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Reliability analysis for components using Fuzzy Membership functions

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Abstract- This paper presents reliability analysis for components using fuzzy operations. Probability assumption and Fuzzy State assumption (PROFUST) reliability theory is used to find out the reliability of each system component. Here, the reliability of each system component is presented by trapezoidal fuzzy number. The proposed method is used to simplify fuzzy arithmetic operations of fuzzy numbers. Finally, a numerical example is illustrated to verify the efficiency of used operations in the functions.
Keywords- Reliability, Fuzzy numbers, Trapezoidal membership function, Functions of fuzzy numbers

Introduction

Reliable engineering is one of the important engineering tasks in design and development of a technical system. In the last 30 years, much effort has been made in the design and development of reliable large-scale systems for space science, Military applications, and power distribution [8]. In the real world problems, the collected data or system parameters are often imprecise, because of incomplete or non-obtainable information. The probabilistic approach to the conventional reliability analysis is inadequate to account for such built-in uncertainties in data. For this reason the concept of fuzzy reliability have been introduced and formulated as a transition from fuzzy success state to fuzzy failure state [3].The reliability of system is defined as the probability that the system performs its assigned function properly during a predefined period [12]. Most of the research in classical reliability theory is based on binary state assumption for states. For multi-component system with parallel redundancy, graceful degradation describes a smooth change to lower performance level of the system as individual component fail [6]. For such systems, it is therefore unrealistic to assume that the system possess only two stages, i.e., "Working" or "Failed". Such systems may be considered working to a certain degree at different states of its performance degradation during its transition from fully working state to completely failed state. The degree may be any real number '0' (to indicate the system is in failed state) and '1' (to indicate the system is in working state). The new concept of fuzzy sets in reliability analysis is suggested in [10]. Fuzzy sets can express the gradual transition of the system from a working state to a failed state. The crisp set theory [11] only discuss the system in to a working state and failed state while fuzzy set theory can handle all possible states between fully working state and completely failed state. This approach to the reliability theory is known as PROFUST reliability theory [6]. In PROFUST reliability theory the binary state assumptions are replaced by fuzzy state assumptions. In this paper, we present a new method to analyze fuzzy system reliability using fuzzy number arithmetic operations. Suggested method is based on PROFUST reliability theory, where the reliability of each system component is presented by trapezoidal fuzzy number. The proposed method uses simplified fuzzy arithmetic operations of fuzzy numbers.

Fuzzy arithmetic operations for reliability analysis

We briefly review some basic definitions of fuzzy sets from [3, 4]. Let U be the universe of discourse, $U = \{u_1, u_2, u_3, \ldots, u_n\}$ .Let set $\tilde{R}$ of U is a set of ordered pairs $\{(u_1, f_{\tilde{R}}(u_1)), (u_2, f_{\tilde{R}}(u_2)) \ldots (u_n, f_{\tilde{R}}(u_n))\}$ where $f_{\tilde{R}}, f_{\tilde{R}} : U \to [0,1]$ is the membership function of U_i in $\tilde{R}$, and $f_{\tilde{R}}(u_i)$ indicates the grade of membership U_i in $\tilde{R}$. To formulate the fuzzy numbers or parameters, we can use either membership functions or possibility distributions [4, 9]. In this work we use the trapezoidal membership function. The membership function curve and characteristic of the trapezoidal fuzzy number $\tilde{R}_i = (\alpha_i, \beta_i, \gamma_i, \delta_i)$, $1 \le i \le n$. are expressed in Fig. (1) and eqn. (1) Where α_i and δ_i are called the left and right spreads of the curve, respectively. $f_{\tilde{R}}(u_i)$

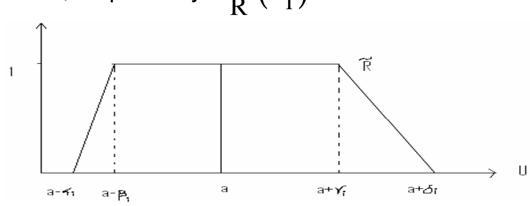

Fig. 1- Trapezoidal membership function

The fuzzy trapezoidal distribution is:

$$\mu_{\tilde{R}}(U) = \begin{cases} 0 & U \le a - \alpha_i \\ \alpha_i + U - a/\alpha_i - \beta_i & a - \alpha_i < U \le a - \beta_i \\ 1 & a - \beta_i \le U < a + \gamma_i, \text{ where } 0 < \beta_i < \gamma_i \text{ and } a \in \square \\ \delta_i - U + a/\delta_i - \gamma_i & a + \gamma_i \le U \le a + \delta_i \\ 0 & a + \delta_i < U \end{cases} \quad (1)$$

Definitions

Let $\tilde{A}$ and $\tilde{B}$ be two fuzzy numbers of the universe of discourse U with the membership functions $f_{\tilde{A}}$ and $f_{\tilde{B}}$, respectively, where $f_{\tilde{A}} : U \rightarrow [0,1]$ and $f_{\tilde{B}} : U \rightarrow [0,1]$. Let X and Y be two real numbers in U, according to [3] the fuzzy number arithmetic operations are:

- Fuzzy number addition:

$$f_{\tilde{A}+\tilde{B}}(Z) = \vee_{Z=x+y}\left[f_{\tilde{A}}(x) \wedge f_{\tilde{A}}(y)\right]$$

- Fuzzy number subtraction:

$$f_{\tilde{A}-\tilde{B}}(Z) = \vee_{Z=x+y}\left[f_{\tilde{A}}(x) \wedge f_{\tilde{A}}(y)\right]$$

- Fuzzy number multiplication:

$$f_{\tilde{A}*\tilde{B}}(Z) = \vee_{Z=x+y}\left[f_{\tilde{A}}(x) \wedge f_{\tilde{A}}(y)\right]$$

- Fuzzy number division:

$$f_{\tilde{A}/\tilde{B}}(Z) = \vee_{Z=x+y}\left[f_{\tilde{A}}(x) \wedge f_{\tilde{A}}(y)\right]$$

Let $\tilde{A}$ and $\tilde{B}$ be two trapezoidal fuzzy numbers parameterized by (a_1, b_1, c_1, d_1) and (a_2, b_2, c_2, d_2), respectively, where,

1. $a_1 \le a_2$, $b_1 \le b_2$, $c_1 \le c_2$ and $d_1 \le d_2$
2. $d_1/d_2 \ge a_2/a_1$

According to [11], the fuzzy arithmetic operations of trapezoidal fuzzy number $\tilde{A}$ and $\tilde{B}$ can be expressed as follows.

- Fuzzy number addition:

$$(a_1,b_1,c_1,d_1)+(a_2,b_2,c_2,d_2)=(a_1+a_2,b_1+b_2,c_1+c_2,d_1+d_2)$$

- Fuzzy number subtraction

$$(a_1,b_1,c_1,d_1)-(a_2,b_2,c_2,d_2)=(a_1-a_2,b_1-b_2,c_1-c_2,d_1-d_2)$$

- Fuzzy number multiplication:

$$(a_1,b_1,c_1,d_1)*(a_2,b_2,c_2,d_2)=(a_1*a_2,b_1*b_2,c_1*c_2,d_1*d_2)$$

Fuzzy number division:

$$(a_1,b_1,c_1,d_1)/(a_2,b_2,c_2,d_2)=(a_1/a_2,b_1/b_2,c_1/c_2,d_1/d_2)$$

The results of the above fuzzy number operations are shown in fig (2).

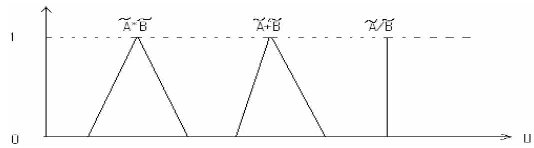

Fig. 2- Fuzzy number operations

The methodology for fuzzy system reliability analysis

In this section, we present the technique for analyzing system reliability using fuzzy arithmetic operations. The serial system and parallel system are shown in fig. (3) and fig. (4).

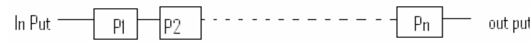

Fig. 3- Configuration of a Serial system

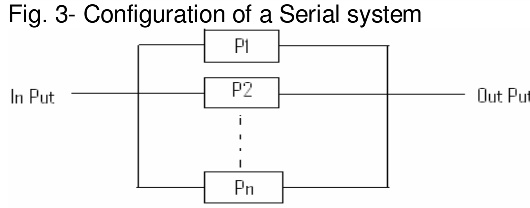

Fig. 4- Configuration of a Parallel system

Where P_i 's are subsystems of equipment. The subsystems P_i 's are represented by the trapezoidal fuzzy number. Suppose that the event model contains logical operators "OR" and "AND" then, the reliability function of the system can be obtained replacing the logical operators by the algebraic addition and multiplication. From the Boolean event of the system, a probabilistic model of the structure is obtained with the difference that the logical OR and AND operators. Creating the nodes of the model, are replaced by the appropriate probability operators like fig. (5).

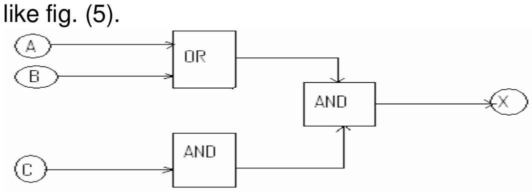

Fig. 5- The probabilistic model of system structure

The probability function of AND and OR operators are:

$$P_X^{AND} = \prod_{i=1}^{n} R_i \quad (2)$$

$$P_X^{OR} = 1-(1-R_1)(1-R_2)\ldots\ldots(1-R_n) = 1-\prod_{i=1}^{n}(1-R_i) \quad (3)$$

Where P_i 's are the probabilities of input events, P_X those of out put events

- Fuzzy operators of reliability analysis-

The membership function of the fuzzy AND & OR operators can be obtained, considering the variables in equation (2) and (3) as fuzzy variables and substituting the algebraic operations given in section 2.

The fuzzy form of AND operator function is:

$$\tilde{P}_X^{AND} = \prod_{i=1}^{n} \tilde{R}_i = AND\left(\tilde{R}_1, \tilde{R}_2, \ldots, \tilde{R}_n\right) \quad (4)$$

and the fuzzy form of OR operator function is:

$$\tilde{P}_X^{OR} = 1 - \prod_{i=1}^{n}\left(1 - \tilde{R}_i\right) = OR\left(\tilde{R}_1, \tilde{R}_2, \ldots, \tilde{R}_n\right) \quad (5)$$

The reliability of the serial system shown in fig. (3) Can be evaluated and is equal to:

$$\tilde{R}_1 \otimes \tilde{R}_2 \otimes \ldots \otimes \tilde{R}_n = \left(\alpha_1,\beta_1,\gamma_1,\delta_1\right) \otimes \left(\alpha_2,\beta_2,\gamma_2,\delta_2\right) \otimes \ldots \otimes \left(\alpha_n,\beta_n,\gamma_n,\delta_n\right) \quad (6)$$
$$= \left[\prod_{i=1}^{n}(\alpha_i), \prod_{i=1}^{n}(\beta_i), \prod_{i=1}^{n}(\gamma_i), \prod_{i=1}^{n}(\delta_i)\right]$$

Furthermore, consider the parallel system shown in fig. (4), where the reliability of the subsystem P_i is represented by the fuzzy number $\tilde{R}_i$ shown in fig. (1). The reliability of parallel system can be evaluated and is equal to:

$$1 - \prod_{i=1}^{n}\left(1 - \tilde{R}_i\right) = 1 - \prod_{i=1}^{n}\left[1 - \left(\alpha_i,\beta_i,\gamma_i,\delta_i\right)\right]$$
$$= 1 - \Big\{\left[\left(1-\alpha_1\right),\left(1-\beta_1\right),\left(1-\gamma_1\right)\left(1-\delta_1\right)\right] \otimes \quad (7)$$
$$\left[\left(1-\alpha_2\right),\left(1-\beta_2\right),\left(1-\gamma_2\right)\left(1-\delta_2\right)\right] \otimes$$
$$\ldots\ldots$$
$$\left[\left(1-\alpha_1\right),\left(1-\beta_2\right),\left(1-\gamma_3\right)\left(1-\delta_4\right)\right]\Big\}$$
$$= \left(1,1,1,1\right) - \left(\prod_{i=1}^{n}\left[1-\left(\alpha_i\right)\right], \prod_{i=1}^{n}\left[1-\left(\beta_i\right)\right], \prod_{i=1}^{n}\left[1-\left(\gamma_i\right)\right], \prod_{i=1}^{n}\left[1-\left(\delta_i\right)\right],\right)$$
$$= \left(1 - \prod_{i=1}^{n}\left[1-\left(\alpha_i\right)\right], 1 - \prod_{i=1}^{n}\left[1-\left(\beta_i\right)\right], 1 - \prod_{i=1}^{n}\left[1-\left(\gamma_i\right)\right],\right) 1 - \prod_{i=1}^{n}\left[1-\left(\delta_i\right)\right]$$

A technical example

Two grinding machines are working next to each other. What is the possibility that people coming in the vicinity of the machines are injured mainly by getting a chip into the eye? It is obvious that the most endangered persons are the operators, who are obliged to wear safety glasses but often fail to do this. Furthermore, endangered are persons coming in the vicinity of the machines, bringing and carrying away items, and those entering the area for other reasons. The fault tree for the main event that somebody will be injured can be constructed as shown in fig. (6).

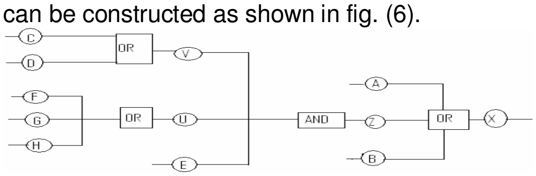

Fig. 6- Fault tree of the example

The basic events contributing to the accident are summarized in table (1) (Data sources [7]).

Table 1-The basic events contributing to the accident

Symbol	Basic events
A	Operator 1 fails to wear safety glasses.
B	Operator 2 fails to wear safety glasses.
C	Machine 1 is operating.
D	Machine 2 is operating.
E	Persons entering the area without safety glasses.
F	Persons entering the endangered area bringing material.
G	Persons entering area carrying away made product.
H	Persons entering the area for other reasons.

Assume that the basic events are mutually independent and reliability of the basic events is represented by trapezoidal fuzzy numbers parameterized by $\left(\alpha_i, \beta_i, \gamma_i, \delta_i\right)$. Then we can see that

$$\tilde{R}_A = \left(0.00888, 0.01444, 0.02, 0.0255\right)$$
$$\tilde{R}_B = \left(0.00888, 0.01444, 0.02, 0.0255\right)$$
$$\tilde{R}_C = \left(0.75552, 0.77776, 0.8, 0.82224\right)$$
$$\tilde{R}_D = \left(0.75552, 0.77776, 0.8, 0.82224\right)$$
$$\tilde{R}_E = \left(0.94434, 0.97217, 1.0, 1.02783\right)$$
$$\tilde{R}_F = \left(0.04722, 0.04861, 0.05, 0.05139\right)$$
$$\tilde{R}_G = \left(0.04722, 0.04861, 0.05, 0.05139\right)$$
$$\tilde{R}_H = \left(0.0944, 0.00972, .000974, 0.009776\right)$$

From fig. (6), the truth function of the main event X can be written as follows:

U=F+G+H

V=C+D

Z=E+U+V

X=A+B+Z

From equation (6) and (7), we can get

$$\tilde{R}_U = 1 - \left(1 - \tilde{R}_F\right) \otimes \left(1 - \tilde{R}_G\right) \otimes \left(1 - \tilde{R}_H\right)$$
$$= 1 - \left[0.8220635, 0.89344942, 0.89370243, 0.89063891\right]$$
$$= \left[0.10936109, 0.10629757, 0.10365505, 0.1779365\right]$$
$$\tilde{R}_v = 1 - \left(1 - \tilde{R}_C\right) \otimes \left(1 - \tilde{R}_D\right)$$
$$= 1 - \left[0.05977, 0.0493906, 0.04, 0.031598617\right]$$
$$= \left[0.96840138, 0.96, 0.9506094, 0.94023\right]$$
$$\tilde{R}_Z = \tilde{R}_E \otimes \tilde{R}_U \otimes \tilde{R}_V$$
$$= \left[0.100010, 0.099205736, 0.0985354, 0.0169438\right]$$
$$\tilde{R}_X = 1 - \left(1 - \tilde{R}_A\right) \otimes \left(1 - \tilde{R}_B\right) \otimes \left(1 - \tilde{R}_c\right)$$
$$= 1 - \left[0.8840771, 0.8749671, 0.8657665, 0.5447422\right]$$
$$= \left[0.115922, 0.125038, 0.1342334, 0.45525\right]$$

It is obvious that the above results coincide with the once presented in [7], [8]. However, from the above procedure, we can see that the proposed method has the advantage of low computation complexity compared to [7, 8].

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Conclusion

Reliability of each system component involves uncertainty in the outcome. This kind of uncertainty is referred as fuzziness. While the probability theory characterizes randomness, fuzzy set theory deals with fuzziness. PROFUST reliability theory provides more realistic estimates than PROBIST reliability theory. In this paper, we have developed a new method to analyze fuzzy system reliability using fuzzy number arithmetic operations and fuzzy logic operators. The reliability of each system component is represented by a trapezoidal fuzzy number. The proposed method uses simplified fuzzy arithmetic operations of fuzzy numbers rather than complicated interval arithmetic operations of fuzzy numbers. Execution of the developed method is faster than the one presented in the above literature.

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