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On gauge conditions for a selfgravitating spherically symmetric charged scalar field

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Abstract. We consider complex scalar field minimally coupled to gravity and to electromagnetism. This note provides an examination of the spherically symmetric Einstein-Maxwell-Klein-Gordon system in a sufficiently general form supporting the maximal gauge freedom for metric functions and the electromagnetic potential. We study in detail gauge invariance and gauge transformations in the spherically symmetric case. Our method of reduction is based on the characteristic function, which is invariantly defined, up to sign, as the squared norm of the differential of the 'area' metric function. For static configuration, one can extract a subsystem of three equations for the characteristic function, the electromagnetic potential, and the modulus of a scalar field.

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1. Introduction

This note deal with a complex scalar field endowed with the $U(1)$ gauge symmetry and minimally coupled to gravity. Over the last fifteen years spherically symmetric configurations of a selfgravitating complex scalar field are being intensively studied as models of some hypothetical astrophysical objects, the so-called boson stars [1, 2, 3]. There are some justified reasons to think that either scalar fields exist in nature as a fundamental substance or they are useful as a mathematical tool for the phenomenological description of the actual stars. In this connection, a number of explorations of the mathematical structure of the problem have been done [4, 5, 6, 7].

We consider the gauge symmetry as an adequate base to reduce the full Einstein-Maxwell-Klein-Gordon system to another one from the point of view of extracting a minimal independent subsystem. Such a reduction is a natural starting point for analytical and numerical studies of spherically symmetric boson stars.

The outline of this note is as follows. In Section 2, we rewrite the general equations governing a selfgravitating nonlinear complex scalar field, namely the Einstein equations, Maxwell equations, and the Klein–Gordon equation with arbitrary selfinteraction potential. This section is completed by a brief discussion of gauge invariance of the Maxwell–Klein–Gordon equations. Section 3 deals with the spherically symmetric spacetime. The subsection 3.1 introduces a spherically symmetric metric and the corresponding (pseudo)orthonormal tetrad, connection, and curvature for the geometry description. In the subsection 3.2 we obtain a full Einstein-Maxwell-Klein-Gordon system of independent equations preserving the gauge invariance both for the metric functions and the electromagnetic potential. In the subsection 3.3 we reduce this system to another system that is especially useful in the static case (also see Ref. [8] to compare with the case of a real field).

Throughout this note, Roman indices run from 0 to 3 and Greek ones take the values $1, 2, 3$ only. We use the signature convention $(+\,-\,-\,-)$ for the metric and geometric system of units with $G = 1, c = 1$, such that mass and charge have the dimension of length.

2. The Einstein-Maxwell-Klein-Gordon system

Mathematically, a selfgravitating charged scalar field $\phi(x)$ is a smooth section of a one-dimensional complex vector bundle with the gauge group $U(1)$. In accordance with the principle of local gauge invariance, the bundle is endowed with a local complex-valued connection 1-form $\mathcal{A}$ and its curvature 2-form $\mathcal{F}$ or, in other words, with an electromagnetic (gauge) potential and the corresponding electromagnetic (gauge) field. Therefore, the action for a selfgravitating minimally coupled complex scalar field with a selfinteraction potential $V(\phi\bar{\phi})$ has the form

$$\Sigma = \frac{1}{8\pi}\int\bigl(-S/2 + \langle D\phi, \overline{D\phi}\rangle - 2V(\phi\bar{\phi}) - (1/2)\,\langle\mathcal{F},\mathcal{F}\rangle\bigr)\sqrt{|\,g|}\,d^4x\,, \tag{1}$$

where S is the scalar curvature and angle brackets $\langle\ ,\rangle$ denote the inner product with respect to a given spacetime metric, so that

$$\left\langle D\phi, \overline{D\phi}\right\rangle = g^{kl} D_k\phi\, \overline{D_l\phi}\,, \quad \langle \mathcal{F}, \mathcal{F}\rangle = \mathcal{F}_{kl}\mathcal{F}^{kl}\,.$$

$$D\phi = d\phi + ie\mathcal{A}\phi = D_k\phi\, dx^k, \quad D_k = \partial_k + ie\mathcal{A}_k\,,$$

$$\mathcal{A} = \mathcal{A}_k\, dx^k, \quad \mathcal{F} = d\mathcal{A} = \frac{1}{2}\mathcal{F}_{kl}\, dx^k\wedge dx^l, \quad \mathcal{F}_{kl} = \partial_k\mathcal{A}_l - \partial_l\mathcal{A}_k\,.$$

Alternatively, the gauge coupling constant e may be included in $\mathcal{A}$, $e\mathcal{A}\to\mathcal{A}$, but, in this case, the electromagnetic part in the action (1) must be multiplied by the factor $1/e^2$.

The standard variational procedure leads to dynamical equations for the fields and metric. A scalar field is contained only in the Lagrangian density

$$\mathcal{L}_\phi = 1/(8\pi)\sqrt{|\,g|}\big(\langle D\phi, \overline{D\phi}\rangle - 2V(\phi\bar\phi)\big),$$

so that we obtain the Klein-Gordon equation

$$\frac{1}{\sqrt{|\,g|}}\, D_k\Big(\sqrt{|\,g|}\, g^{kl} D_l\phi\Big) \;+\; 2\,\frac{dV}{d\big(\phi\bar\phi\big)}\,\phi \;=\; 0\,. \tag{2}$$

In the Lagrangian density

$$\mathcal{L} = \mathcal{L}_\phi + \mathcal{L}_{em}\,, \quad \mathcal{L}_{em} = -1/(16\pi)\sqrt{|\,g|}\,\langle\mathcal{F},\mathcal{F}\rangle\,,$$

one varies $\mathcal{A}$. This gives the Maxwell equations ${\mathcal{F}^{kl}}_{;k} = j^l$, which can be written in detail as follows:

$$\frac{1}{\sqrt{|\,g|}}\,\partial_k\Big(\sqrt{|\,g|}\,\mathcal{F}^{kl}\Big) = j^l\,, \quad j^l = \frac{i}{2}\, e g^{lm}\big(\bar\phi\, D_m\phi - \phi\,\overline{D_m\phi}\big)\,, \tag{3}$$

where the current 1-form j is real, since it is invariant under complex conjugation, and its covariant divergence obviously vanishes identically, that is, ${j^k}_{;k} = 0$.

Equations (2) and (3), together with the Einstein equations

$$\mathcal{R}_{ij} - \frac{1}{2}g_{ij}S = \mathcal{T}_{ij}, \tag{4}$$

make up a closed system of equations for the problem under consideration. The energy-momentum tensor consists of two parts, $\mathcal{T} = \mathcal{T}^{(\phi)} + \mathcal{T}^{(em)}$, such that in components one has

$$\mathcal{T}^{(\phi)}_{ij} = \frac{1}{8\pi}\big\{D_i\phi\overline{D_j\phi} + D_j\phi\overline{D_i\phi} - g_{ij}\big(g^{kl}D_k\phi\overline{D_l\phi} - 2V(\phi\bar\phi)\big)\big\}\,, \tag{5}$$

$$\mathcal{T}^{(em)}_{ij} \;=\; \frac{1}{8\pi}\bigg(-\,2\mathcal{F}_{ik}\mathcal{F}_{jl}g^{kl} \;+\; \frac{1}{2}\,\mathcal{F}_{kl}\mathcal{F}^{kl}g_{ij}\bigg)\,. \tag{6}$$

There is a local gauge freedom to change ϕ and $\mathcal{A}$ without changing the action (1), field equations (2), (3), and components (5) and (6) of the energy-momentum tensor. Namely, they are invariant under the gauge transformations

$$\phi \to \phi \exp(-ie\lambda), \quad \mathcal{A} \to \mathcal{A} + d\lambda, \tag{7}$$

where $\lambda = \lambda(x)$ is an arbitrary smooth function. In particular this means that for any smooth complex scalar field $\phi = \psi \exp(ie\omega)$, where $\psi(x)$ and $\omega(x)$ are real functions, one can choose $\lambda = \omega$ or, in other words, to impose the gauge condition $\omega = 0$. For this choice, a scalar field will be described by a single real function. Another choice consists in an appropriate reduction of the electromagnetic potential $\mathcal{A}$ to some simple form.

3. Selfgravitating spherically symmetric scalar fields

3.1 The metric and the curvature

In this section we consider a spherically symmetric spacetime with the metric

$$ds^2 = A^2 dt^2 - B^2 dr^2 - C^2(d\theta^2 + \sin^2\theta\, d\varphi^2), \tag{8}$$

where the functions A, B and C depend only on the coordinates t and r. Note that for the sake of generality there remains the possibility to impose a gauge condition on the metric functions in (8). Depending on the specific problem one can adopt some suitable gauge condition, for instance, a static form for the function $C(t,r)$, such as $C = r$, or the condition $A = B^{-1}$.

The freedom in the choice of the gauge potential and the metric functions leads to cumbersome calculations even for the explicit expressions of field equations (2), (3), and (4). In order to simplify these calculations, in what follows we will use the (pseudo) orthonormal basis

$$\mathrm{e}_0 = \frac{1}{A}\,\partial_t, \quad \mathrm{e}_1 = \frac{1}{B}\,\partial_r, \quad \mathrm{e}_2 = \frac{1}{C}\,\partial_\theta, \quad \mathrm{e}_3 = \frac{1}{C\sin\theta}\,\partial_\varphi\,, \tag{9}$$

associated with the metric (8), the dual basis formed by the 1-forms

$$\mathrm{e}^0 = A\,dt\,, \quad \mathrm{e}^1 = B\,dr\,, \quad \mathrm{e}^2 = C\,d\theta\,, \quad \mathrm{e}^3 = C\sin\theta\,d\varphi\,, \tag{10}$$

and the standard basis of the 2-forms

$$\alpha^1 = \mathrm{e}^0\wedge \mathrm{e}^1,\ \alpha^2 = \mathrm{e}^0\wedge \mathrm{e}^2,\ \alpha^3 = \mathrm{e}^0\wedge \mathrm{e}^3,\ *\alpha^1 = \mathrm{e}^3\wedge \mathrm{e}^2,\ *\alpha^2 = \mathrm{e}^1\wedge \mathrm{e}^3,\ *\alpha^3 = \mathrm{e}^2\wedge \mathrm{e}^1.$$

For the derivatives of functions with respect to the basis vectors, we will use the notations

$$f_{(0)} \equiv \mathrm{e}_0 f = \frac{1}{A}\partial_t f, \quad f_{(1)} \equiv \mathrm{e}_1 f = \frac{1}{B}\partial_r f, \quad f_{(1)(0)} \equiv \mathrm{e}_0\mathrm{e}_1 f = \frac{1}{A}\,\partial_t\left\{\frac{1}{B}\,\partial_r C\right\},$$

and the identity

$$f_{(1)(0)} - f_{(0)(1)} = f_{(0)}\,\frac{A_{(1)}}{A} - f_{(1)}\,\frac{B_{(0)}}{B}. \tag{11}$$

The corresponding connection forms are defined by the relations $\nabla_X \mathrm{e}_j = \omega^i_j(X)\mathrm{e}_i$, which must hold for an arbitrary vector field X, and they obey the conditions $\omega^0_\alpha = \omega^\alpha_0$, $\omega^\alpha_\beta = -\omega^\beta_\alpha$. By using the Cartan structural equations, we can calculate algebraically independent components of the connection and the spacetime curvature. They are

$$\begin{aligned}
\omega^0_1 = \frac{A_{(1)}}{A}\,\mathrm{e}^0 + \frac{B_{(0)}}{B}\,\mathrm{e}^1, \quad \omega^0_2 &= -\frac{C_{(1)}}{C}\,\mathrm{e}^3, \quad \omega^0_3 = \frac{C_{(0)}}{C}\,\mathrm{e}^3,\\
\omega^1_2 = -\frac{C_{(1)}}{C}\,\mathrm{e}^2, \quad \omega^1_3 &= -\frac{C_{(1)}}{C}\,\mathrm{e}^3, \quad \omega^2_3 = \frac{\cot\theta}{C}\,\mathrm{e}^3,
\end{aligned} \tag{12}$$

and

$$\begin{aligned}
R = &\left(\frac{B_{(0)(0)}}{B} - \frac{A_{(1)(1)}}{A}\right)\alpha^1\otimes\alpha^1 + \left(\frac{C_{(0)(0)}}{C} - \frac{A_{(1)}C_{(1)}}{AC}\right)(\alpha^2\otimes\alpha^2 + \alpha^3\otimes\alpha^3)\\
&+ \frac{C^2_{(1)} - C^2_{(0)} - 1}{C^2}\,{*\alpha^1}\otimes{*\alpha^1} + \left(\frac{C_{(1)(1)}}{C} - \frac{B_{(0)}C_{(0)}}{BC}\right)({*\alpha^2}\otimes{*\alpha^2} + {*\alpha^3}\otimes{*\alpha^3})\\
&+ \left(\frac{C_{(1)(0)}}{C} - \frac{A_{(1)}C_{(0)}}{AC}\right)(\alpha^3\otimes{*\alpha^2} + {*\alpha^2}\otimes\alpha^3 - \alpha^2\otimes{*\alpha^3} - {*\alpha^3}\otimes\alpha^2).
\end{aligned} \tag{13}$$

3.2 The field equations

In any spherically symmetric spacetime, a charge selfgravitating scalar field and a gauge potential can be written as

$$\phi = \psi(t,r)\exp\bigl(ie\omega(t,r)\bigr), \quad \mathcal{A} = \sigma(t,r)\mathrm{e}^0 + \tau(t,r)\mathrm{e}^1. \tag{14}$$

In fact, it is easy to show that the expressions (14) give us the most general form of ϕ and $\mathcal{A}$ compatible with the spherically symmetric geometry in general relativity.

Substituting the ansatz (14) in the Maxwell equations (3) and equation (2) for a scalar field, we find that they become

$$\frac{1}{C^2}\left[C^2\left(\sigma_{(1)} - \tau_{(0)} + \sigma\frac{A_{(1)}}{A} - \tau\frac{B_{(0)}}{B}\right)\right]_{(0)} = e^2\bigl(\omega_{(1)} + \tau\bigr)\psi^2\,, \tag{15}$$

$$\frac{1}{C^2}\left[C^2\left(\sigma_{(1)} - \tau_{(0)} + \sigma\frac{A_{(1)}}{A} - \tau\frac{B_{(0)}}{B}\right)\right]_{(1)} = e^2\bigl(\omega_{(0)} + \sigma\bigr)\psi^2\,, \tag{16}$$

and

$$\begin{aligned}
\psi_{(0)(0)} - \psi_{(1)(1)} + \psi_{(0)}\frac{(BC^2)_{(0)}}{BC^2} - \psi_{(1)}\frac{(AC^2)_{(1)}}{AC^2} - e^2\psi\Bigl[\bigl(\omega_{(0)} + \sigma\bigr)^2 - \bigl(\omega_{(1)} + \tau\bigr)^2\Bigr]\\
+ 2\frac{dV}{d(\psi^2)}\,\psi + ie\psi\Bigl\{\bigl(\omega_{(0)} + \sigma\bigr)_{(0)} - \bigl(\omega_{(1)} + \tau\bigr)_{(1)}
\end{aligned} \tag{17}$$

$$+ \left(\omega_{(0)} + \sigma\right) \left[\frac{(BC^2)_{(0)}}{BC^2} + 2\,\frac{\psi_{(0)}}{\psi} \right] - \left(\omega_{(1)} + \tau\right) \left[\frac{(AC^2)_{(1)}}{AC^2} + 2\,\frac{\psi_{(1)}}{\psi} \right] \Bigg\} = 0,$$

Substituting the curvature (13) and the expressions (14) into the Einstein equations (4) yields the three independent equations

$$- 2\frac{C_{(1)(1)}}{C} + 2\frac{B_{(0)}C_{(0)}}{BC} - \frac{C_{(1)}^2 - C_{(0)}^2 - 1}{C^2} = \left(\sigma_{(1)} - \tau_{(0)} + \sigma\frac{A_{(1)}}{A} - \tau\frac{B_{(0)}}{B}\right)^2 \\ + \left(\psi_{(0)}^2 + \psi_{(1)}^2 + e^2\psi^2\left[\left(\omega_{(0)} + \sigma\right)^2 + \left(\omega_{(1)} + \tau\right)^2\right]\right) + 2V, \quad (18)$$

$$- 2\frac{C_{(0)(0)}}{C} + 2\frac{A_{(1)}C_{(1)}}{AC} + \frac{C_{(1)}^2 - C_{(0)}^2 - 1}{C^2} = - \left(\sigma_{(1)} - \tau_{(0)} + \sigma\frac{A_{(1)}}{A} - \tau\frac{B_{(0)}}{B}\right)^2 \\ + \left(\psi_{(0)}^2 + \psi_{(1)}^2 + e^2\psi^2\left[\left(\omega_{(0)} + \sigma\right)^2 + \left(\omega_{(1)} + \tau\right)^2\right]\right) - 2V, \quad (19)$$

$$- 2\frac{C_{(0)(1)}}{C} + 2\frac{B_{(0)}C_{(1)}}{BC} \equiv -2\frac{C_{(1)(0)}}{C} + 2\frac{A_{(1)}C_{(0)}}{AC} = \\ 2\left\{\psi_{(0)}\psi_{(1)} + e^2\psi^2\left(\omega_{(0)} + \sigma\right)\left(\omega_{(1)} + \tau\right)\right\}. \quad (20)$$

They are related to 00, 11, and 01 components of the Einstein equations, while the remaining nontrivial equations (22 and 33 components) are dependent.

3.3 A reduction of the field equations

In view of the identity (11), the substitutions $\tau \to \omega_{(1)} + \tau$ and $\sigma \to \omega_{(0)} + \sigma$ leave the left-hand sides of equations (15) and (16) unchanged. Therefore, the gauge potential components τ and σ appear in equations (15)–(20) only in the combinations $\omega_{(1)} + \tau$ and $\omega_{(0)} + \sigma$ respectively. This means that without loss of generality, we can take ω to be an arbitrary smooth function and, in so doing, there remains the gauge freedom for τ and σ. For simplicity we suppose $\omega = 0$ from now on.

Denoting, for a moment, the left-hand sides of the Maxwell equations (15) and (16) by $\mu_{(0)}/C^2$ and $\mu_{(1)}/C^2$ respectively, and substituting them instead of the corresponding right-hand sides into equation (18), we find that the imaginary part of the latter vanishes identically. Thus, we see that the matter field equations can be rewritten in the form

$$\frac{1}{C^2}\left[C^2\left(\sigma_{(1)} - \tau_{(0)} + \sigma\frac{A_{(1)}}{A} - \tau\frac{B_{(0)}}{B}\right)\right]_{(0)} = e^2\tau\psi^2\,, \quad (21)$$

$$\frac{1}{C^2}\left[C^2\left(\sigma_{(1)} - \tau_{(0)} + \sigma\frac{A_{(1)}}{A} - \tau\frac{B_{(0)}}{B}\right)\right]_{(1)} = e^2\sigma\psi^2\,. \quad (22)$$

$$\psi_{(0)(0)} - \psi_{(1)(1)} + \psi_{(0)}\frac{(BC^2)_{(0)}}{BC^2} - \psi_{(1)}\frac{(AC^2)_{(1)}}{AC^2} - e^2\psi\left(\sigma^2 - \tau^2\right) + \frac{dV}{d\psi} = 0\,. \quad (23)$$

Our method for studying geometrically distinct classes of solutions is based on the function

$$f \equiv -\langle dC, dC\rangle = C_{(1)}^2 - C_{(0)}^2, \quad (24)$$

which will hereafter be referred to as *the characteristic function*. In spherically symmetric spacetimes the metric function C is invariantly defined in the sense that it does not change under coordinate transformations that leave the angle coordinates unchanged. The characteristic function f is minus the squared norm of the 1-form dC, and so is invariant too, in contrast to the metric functions A and B.

Now substituting equation (20), written in the two equivalent forms as

$$\frac{B_{(0)}C_{(1)}}{BC} = \{\psi_{(0)}\psi_{(1)} + e^2\psi^2\sigma\tau\} + \frac{C_{(0)(1)}}{C}\,, \qquad \frac{A_{(1)}C_{(0)}}{AC} = \{\psi_{(0)}\psi_{(1)} + e^2\psi^2\sigma\tau\} + \frac{C_{(1)(0)}}{C}\,,$$

into equations (18) and (19) multiplied by $C_{(1)}$ and $C_{(0)}$ respectively, we obtain for the characteristic function f the equations

$$\begin{aligned} &-\frac{f_{(1)}}{C} - \frac{f-1}{C^2}C_{(1)} = -2C_{(0)}\left(\psi_{(0)}\psi_{(1)} + e^2\psi^2\sigma\tau\right) \\ &\quad + C_{(1)}\left\{\left(\sigma_{(1)} - \tau_{(0)} + \sigma\frac{A_{(1)}}{A} - \tau\frac{B_{(0)}}{B}\right)^2 + \psi_{(0)}^2 + \psi_{(1)}^2 + e^2\psi^2\left(\sigma^2+\tau^2\right) + 2V\right\}, \end{aligned} \tag{25}$$

$$\begin{aligned} &-\frac{f_{(0)}}{C} - \frac{f-1}{C^2}C_{(0)} = 2C_{(1)}\left(\psi_{(0)}\psi_{(1)} + e^2\psi^2\sigma\tau\right) \\ &\quad + C_{(0)}\left\{\left(\sigma_{(1)} - \tau_{(0)} + \sigma\frac{A_{(1)}}{A} - \tau\frac{B_{(0)}}{B}\right)^2 - \psi_{(0)}^2 - \psi_{(1)}^2 - e^2\psi^2\left(\sigma^2+\tau^2\right) + 2V\right\}. \end{aligned} \tag{26}$$

For any 1-form $\alpha = \alpha_0 \mathrm{e}^0 + \alpha_1 \mathrm{e}^1$, one has $\langle\alpha,\alpha\rangle = \alpha_0^2 - \alpha_1^2$ and

$$\begin{aligned} C_{(1)}\alpha_0^2 + C_{(1)}\alpha_1^2 - 2C_{(0)}\alpha_0\alpha_1 &= C_{(1)}\langle\alpha,\alpha\rangle - 2\alpha_1\langle dC,\alpha\rangle, \\ -C_{(0)}\alpha_0^2 - C_{(0)}\alpha_1^2 + 2C_{(1)}\alpha_0\alpha_1 &= C_{(0)}\langle\alpha,\alpha\rangle - 2\alpha_0\langle dC,\alpha\rangle. \end{aligned}$$

Applying these identities to the 1-forms $d\psi$ and $\mathcal{A}$ in equations (25) and (26) multiplied by e^1 and e^0 respectively, and then adding the equations together, we obtain

$$\begin{aligned} &\frac{df}{C} + \left[\frac{f-1}{C^2} + \left(\sigma_{(1)} - \tau_{(0)} + \sigma\frac{A_{(1)}}{A} - \tau\frac{B_{(0)}}{B}\right)^2 + \right. \\ &\quad \left. + \langle d\psi, d\psi\rangle + e^2\psi^2\langle\mathcal{A},\mathcal{A}\rangle + 2V\right] dC - 2\langle dC, d\psi\rangle d\psi - 2e^2\psi^2\langle dC,\mathcal{A}\rangle\mathcal{A} = 0\,, \end{aligned} \tag{27}$$

where the equality $\langle\mathcal{A},\mathcal{A}\rangle = \sigma^2 - \tau^2$ has been used.

Thus we reduce Einstein-Maxwell-Klein-Gordon system to five equations (21), (22), (23), (24), and (27) for the seven unknown functions A, B, C, σ, τ, f, and ψ. In order to obtain a definite solution we have to impose two gauge conditions, e.g. for the metric functions and the electromagnetic potential $\mathcal{A}$. Note that static configurations should be treated in another way: e.g. one can choose the metric function C to be the coordinate r (so that $B^2 = 1/f$) and eliminate the term $A_{(1)}/A$ from the other equations by using equation (19). By continuing in this way, one will have to solve only three independent equations for the unknown functions σ, f, and ψ, and then to solve equation (19) for the metric function A.

4. Conclusions

In the present note we have considered gauge symmetries of the full Einstein-Maxwell-Klein-Gordon system and have revisited the question of the optimal reduction of the system for the spherically symmetric spacetime. Key features of our method were the use of the gauge symmetries to reduce the system, the generality of our analysis (we have kept the possibility to impose a gauge condition on the metric functions and the gauge potential), and the observation that in the spherically symmetric case a scalar field can be described by only its modulus. We have presented, in the mathematically closed and general form, a reduced system that has exactly the same gauge degrees of freedom as the initial system. We have shown that for static configuration, one can extract a subsystem of three equations for the characteristic function, the electromagnetic potential, and the modulus of a scalar field. These equations can be considered as a natural starting point for analytical and numerical studies of spherically symmetric boson stars.

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