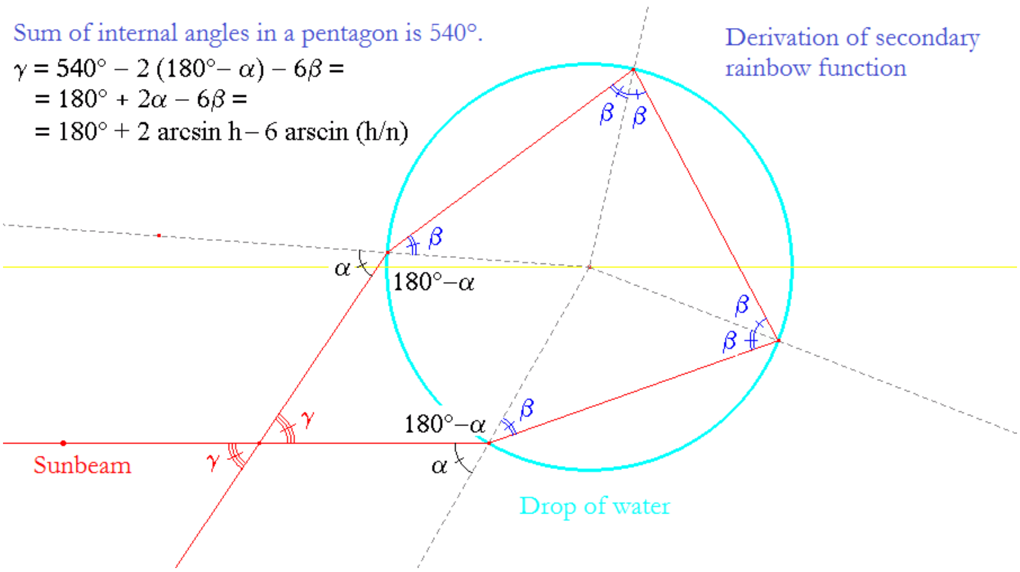


Figure 3: Derivation formula for the secondary rainbow function.

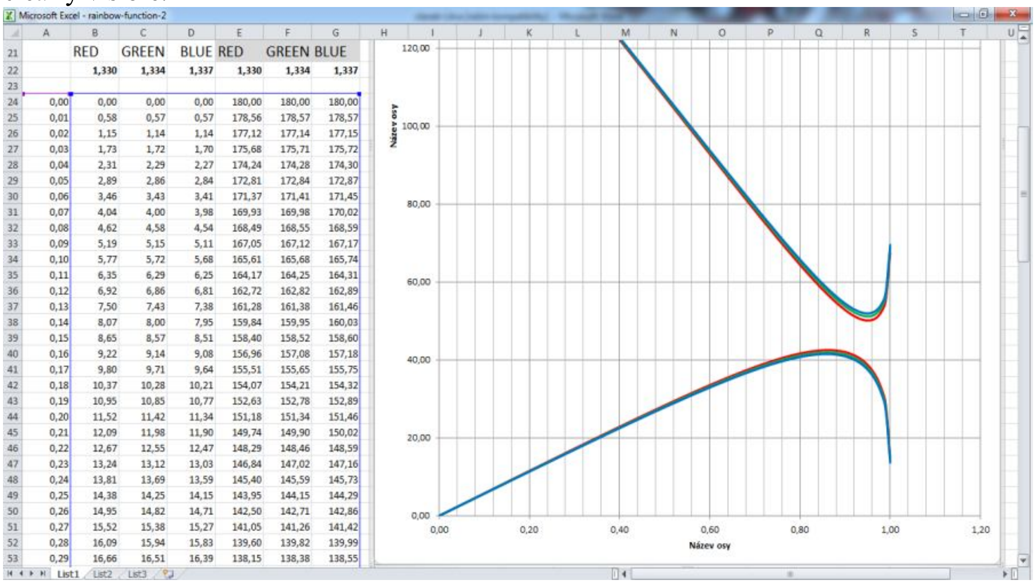
We can similarly derive a formula for a rainbow function of the secondary rainbow:

$$\gamma = 180° + 2\arcsin(h) - 6\arcsin\left(\frac{h}{n}\right).$$

The formula in the next three columns E, F, and G is: =180+(2*ARCSIN(h)-6*ARCSIN(h/n))*180/PI().

The dynamic geometrical notebook is in the explanation of the secondary rainbow formation more suitable than a static picture. Very illustrative is also a common graph of both rainbow functions (primary and secondary). We can see there the opposite order of the colours of the secondary rainbow, we can guess a size of a minimal angle to 50º and we can see that there are not any light rays reflected in the area between a maximal angle of the primary rainbow and a minimal angle of the secondary rainbow. And that is why there is formed the darker area between both rainbow arcs. The phenomenon was described for the first time by Alexander of Aphrodisias and the arc is called Alexander's arc.

When looking at a chart for the rainbow function, we can see that the extreme of the secondary rainbow function arises in the relative entering height 0.95 and reaches amount of 50.1º for red light, 51.2º for green light, and 51.9º for blue light. There has to be a good visibility because the secondary rainbow is always less bright and less contrastive than the primary rainbow. If we watch both rainbows, the dark Alexander's arc is clearly visible.



	A	B	C	D	E	F	G
21		RED	GREEN	BLUE	RED	GREEN	BLUE
22		1,330	1,334	1,337	1,330	1,334	1,337
23							
24	0,00	0,00	0,00	0,00	180,00	180,00	180,00
25	0,01	0,58	0,57	0,57	178,56	178,57	178,57
26	0,02	1,15	1,14	1,14	177,12	177,14	177,15
27	0,03	1,73	1,72	1,70	175,68	175,71	175,72
28	0,04	2,31	2,29	2,27	174,24	174,28	174,30
29	0,05	2,89	2,86	2,84	172,81	172,84	172,87
30	0,06	3,46	3,43	3,41	171,37	171,41	171,45
31	0,07	4,04	4,00	3,98	169,93	169,98	170,02
32	0,08	4,62	4,58	4,54	168,49	168,55	168,59
33	0,09	5,19	5,15	5,11	167,05	167,12	167,17
34	0,10	5,77	5,72	5,68	165,61	165,68	165,74
35	0,11	6,35	6,29	6,25	164,17	164,25	164,31
36	0,12	6,92	6,86	6,81	162,72	162,82	162,89
37	0,13	7,50	7,43	7,38	161,28	161,38	161,46
38	0,14	8,07	8,00	7,95	159,84	159,95	160,03
39	0,15	8,65	8,57	8,51	158,40	158,52	158,60
40	0,16	9,22	9,14	9,08	156,96	157,08	157,18
41	0,17	9,80	9,71	9,64	155,51	155,65	155,75
42	0,18	10,37	10,28	10,21	154,07	154,21	154,32
43	0,19	10,95	10,85	10,77	152,63	152,78	152,89
44	0,20	11,52	11,42	11,34	151,18	151,34	151,46
45	0,21	12,09	11,98	11,90	149,74	149,90	150,02
46	0,22	12,67	12,55	12,47	148,29	148,46	148,59
47	0,23	13,24	13,12	13,03	146,84	147,02	147,16
48	0,24	13,81	13,69	13,59	145,40	145,59	145,73
49	0,25	14,38	14,25	14,15	143,95	144,15	144,29
50	0,26	14,95	14,82	14,71	142,50	142,71	142,86
51	0,27	15,52	15,38	15,27	141,05	141,26	141,42
52	0,28	16,09	15,94	15,83	139,60	139,82	139,99
53	0,29	16,66	16,51	16,39	138,15	138,38	138,55

Figure 4: Calculation and display of both rainbow functions (primary and secondary).

Results and Summing-up

Unlike most of the high school textbooks the modern university textbook of general physics deals briefly with the rainbow formation (Halliday – Resnick – Walker, 1997). This textbook describes the primary rainbow formation within explanation and the secondary rainbow formation within the tasks for practicing. The principle is explained there by pictures but the rainbow function is not derived there and the amount of the limit angle of the primary rainbow 42º is stated there without any justification. In the explanation part there is a sentence which is uncommonly poetic and that interested me: "Your rainbow is only yours because another observer can see the light coming from other raindrops". It is obvious that the authors don't deal with this issue in details. The derivation of the rainbow functions is not too difficult and is more suitable for high school students. The material dedicated to a rainbow is moreover an ideal material for revision of geometry and practice of usage of software for creation of a model of an interesting physical phenomenon. The dynamic geometrical notebook helps to understand a practical value of the rainbow functions even to students with a lower level of mathematical intuition, meanwhile for talented students is a construction of dynamic model an interesting topic for individual work.

From the didactic point of view the topic of the rainbow formation can be placed among the enlarging themes of the high school physics supposing it is taught with the ICT support.

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