COMPOSITE MATERIALS IN AUTOMOTIVE ENGINEERING – MECHANICAL BEHAVIOR OF ANISOTROPIC MEDIA

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1. INTRODUCTION

Composite materials consist two or more constituents such as fibres and matrix which make layers mutually bonded to form multilayered composite called laminate. Fibres carry loads giving strength of composite, and matrix bond fibres together and have role in transfer loads to fibre, forms outer shape of composite and between other properties defines its behavior influenced by environment. Fibres are made of carbon, glass, aramid (such as kevlar) or metal, and the most often they are 60-70% of composite volume. Matrices may be made of polymers, such as thermosets or thermoplastics, metals, such as aluminum alloys or magnesium, ceramics etc.

2. GOVERNING EQUATIONS

Governing equations of elastic materials in small strain conditions are developed in the beginning of nineteenth Century. If strains are small enough equations are linear and relation connecting stress and strain are generalized Hooke's Law given as

$$\sigma_{ij} = C_{ijkl}\varepsilon_{kl} \quad (i, j, k, l = 1,2,3), \tag{1}$$

which is postulated by Cauchy. This law is base of linear elasticity. Coefficients C_{ijkl} are stiffness coefficients. That is tensor of fourth rank whose coefficients, in general, vary from point to point of elastic body. If these coefficients are independent on position than elastic body is homogeneous. In direct notation equation (1) may be expressed as

$$\boldsymbol{\sigma} = \boldsymbol{C}\,\boldsymbol{\varepsilon}\,. \tag{2}$$

Strain energy function W is specific energy of deformation and it is positive definite function. Stiffness coefficients C_{ijkl}, taking in account symmetry of stress and strain, have 21 independent components in body with general anisotropy. Strain energy W, for linear elastic materials, may be defined as quadratic in strain ε_{ij} in form

$$W = \frac{1}{2}C_{ijkl}\varepsilon_{ij}\varepsilon_{kl}, \qquad (i, j, k, l = 1,2,3)\,. \tag{3}$$

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3. LINEAR ELASTICITY – ONE FAMILY OF FIBRES

Material reinforced by one family of fibres has one preferred direction and it is transversely isotropic in relation to that direction. Preferred direction may be defined with field of unit vectors $\boldsymbol{a}$ which may vary from point to point. Trajectories of unit vector field $\boldsymbol{a}$ form lines called fibres and material is transversely isotropic in relation to local fibre direction. Such material is usually treated in coordinate system with one axes coincides with axes of transversal isotropy and study constrains on strain energy function from requirements that stay invariant during rotations around that axes. Here, however, we are going to use coordinate free constitutive equations following Spencer [1].

In such approach strain energy W is function of both strain $\boldsymbol{\varepsilon}$ and fibre $\boldsymbol{a}$ direction, that is

$$W = W(\boldsymbol{\varepsilon}, \boldsymbol{a}) . \tag{4}$$

The most general quadratic form of strain energy function is given as

$$W = \frac{1}{2}\lambda (tr\,\boldsymbol{\varepsilon})^2 + \mu_{\mathrm{T}} tr\,\boldsymbol{\varepsilon}^2 + \alpha(\boldsymbol{a}\cdot\boldsymbol{\varepsilon}\cdot\boldsymbol{a}) tr\,\boldsymbol{\varepsilon} + 2(\mu_{\mathrm{L}} - \mu_{\mathrm{T}})\boldsymbol{a}\cdot\boldsymbol{\varepsilon}^2\cdot\boldsymbol{a} + \frac{1}{2}\beta(\boldsymbol{a}\cdot\boldsymbol{\varepsilon}\cdot\boldsymbol{a})^2 , \tag{5}$$

where λ, μ_{T}, μ_{L}, α, β represent elastic constants.

Constitutive relation then may be expressed as

$$\begin{aligned} \sigma_{ij} = \frac{\partial W}{\partial \varepsilon_{ij}} &= \lambda \varepsilon_{rr}\delta_{ij} + 2\mu_{\mathrm{T}}\varepsilon_{ij} + 2(\mu_{\mathrm{L}} - \mu_{\mathrm{T}})\left(a_i \varepsilon_{jn} a_n + a_n \varepsilon_{ni} a_j\right) + \\ &\alpha\left(\varepsilon_{rr} a_i a_j + a_m \varepsilon_{mn} a_n \delta_{ij}\right) + \beta(a_m \varepsilon_{mn} a_n) a_i a_j . \end{aligned} \tag{6}$$

Stiffness tensor then may be calculated, as shown in [2], in following way

$$\begin{aligned} \mathrm{C}_{ijkl} = \frac{\partial^2 W}{\partial \varepsilon_{ij} \partial \varepsilon_{ij}} &= \lambda \delta_{ij}\delta_{kl} + \mu_{\mathrm{T}}\left(\delta_{ik}\delta_{jl} + \delta_{jk}\delta_{il}\right) + \\ &(\mu_{\mathrm{L}} - \mu_{\mathrm{T}})\left(a_i a_k \delta_{jl} + a_i a_l \delta_{jk} + a_j a_k \delta_{il} + a_j a_l \delta_{ik}\right) + \\ &\alpha\left(a_k a_l \delta_{ij} + a_i a_j \delta_{kl}\right) + \beta a_i a_j a_k a_l , \end{aligned} \tag{7}$$

showing obvious dependence on fibre direction.

Expression (7) is in agreement with well known expressions for transversally isotropic linear elastic material. Stiffness coefficients may be expressed in relation to other engineering constants more suitable for direct measuring. Material constant μ_{L} represents shear modulus along the fibre direction $\boldsymbol{a}$, while μ_{T} represents shear modulus perpendicular to the fibre direction $\boldsymbol{a}$. Remained material constants λ, α, β may be connected to other modulus such as extension modulus, Yung's modulus or Poisson ratio.

Taking fibre direction to be along axes x_1 of Cartesian coordinate system leads to expression of unit fibre direction vector $(a_i) = (a_1,\ 0,\ 0) = (1,\ 0,\ 0)$ and using Voight notation in (7) one obtains

$$\begin{aligned}
C_{11} &= \lambda + 2\alpha + \beta + 4\mu_L - 2\mu_T, \\
C_{12} &= \lambda + \alpha = C_{21}, \\
C_{13} &= \lambda + \alpha = C_{31}, \\
C_{22} &= \lambda + 2\mu_T, \\
C_{23} &= \lambda = C_{32}, \\
C_{33} &= \lambda + 2\mu_T, \\
C_{44} &= \mu_T, \\
C_{55} &= \mu_L, \\
C_{66} &= \mu_L,
\end{aligned} \tag{8}$$

and all remained constants vanish, that is

$$C_{14} = C_{15} = C_{16} = C_{24} = C_{25} = C_{26} = C_{34} = C_{35} = C_{36} = C_{45} = C_{46} = C_{56} = 0. \tag{9}$$

In Voight notation indices 11, 22 and 33 take values 1, 2 and 3, respectively, and indices 23, 13 and 12 take values 4, 5 and 6, respectively.

4. BULK WAVES

Mechanical behavior of anisotropic media may be seen through examination of bulk waves. These waves propagate through unbounded media without perturbations caused by boundaries and inter layers. Bulk waves may be decomposed in finite plane waves which propagate along arbitrary direction $\boldsymbol{n}$ inside solid.

Properties of these waves, according to [3], are determined with propagation direction and constitutive properties of media. In general, it is possible to generate three types of such waves, which are determined with three displacement vectors $\boldsymbol{U}^{(k)},\ k = 1,2,3$ representing acoustical polarization. These polarization vectors, with propagation directions, are shown in sketch in Figure 1. Three polarization vectors are mutually orthogonal, but usually any of them are neither parallel nor perpendicular to propagation direction $\boldsymbol{n}$.

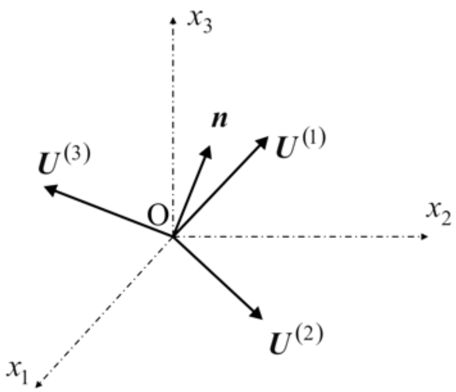

Figure 1 *Typical scheme of polarization in anisotropic media*

In anisotropic media "pure" modes may appear for some particular propagation directions only, depending on degree of symmetry of considered material.

3.1 ***Propagation conditions***

Propagation of elastic waves may be examined according to first Cauchy law of motion. Taking fact that stiffness tensor C_{ijkl} possess symmetry in relation to second pair of indices one may write equations

$$\rho \frac{\partial^2 u_i}{\partial t^2} = C_{ijkl} \frac{\partial^2 u_l}{\partial x_j \partial x_k} , \tag{10}$$

which represent system of homogeneous linear differential equations of second order in relation to displacement vectors. Solution of such system of equations may be supposed as plain wave solution with wave normal with components $(n_i) = (n_1, n_2, n_3)$ with displacement vector given as

$$u_i = U_i e^{i\left(kn_j x_j - \omega t\right)} = U_i e^{i\varphi} \tag{11}$$

Expression (11) gives complex displacement, although displacement should be real. Vector (11) will satisfy system of homogeneous linear differential equations (10) if both real and imaginary parts satisfy (11). Substituting (11) in (10), taking into account $\partial u_i / \partial x_j = ikn_j u_i$, leads to

$$\left(C_{ijkl} n_k n_j - \rho v^2 \delta_{il}\right) U_l = 0 \tag{12}$$

Second order tensor Λ_{il} , introducing $\lambda_{ijkl} = C_{ijkl} / \rho$, which may be expressed as

$$\Lambda_{il} = \lambda_{ijkl} n_k n_j \tag{13}$$

is called Riemann-Christoffel's tensor, and equation (13) may be written as

$$\left(\Lambda_{il} - v^2 \delta_{il}\right) U_l = 0 . \tag{14}$$

This equation is known as Riemann-Christoffel's equation and represents system of three homogeneous linear equations in relation to displacement amplitudes U_l. Equation (14) in matrix form is

$$\begin{bmatrix} \Lambda_{11}-v^2 & \Lambda_{12} & \Lambda_{13} \\ \Lambda_{12} & \Lambda_{22}-v^2 & \Lambda_{23} \\ \Lambda_{13} & \Lambda_{23} & \Lambda_{33}-v^2 \end{bmatrix} \begin{Bmatrix} U_1 \\ U_2 \\ U_3 \end{Bmatrix} = 0\,, \tag{15}$$

where elements Λ_{il}, using relation (13), may be expressed in the following way

$$\begin{aligned} \rho\Lambda_{il} = C_{ijkl}n_k n_j = C_{i11l}n_1n_1 + (C_{i12l}+C_{i21l})n_1n_2 + (C_{i13l}+C_{i31l})n_1n_3 \\ + C_{i22l}\,n_2n_2 + (C_{i23l}+C_{i32l})n_1n_3 + C_{i33l}n_3n_3. \end{aligned} \tag{16}$$

5. NUMERICAL EVALUATION OF SLOWNESS SURFACES

Materials reinforced by one family of fibres posses transversal isotropy and, without loss of generality, one may choose one of Cartesian axis, say x_1, to coincide with fibre direction. Thus, unit vector of fibre direction may be written as $(a_i) = (1,0,0)$, and components of acoustic tensor (16), with use of (8), may be expressed as

$$\begin{aligned} \rho\Lambda_{11} &= (\lambda+2\alpha+\beta+4\mu_L-2\mu_T)n_1^2+\mu_L n_2^2+\mu_L n_3^2, \\ \rho\Lambda_{12} &= (\lambda+\alpha+\mu_L)n_1n_2, \\ \rho\Lambda_{13} &= (\lambda+\alpha+\mu_L)n_1n_3, \\ \rho\Lambda_{22} &= \mu_L n_1^2+(\lambda+2\mu_T)n_2^2+\mu_T n_3^2, \\ \rho\Lambda_{23} &= (\lambda+\mu_T)n_2n_3, \\ \rho\Lambda_{33} &= \mu_L n_1^2+\mu_T n_2^2+(\lambda+2\mu_T)n_3^2. \end{aligned} \tag{17}$$

Let suppose that propagation direction is in coordinate plane (x_1, x_3), than wave normal vector is $(n_i) = (n_1, 0, n_3)$, and in (17) Λ_{12} and Λ_{23} vanish leading to

$$\begin{bmatrix} \Lambda_{11}-v^2 & 0 & \Lambda_{13} \\ 0 & \Lambda_{22}-v^2 & 0 \\ \Lambda_{13} & 0 & \Lambda_{33}-v^2 \end{bmatrix} \begin{Bmatrix} U_1 \\ U_2 \\ U_3 \end{Bmatrix} = 0 \tag{18}$$

Riemann-Christoffel's equation may be solved analytically for very simple material symmetries only. In general, it is necessary to calculate wave surfaces numerically. Having in minds that crystallographic axes are known the simplest way of calculation is to coincide axes of symmetry with coordinate axes.

Let us choose Cartesian system (ξ_1, ξ_2, ξ_3) whose axes coincide with axes of material symmetries. We may imagine vertical plane, which coincide initially with coordinate plane (ξ_1, ξ_3) and rotate around vertical axis ξ_3 for arbitrary angle θ. That plane is referred as sagittal plane. To simplify analysis and calculations it is useful to consider slowness surfaces, representing inversed velocities. These surfaces may be obtained by calculating phase velocities for chosen propagation direction, and then calculating of slowness as inverse of phase velocity. Slowness then may be drawn as function of propagation direction.

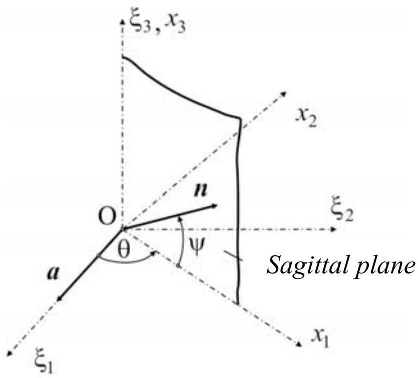

Figure 2 *One family of fibres*

In slowness surface is curved line, as intersection of slowness surface and sagittal plane, and each calculation leads to one point of all quasi-longitudinal and two quasi-transversal slowness curves. By choosing propagation direction to be in sagittal plane and adding certain increment to angle of wave normal to horizontal axes it may be drawn complete slowness curve in sagittal plane. Rotation of sagittal plane around vertical x_3 axes, for certain increment, one may obtain complete slowness surface for all three waves. That is illustrates in figure 2.

Slowness curves in sagittal plane, in general are three closed curves which may intersect each other. Two slower wave speeds, represented with outer curves are quasi shear curves. Depending on material symmetry these curves may intersect each other or, in case of isotropic material, to coincide. On the contrary, inner slowness curve, which is separated of other two, is convex and represents quasi-longitudinal waves which travel with highest speed.

For numerical calculation is employed carbon fibre epoxy resin composite which represents strongly anisotropic material. Material constants, for such materials are measured by ultrasound methods and reported in [4], have values

$$\begin{aligned} &\lambda = 5{,}65 \cdot 10^9\, Nm^{-2}, \quad \mu_T = 2{,}46 \cdot 10^9\, Nm^{-2}, \quad \mu_L = 5{,}66 \cdot 10^9\, Nm^{-2}, \\ &\alpha = -1{,}28 \cdot 10^9\, Nm^{-2}, \quad \beta = 220{,}90 \cdot 10^9\, Nm^{-2}, \end{aligned} \tag{19}$$

where as density is given as $\rho = 1{,}60 \cdot 10^3\, kg / m^3$.

By varying angle ψ between 0 and 2π may be calculated slowness curve in sagittal plane whereas by rotating of sagittal plane around axes x_3 for angle θ slowness surface may be completed. Slowness surfaces, for material reinforced by one family of fibres, are calculated in program pack MATLAB and presented in figures 3 to 6, for angles θ varies as 0^0, 30^0, 45^0 and 90^0, respectively. In these figures quasi-longitudinal waves are represented with solid lines, whereas quasi-transversal waves are represented with broken lines.

In figure 3, for $\theta = 0^0$, sagittal plane contains axis ξ_1, which coincide with axis x_1, and, therefore coincide with preferred direction, that is with fibre direction. All three modes are clearly distinguished in sagittal plane except when propagating direction is along x_1 axis, for $\psi = 0^0$, in which case two quasi-transversal waves have same speed, that is they meet each other. That is clear considering that propagation direction is along fibre direction and in plane perpendicular to fibres material behaves as isotropic.

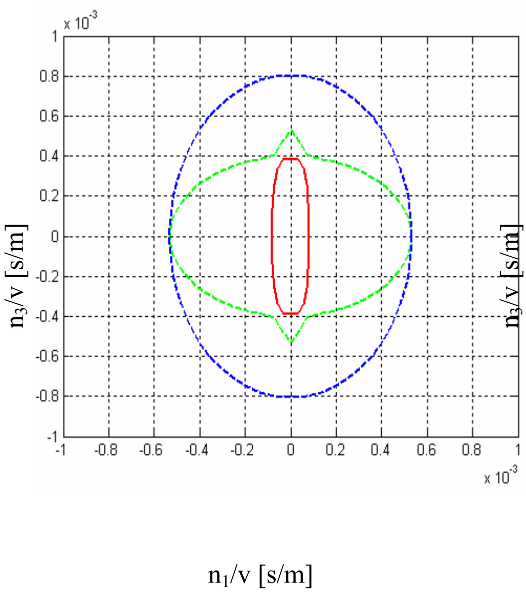

Figure 3 *One family of fibres* $\theta = 0°$

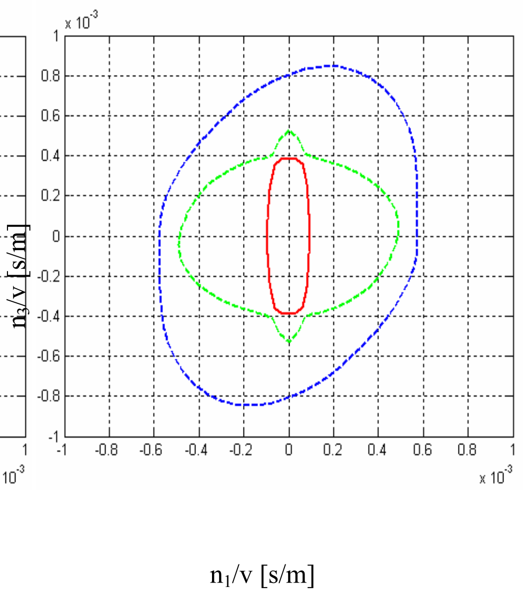

Figure 4 *One family of fibres* $\theta = 30°$

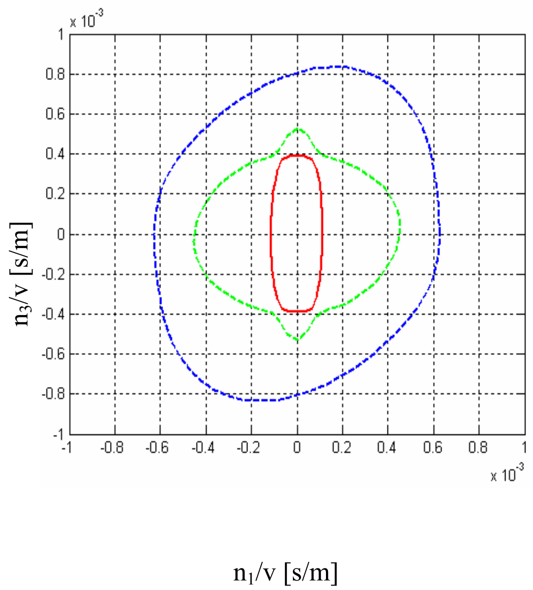

Figure 5 *One family of fibres* $\theta = 45°$

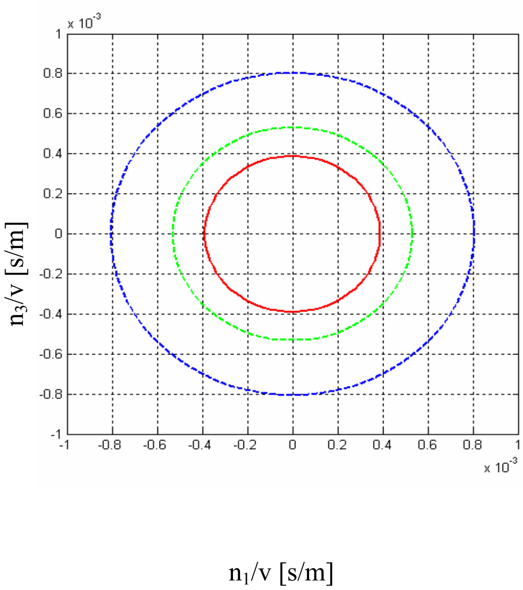

Figure 6 *One family of fibres* $\theta = 90°$

Increasing of sagittal plane rotation angle implies, as may be noticed in figures 4, $\theta = 30^0$ and 5, $\theta = 45^0$, that slowness curve of quasi-longitudinal wave have elliptic shape, whereas quasi-transversal waves behave in such way that “faster” wave deviate from elliptic shape in regions in which propagation direction approaches to normal to fibre.

For sagittal plane angle, $\theta = 90^0$, as shown in figure 6, wave propagates normal to fibre and fibres are “embedded” in wave surface and, therefore one may observe three “pure” modes along symmetry lines, which propagate with constant intensity speeds, which may be concluded from the fact that slowness curves are circular.

6. CONCLUSIONS

Mechanics of continuum treats material on macroscopic level in which microscopic level may be used as preparation for homogenization purposes. Anisotropy has different effects on wave propagation as well as on complete elastic behavior of media, which may be observed trough fact that wave front deviate from spherical shape. General conclusions about anisotropic material behavior, in mechanical sense, are taken from considering of bulk waves propagation.

For considered material acoustic tensor, as propagation condition, has been formed, and determined for different directions of wave propagation. For particular material reinforced by one family of fibres components of that tensor are calculated.

These calculations has practical significance, because it has been formed easy mathematical approach which may give fast answer about material behavior in dynamic circumstances, which often appear in parts of motor vehicles.

This approach may be used as first approximation of dynamical behavior of real parts with anisotropic characteristics that appears very often in consideration of vehicle construction parts.

ACKNOWLEDGMENTS

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