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Estimation of Bouguer Density Precision: Development of Method for Analysis of La Soufriere Volcano Gravity Data

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Abstract

The precision of topographic density (Bouguer density) estimation by the Nettleton approach is based on a minimum correlation of Bouguer gravity anomaly and topography. The other method, the Parasnis approach, is based on a minimum correlation of Bouguer gravity anomaly and Bouguer correction. The precision of Bouguer density estimates was investigated by both methods on simple 2D syntetic models and under an assumption free-air anomaly consisting of an effect of topography, an effect of intracrustal, and an isostatic compensation. Based on simulation results, Bouguer density estimates were then investigated for a gravity survey of 2005 on La Soufriere Volcano-Guadeloupe area (Antilles Islands). The Bouguer density based on the Parasnis approach is 2.71 g/cm^3 for the whole area, except the edifice area where average topography density estimates are 2.21 g/cm^3 where Bouguer density estimates from previous gravity survey of 1975 are 2.67 g/cm^3. The Bouguer density in La Soufriere Volcano was uncertainly estimated to be 0.1 g/cm^3. For the studied area, the density deduced from refraction seismic data is coherent with the recent Bouguer density estimates. New Bouguer anomaly map based on these Bouguer density values allows to a better geological intepretation.

Keywords: Bouguer density, free-air anomaly, Bouguer anomaly

Sari

Perhitungan ketelitian estimasi rapat massa Bouguer akan dikembangkan berdasarkan metode estimasi rapat massa cara Nettleton, yaitu dengan meminimumkan korelasi antara anomali Bouguer dan topografi. Metode lain, yaitu metode Parasnis, berprinsip pada peminimuman korelasi antara anomali Bouguer dan koreksi Bouguer. Perhitungan ketelitian rapat massa didasarkan pada hasil perhitungan efek gravitasi kasus model sintetik sederhana dua dimensi. Perhitungan ketelitian rapat massa didasarkan juga pada asumsi jika free-air anomaly disebabkan oleh gabungan efek topografi, ketidakhomogenan kerak bumi, dan isostasi. Berdasarkan hasil simulasi estimasi, ketelitian rapat massa selanjutnya diselidiki untuk penentuan rapat massa Bouguer di daerah Gunung Api La Soufriere-Guadeloupe (Kepulauan Antille) dari hasil survei gaya berat tahun 2005. Hasilnya, ditemukan harga rapat massa Bouguer 2,71 g/cm^3 untuk keseluruhan daerah survei kecuali di daerah tubuh Gunung La Soufriere yang harga rata-rata rapat massa topografinya sebesar 2,21 g/cm^3. Hasil ini berbeda dengan hasil perhitungan rapat massa Bouguer sebelumnya, hasil survei tahun 1975, yaitu sebesar 2,67 g/cm^3. Diperkirakan ketelitian rapat massa Bouguer di daerah Gunung Api La Soufriere sebesar 0,1 g/cm^3. Rapat massa untuk daerah survei Gunung Api La Soufriere dan sekitarnya yang dideduksi dari data seismik refraksi cocok dengan hasil estimasi rapat massa Bouguer. Dengan peta anomali Bouguer baru tersebut hasilnya dapat memberikan interpretasi geologi relatif lebih baik.

***Kata kunci**: rapat massa Bouguer, free-air anomaly, anomali Bouguer*

Introduction

New higher resolution gravity survey at La Soufriere volcano area in Guadeloupe (Antilles Islands) (Gunawan, 2005) in comparison to previous gravity survey (Coron *et al.*, 1975) provides a possibility to obtain more precise terrain density estimates, and thus may improve volcanological and geological intepretation. It implicates that Bouguer anomaly map at La Soufriere Volcano edifice area could provide different geologic intepretations. In general condition, a previous land gravity survey delivered 2.67 g/cm^3 value as an appropriate density Bouguer (Coron *et al.*, 1975), that considerably differs from the two average Bouguer density values, 2.21 g/cm^3 and 2.71 g/cm^3, estimated for the recent high resolution gravity survey (Gunawan, 2005). In fact, the complete coverage of gravity survey was carried out by oceanographic surveys AGUADOMAR (Deplus *et al.*, 2001). Other geophysical studies, seismic refraction and aeromagnetic survey provide some constraints for the study result to determine a superficial crust model beneath La Soufriere Volcano edifice. An interesting result of the one from seismic refraction survey, Dorel (1978) showsed that the average seismic velocity of P wave is 3.0 km/s except beneath La Soufriere Volcano edifice, 2.7 km/s. However, the low seismic velocity in this area shows a positive magnetic anomaly. This positive magnetic anomaly represents several older andesitic dykes or volcanic rocks (Late Pliocene) (Le Mouël *et al.*, 1979). Further result, the study of volcanic complexes in Guadeloupe (Feuillet, 2000; and Feuillet *et al.*, 2002), puts those geophysical surveyproducts in a seismotectonic regional context.

The field survey result stimulates the development of analysis method based on Bouguer anomaly equation in finding Bouguer density precision with the Nettleton and Parasnis approaches. By definition, a Bouguer anomaly at a given position is a different value between a measured and theoretical gravity and corrected by a Bouguer correction. In practise, a free-air anomaly is a Bouguer anomaly without a Bouguer correction. The analysis principal of a Bouguer density estimation is by introducing a free-air anomaly as a combination gravity effect of crustal inhomogeneities, topographic effect, and isostatic compensation.

Background Problem

La Soufriere Volcano, 1467 m high and 16°02’ N and 61°04’ W in position, is located in Basse Terre region on Southern Guadeloupe Island (Figures 1 and 2) which is a part of the Lesser Antilles arc (situated at overriding Caribbean Plate above subducting North American Plate). La Soufriere Volcano is an active one, characterized by several phreatic and magmatic eruptions in the past. The last phreatic eruptions occured in 1976-1977 and the last seismic crisis activity occured in 1992. To mitigate volcanic hazards, Institut de Physique du Globe de Paris (IPGP) conducts continuous volcanological monitoring and geological and geophysical researches in the La Soufriere Volcano area. For their case, the gravity study result would be used as a geophysical constraint on a geological intepretation.

Results of previous gravity study are shown on Figure 3. This map was made based on a gravity survey from Coron *et al*. (1975) and performed using Worden gravitymeter. Coron *et al*. (1975) assumed that a topographic density 2.67 g/cm^3 was an appropriate Bouguer density for the whole Guadeloupe area.

Bouguer anomaly map on Figure 3 does not show any significant Bouguer anomaly in La Soufriere Volcano area, which however does not prove correctness of Bouguer density estimates. When new gravity dataset was used for the studied area (the Parasnis approach was used for Bouguer density calculation), Parasnis curve on Figure 4 shows two lines having different slopes. Curve lines are generated based on a Bouguer anomaly (ordinate) vs Bouguer correction (absis). On this curve, the steeper slope corresponds to 2.71 g/cm^3 of Bouguer density value and the gentler slope is equal to 2.21 g/cm^3.

The final Bouguer anomaly map in Guadeloupe (Figures 5 and 6) is obtained by choosing a Bouguer density 2.71 but with ‘Bouguer density 2.21 correction’ for around volcano edifice area. The Bouguer anomaly map shows a regional anomaly stretching NW at west of La Soufriere Volcano. A similar trend is also seen in Bouguer map made by Coron *et al*. (1975) (Figure 3). There is a contrast situation when the complete Bouguer anomaly map at La Soufriere Volcano edifice (Figure 6) is compared with the previous Bouguer anomaly map on Figure 3, especially around the volcano edifice area. It is

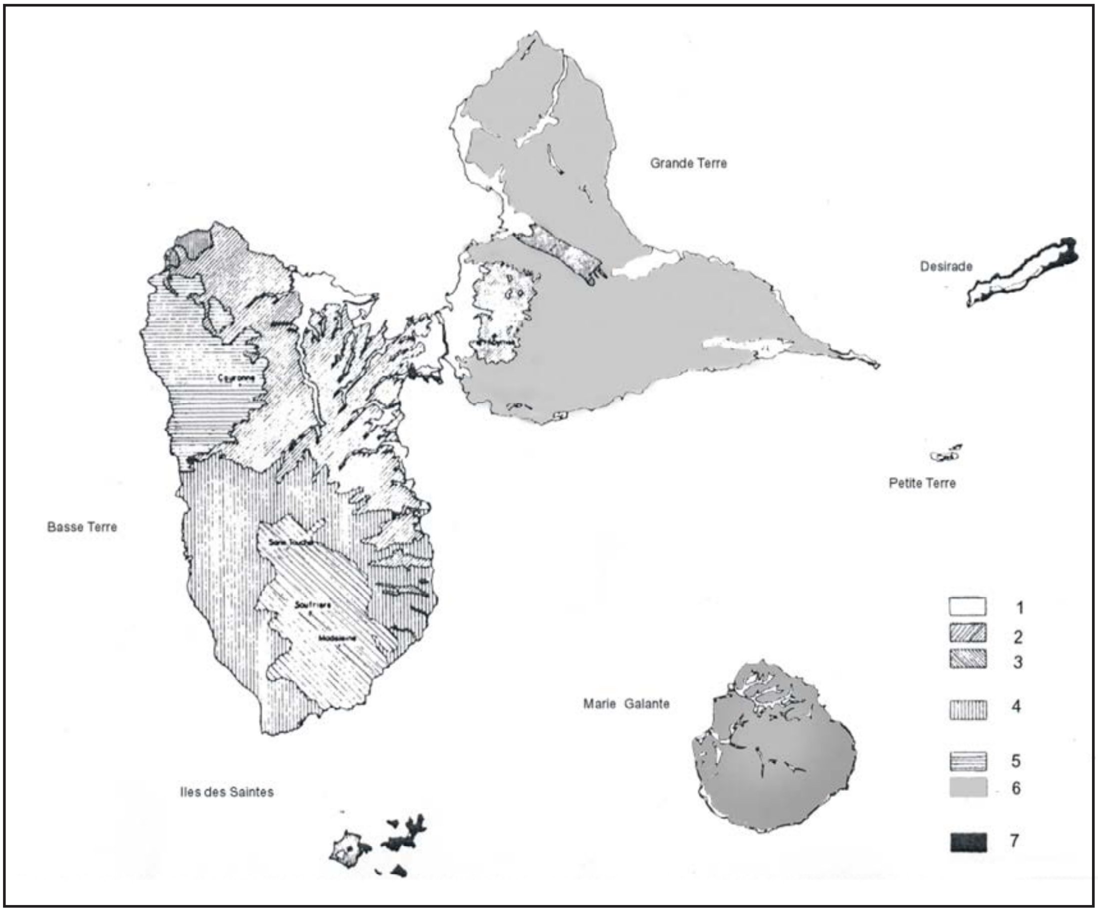

Figure 1. Simple geologic map of Guadeloupe (Quaternary 1: Alluvials, 2: Laterite, 3: Volcanic rock; Pliocene 4: Volcanic rock; Miocene 5: Volcanic rock, 6: Transgressive sediment layers; Pre Miocene 7: Volcanic and intrusive rocks (After Fink, in Gunawan, 2005).

worth noting that NW striking anomaly at west of La Soufriere Volcano has the same strikes as structure of La Soufriere Volcano edifice. In this article, the concern restricts to the analysis of methods in finding precision of Bouguer density estimates. With such analysis result, the recent Bouguer density estimates would be verified.

Method

By definition, a Bouguer anomaly at a given position, is the different value between a measured and theoretical gravities and corrected by a Bouguer correction (eq. 1).

$$\Delta g_{boug} = \Delta g_{mes} - \Delta g_{theor} + 0.3086 z_{mes} - (0.0419 z_{top} - C_{ter})\rho_{top} \quad (1)$$

where Δg_{mes}, Δg_{theor}, Δg_{boug} are measured, theoretical (normal) and Bouguer gravity anomalies respectively; z_{mes} and z_{top} are the height of the gravity stations and topography both above the geoid (when measuring at the earth surface $z_{mes} = z_{top}$); ρ_{top} is the density of the topography (supposed to be uniform) and C_{ter} is the terrain correction. Further, notations are used: free-air anomaly $\Delta g_{fa} = \Delta g_{mes} - \Delta g_{theor} + 0.3086 z_{mes}$ and Bouguer correction with unit density $\Delta g_{topo} = (0.0419 z_{top} - C_{ter})$. Cartesian coordinates are used with *Oz* axis directed downward. The analysis principal for a Bouguer density estimation is by introducing a free-air anomaly as a combination gravity effect of crustal inhomogeneities, topographic effect and isostatic compensation. In finding of ρ_{top} or Bouguer density, the Parasnis approach is equivalent in solving a linear regression between Bouguer gravity

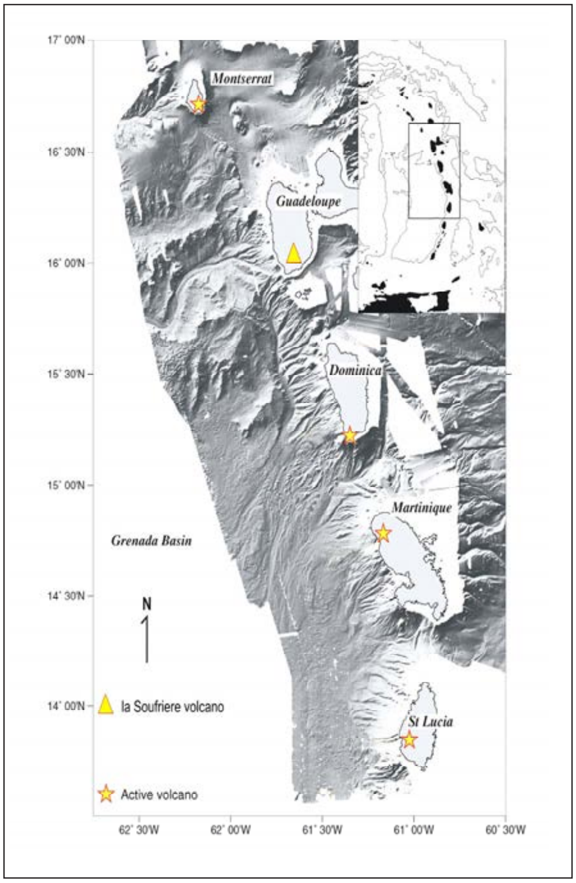

Figure 2. Guadeloupe Island and Antilles Arc map (AGUA-DOMAR survey; after Deplus, personal communication).

anomaly Δg_{boug} and Bouguer correction (see also *e.g.* Telford *et al*. (1976, p.30):

$$\sum_i (\Delta\bar{g}_{fa,i} - \rho_{top}\Delta\bar{g}_{top,i})\Delta\bar{g}_{top,i} = 0 \qquad (2)$$

$$\rho_{top} = \sum_i \Delta\bar{g}_{fa,i}\Delta\bar{g}_{top,i} \Big/ \sum_i \Delta\bar{g}^2_{top,i} \qquad (3)$$

Bouguer density estimation with the Nettleton approach is to solve linear regression between a free-air anomaly Δg_{fa} and a topography. The free-air anomaly is presented as $\Delta g_{fa} = \Delta g_{top}\,\rho_{op} + \Delta g_{geol} + \Delta g_{isost}$. Then the estimation of Bouguer (topography) density is provided by Parasnis:

$$\rho_{parasnis} = \left[\sum_i (\rho_{top}\Delta\bar{g}_{top,i} + \Delta\bar{g}_{geol,i} + \Delta\bar{g}_{isost,i})\Delta\bar{g}_{top,i}\right] \Big/ \sum_I \Delta\bar{g}^2_{top,i}$$

$$= \rho_{top} + \delta\rho_{geol} + \delta\rho_{isost} \qquad (4)$$

To investigate an error $\delta\rho$ in more details, an approximation of a gravity anomaly of a surface density is used, *i.e.* by causative sources distributed within an infinitely thin layer having topography z_{eqv}. Hereafter, to simplify, 2D problem is considered.

Gravity Effect of A Surface Density

Any gravity anomaly can be presented as a gravity effect of a surface density distributed at the surface z_{eqv} situated outside the volume containing causative sources (strictly speaking, outside the volume, containing corresponding singular points). In particular, in 2D case, z_{eqv} could be an infinite surface, that all causative sources are situated above ar below it. When a gravity anomaly consists of several components, this equivalent presentation can be applied for each component seperately using different $z_{eqv}(x)$. The only requirement is that causative sources of corresponding component were situated above or below its surface $z_{eqv}(x)$. The gravity field at the surface $z_{mes}(x)$ caused by surface density $\mu(x)$ and distributed at the surface $z_{eqv}(x)$ when $z_{mes}(x) = z_{eqv}(x)$ is (see also *e.g*. Wahr, 1996, p.189):

$$\Delta g = 2\pi G\mu(x) \qquad (5)$$

where G is Newton's gravitational constant.

Crustal Density Inhomogeneities Effect

The topographic density estimated using formula (3) also contains some additional error when z_{mes} is not constant, *e.g*. when gravity stations are situated on a rough topography. Indeed, suppose that there is a causative body in the upper crust, which gravity effect at a flat surface does not correlate with topography.

Figure 7 illustrates the effect of topography. The lower plot (A) shows the sinusoidal topography $z_{top}(x) = 500\sin(\pi x/3 + \pi/4)$ (x in km, z in m) and plot B presents the geological anomaly caused by a sinusoidal density distributed within a slab of constant thickness situated at a depth z = 0. If estimated on a flat surface, the geological signal is sinusoidal $\Delta g_{geol}(x) = A\sin(2\pi x/3)$ (shown by black line, geological anomaly was calculated at level $z_{mes,0}$ = 2000 m). The amplitude of the geological

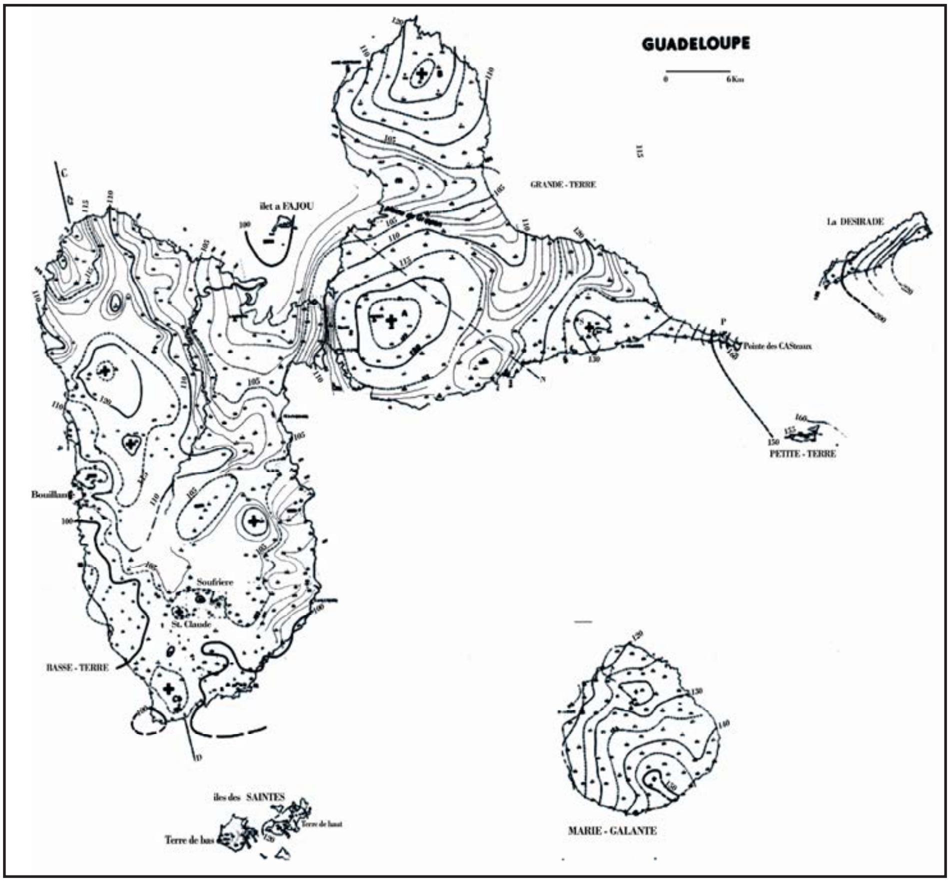

Figure 3. Bouguer anomaly map after Coron *et al*. (1975). Bouguer density is 2.67g/cc.

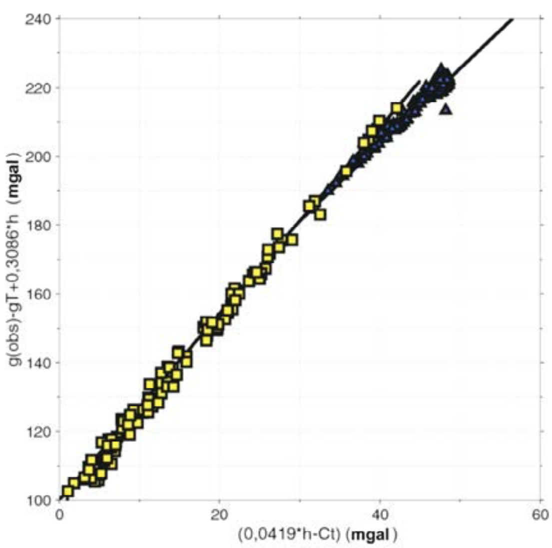

Figure 4. Bouguer density calculation in Guadeloupe region with Parasnis approach. Line with yellow square corresponds to density 2.71 g/cm^3. Line with blue triangle is for density 2.21 g/cm^3.

anomaly is A = 2.5 mgal, the period is two times less than the period of topography and the geologic anomaly is shifted relative to topography at $\pi/4$. If measured on topography, the geological anomaly has the same amplitude at the top of the uplift (because the flat level $z_{mes,0}$ coincides with the top of the uplift) and has a considerably bigger amplitude in a topographic depression (red line on plot B). The total free air anomaly (geological anomaly plus effect of the topography) calculated on a flat level (black line) and on the topography (red line) is shown on plot C.

If a studied area is relatively small, the correlation of the topography with regional gravity field could contaminate topographic density estimates. Figure 8 shows topography (plot A) and regional trends (plot C), both are sinusoidal, but the period of the trend is four times bigger than that of the topography. The free –air anomaly equal to the effect of topography

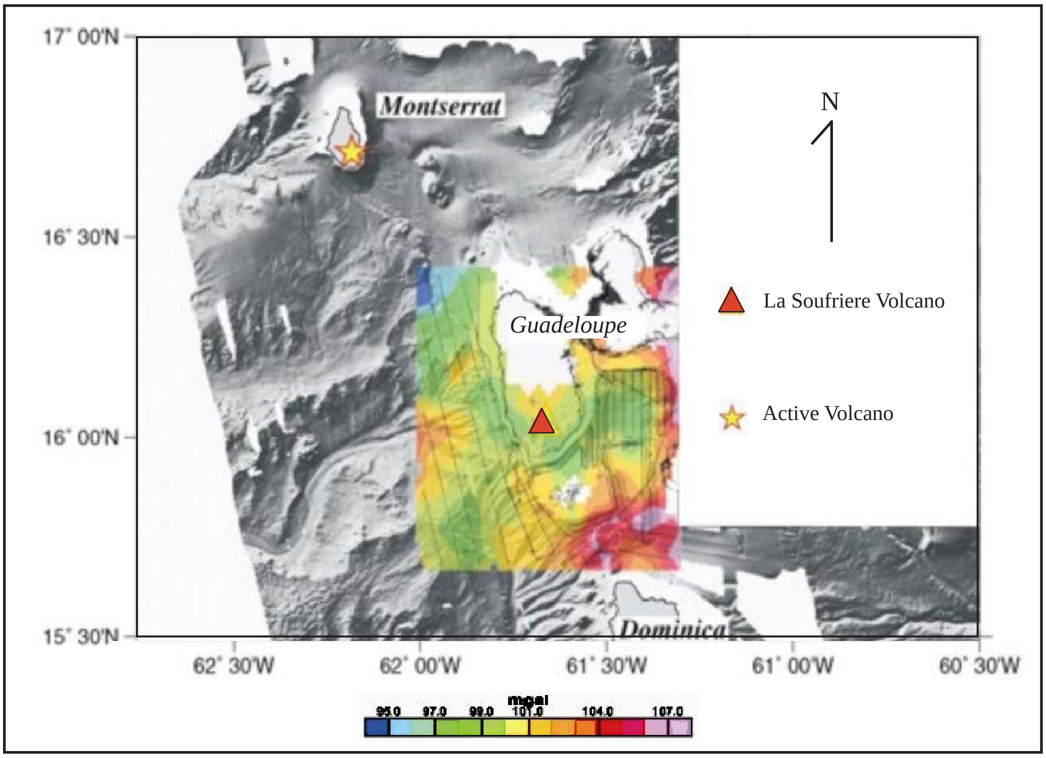

Figure 5. The complete Bouguer anomaly in Guadeloupe with density 2.71 g/cm[3].

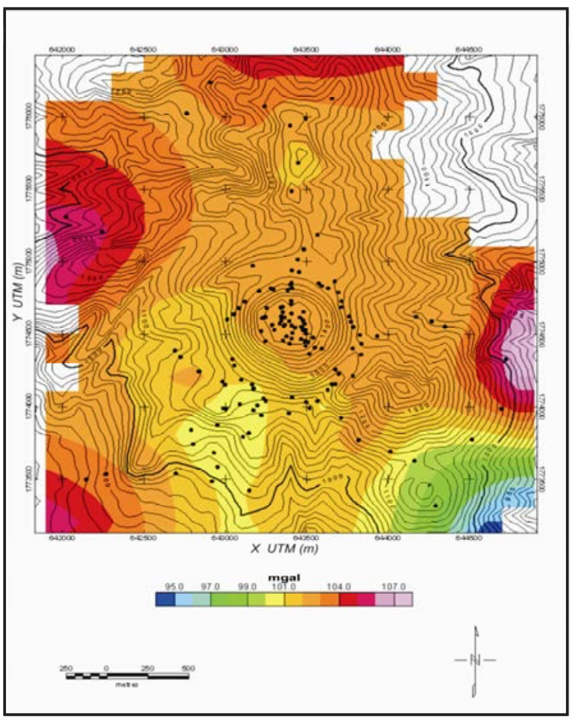

Figure 6. The complete Bouguer anomaly map around La Soufriere Volcano edifice by choosing Bouguer density 2.21 g/cm[3].

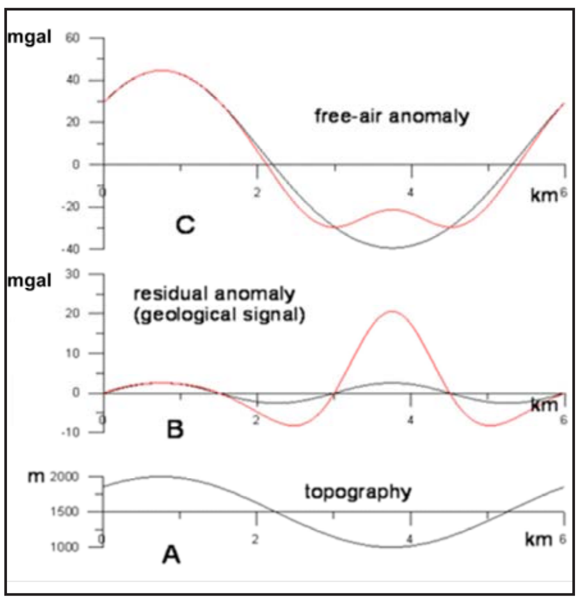

Figure 7. The effect of topography.

plus geological anomaly is shown on plot D. The correlation of free-air anomaly with the topography is shown on plot B.

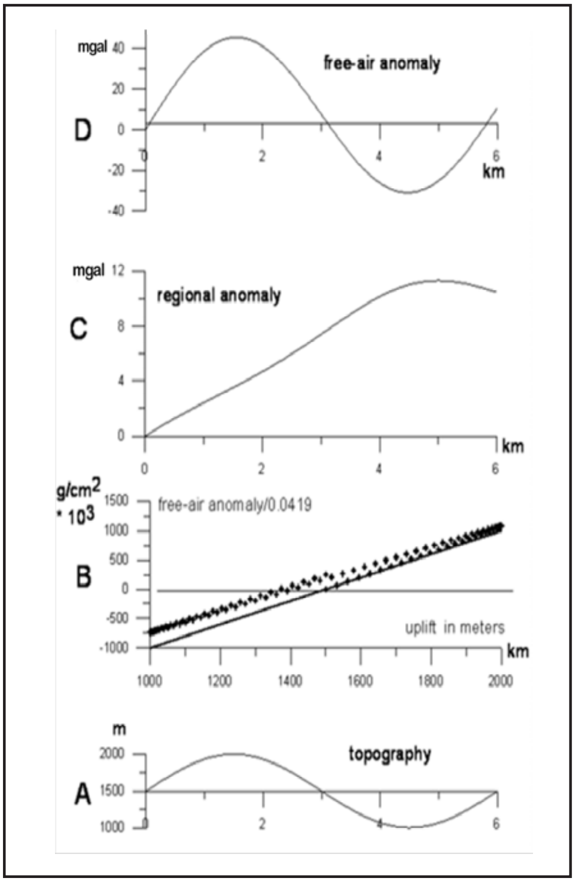

Figure 8. The effect of regional field.

Isotatic Compensation Effect

It is clear that the local isostatic compensation is the worst case, as for the local compensation correlation of Moho topography with the surface topography is maximum. The contribution of a different wavelength in topography:

$z_{top}(x) = z_0 + A\sin(\pi x/L)$; $z_{Moho}(x) = z_{M0} - A\sin(\pi x/L)\rho_{crust}/(\rho_{mantle} - \rho_{crust})$ and $\Delta g_{top}(x) = 2G\pi A\sin(\pi x/L)\exp[-\pi\{z_{top}(x) - z_{mes}(x)\}/L]$ {at a surface $z_{mes}(x)$} with $\mu_{top}(x) = A\sin(\pi x/L)$.

Using (5), the gravity effect of isostatic compensation can be calculated at the surface z_{mes}:

$$\Delta g_{comp}(x) = -2G\pi A\sin(\pi x/L)\,\rho_{crust}\exp[(-\pi\{(z_{Moho}(x) - z_{mes}(x)\}/L] \quad (6)$$

Substituting (6) into (4) we arrive an estimate of the error to a simple result is:

$$\delta\rho_{isost} = -\rho_{crust}\exp\{(-\pi(z_{M0} - z_0)\}/L \quad (7)$$

According to Equation 7, the relative error depends on the “normal” thickness of the crust and on the wavelength of the topography.

Shallow Density Inhomogeneity Effect

Figure 9 presents the effect of the sedimentary basin. Its effect has been modeled by a wire, situated at a depth at 5 km. Plot A shows the topography. Correlating the gravity effect on a flat level $z_{mes,0}$= 1 km (black line) and on the topography (red line) are shown on plot B. Even if the correlation is not strong, the Parasnis approach gives an error equal to 0.11 gr/cm3 using the data on flat level $z_{mes,0}$ = 1 km, and 0.125 gr/cm^3 using the data measured on the topography. It is worth noting, even if the anomaly due to causative body is negative, the error in the estimated topographic density is positive, because the negative causative source is situated below the topographic depression. Thus, shallow density inhomogeneities cause errors in topographic density estimates.

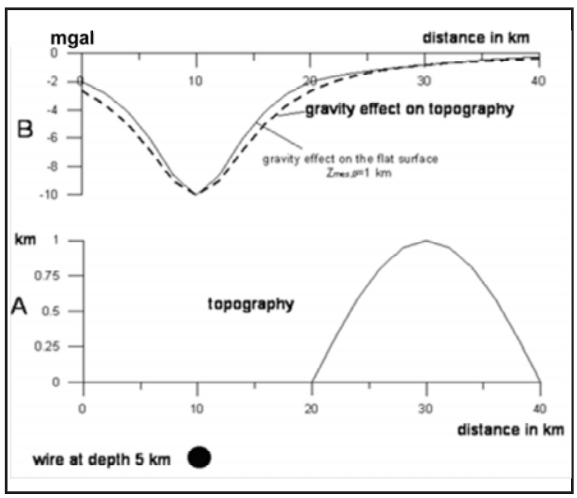

Figure 9. The effect of shallow body (sedimentary basin).

Figure 10 presents the influence of the positive density inhomogeneity (ribbon) situated below the topographic high at a depth at 5 km. The topography is shown on plot A, the geological anomaly on a flat level $z_{mes,0}$ = 1 km (black line) and on the topography (red line) are presented on plot B.

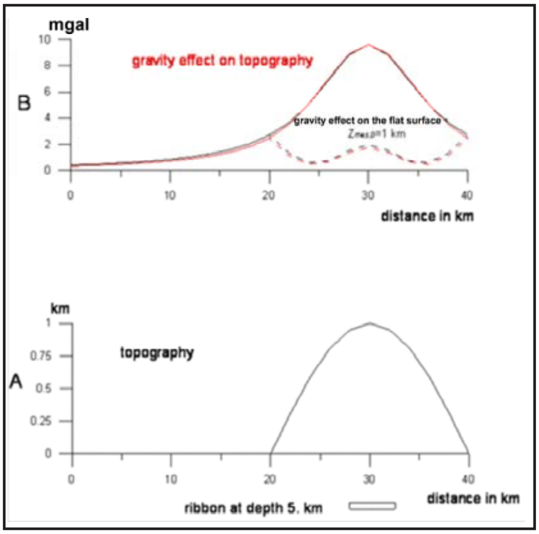

Figure 10. The effect of shallow causative body. The gravity effect of the body on a flat level (black) and on the topography (red).

Discussions

Crustal inhomogeneities case: If Figure 6 is noticed, at the flat level $z_{mes,0}$, the correlation of the topography and the geological signal is zero and the Nettleton approach provides a correct estimate of the topographic density. But by applying this approaches to the gravity anomaly calculated (measured) on the topography, a topographic density with an error equal to 0.22 g/cm^3 is obtained. The Parasnis approach does not depend so strongly on the topography, because when the measurements were made on the topography, the Bouguer correction should be calculated at the gravity stations situated on the same topography. As a result for data measured on the topography, the error of topographic density estimate using the Parasnis approach is equal to 0.07 g/cm^3. In Figure 7, thick black line shows a theoretical value, correlating to topographic density 2.0 g/cm^3, while crosses show a relation of free-air anomaly to the topography. The estimate of the topographic density in this case is shifted to 0.21 g/cm^3 using both the Nettleton and Parasnis approaches. On the contrary, when the wavelength of the geological anomaly is two or more times less than the wavelength of the topography, the error in the topographic density estimated by both the Nettleton and Parasnis approaches is very small.

Isostatic compensation case: If equation 11 is noticed and assuming ρ_{crust} is equal to ρ_{top}, the relative error depends on the "normal" thickness of the crust and the wavelength of the topography. When L = 100 km and H = 40 km the relative error is 0.28 g/cm^3, *i.e*. the estimated density of the topography is 28% less than the "real" one. When L=20 km, the relative error is 0.002 g/cm^3, *i.e*. only 0.2% less. Thus, this error is important for a relatively long wavelength topography (L > H), for which isostatic compensation is always close to the local one (see Turcotte and Schubert, 2002).

Shallow inhomogeneities case: If Figure 9 is noticed, shallow density inhomogeneities cause errors in topographic density estimates. The Parasnis approach gives an error 0.18 g/cm^3 using the data on a flat level $z_{mes,0}$ = 1 km and 0.19 g/cm^3 data measured on the topography. Dashed lines on plot B present residual anomalies on the flat level (black) and on the topography (red). The wavelength of both residual anomalies is small in comparison to the topography and the geological anomaly. These anomalies could be erroneously interpreted as shallow bodies, when a real geological anomaly was caused by one deep body. Short wavelength residual anomalies obtained after the application of the Parasnis approach could be artifacts. Thus, the absence of correlation between residual anomalies and Bouguer correction can not prove a correctness of topographic density estimates.

Conclusions

For the Parasnis approach, the influence of a topography effect, *e.g*. suppose that there is a causative body in the upper crust within a slab of the constant thickness and the gravity anomaly calculated on the topography, gives the error of Bouguer density to 0.07 g/cm^3. Nevertheless, for the Nettleton approach topographic (Bouguer) density estimates will be contaminated with an error equal to 0.22 g/cm^3. It is a reason why the first approach is used in this study. However, when there are deep causative sources, the errors in Bouguer density estimates could reach 0.2 g/cm^3. For La Soufriere Volcano case no regional trend is seen around La Soufriere, thus errors should not be important. By using the Parasnis approach, the estimate effect of an isostatic compensation of a short wavelength topography (L = 20 km), *i.e.* size of La Soufriere Volcano area, can not contaminate Bouguer density estimates. Otherwise, the main errors could be caused by shallow density inhomogeneities using data measured on the topography in topographic density estimates. When the amplitude of gravity anomalies does not exceed 10 mgal and the topography contrast is about 1 km, the main errors is 0.19 g/cm^3. Thus, the total uncertainty is in order ~ 0.1 g/cm^3. In the previous gravity survey, a value of 2.67 g/cm^3 was used as the Bouguer density. It is explained by the absence of the correlation between residual anomalies and a Bouguer correction. Such correlation can not be used as the criterion of correctness of the topographic density estimates. An appropriate Bouguer density around La Soufriere summit area is 2.21 g/cm^3.This result may explain that the Bouguer density of 2.21 g/cm^3 based on the recent gravity survey on La Soufriere Volcano area could be caused by real part density inhomogeneities or shallow density inhomogeneities of the edifice. Therefore, the application of the Bouguer density from the previous result on this area will be overestimated.

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