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SIMPLE WEAKLY STANDARD RINGS

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ABSTRACT

In this paper we prove that a simple weakly standard ring is either (-1, 1) ring or a commutative ring.

KEYWORDS: Weakly Standard Ring, Simple Ring, (-1,1) Ring, Commutative Ring

INTRODUCTION

Paul [1] proved that a (-1, 1) ring with ((x, y), z, w) = 0 satisfies the identity ((x, y, z),w) = 0 and then associative if it is semi prime. The identity ((x, y, z) v, w) = 0 holds in accessible rings under the assumption that the rings are without nilpotent elements in the center. Using this property it is proved that a simple accessible ring is either associative or commutative. Without this assumption we prove the identity ((x, y, y)v, w) = 0 hold in simple weakly standard rings. Using this identity in this paper, we prove that a simple weakly standard ring is either (-1,1) ring or commutative.

PRELIMINARIES

A weakly standard ring R is a non associative ring satisfying the identities,

$(x, y, x) = 0$

$((w, x), y, z) = 0$

$(w, (x, y), z) = 0$

for all w, x, y, z, in R. We define a ring R is commutative if xy = yx. A (-1, 1) ring is non associative ring in which the following identities hold:

$$(x, y, z) + (x, z, y) = 0 \quad (1)$$

$$\text{and } (x, y, z) + (y, z, x) + (z, x, y) = 0. \quad (2)$$

In this paper R represents a weakly standard ring. R is simple if whenever A is an ideal of R then either A = R or A = 0. The nucleus N of R is defined as the set of all elements n in R with the property (n, R, R) = 0. i.e, $N = \{ n \in R / (n, R, R) = 0\}$. The center Z of R is defined as the set of all elements z in N which have the additional property that

$(z, R) = 0.$

i.e, $Z = \{ z \in N / (z, R) = 0\}$.

In an arbitrary ring the identities

$$(wx, y, z) – (w, xy, z) + (w, x, yz) = w(x, y, z) + (w, x, y)z \quad (3)$$

and $(xy, z) = x(y, z) + (x, z)y + (x, y, z) + (z, x, y) – (x, z, y)$ (4)

hold. We know that if R is simple and $R^2 \neq 0$, then R is prime. From (2) we have

(x, y, z) + (y, z, x) + (z, x, y) = 0. Hence (2) is satisfied in R. It remains to show that R is right alternative. Now from (4) the identity

$$(xy, z) = x(y, z) + (x, z)y \quad (5)$$

holds in every simple weakly standard ring.

We now proceed to develop further identities that hold in arbitrary weakly standard rings. The elements u, v, w, x, y, z will denote arbitrary elements of such rings. Through repeated use of (5), we break up

((w, x, y), z) as

((w, x, y), z) = (wx. y - w. xy, z)

= wx. (y, z) + w(x, z).y + (w, z) x.y – (w, z). xy – w. x(y, z) - w.(x.z)y.

= (w, x,(y, z)) + (w,(x, z),y) + ((w, z), x, y).

Since every commutator is in the nucleus of R, we obtain

((w, x, y), z) = 0. (6)

Hence every associator commutes with every element of R. Because of (3) and the fact that every commutator is in the nucleus, we get (v, x)(x, y, z) = ((v, x)x, y, z).

It follows from (5) that (v, x)x = (vx, x). Consequently ((v, x)x, y, z) = ((vx, x), y, z) = 0.

Thus (v, x)(x, y, z) = 0. (7)

MAIN RESULTS

Lemma 1: *In a simple weakly standard ring R, ((x, y, y)v, w) = 0.*

Proof: Linearization of (7) becomes (v, w) (x, y, z) = -(v, x)(w, y, z) (8)

By using the flexible law (y, x, z) = -(z, x, y), we obtain (v, w)(x, y, y) = -(v, w)(y, y, x) = (v, y)(w, y, x) = (v, y) [-(y, x, w)-(x, w, y)] = -(v, y) (y, x, w)-(v, y) (x, w, y) = -(v, y) (y, x, w) + (v, y) (y, w, x) = 0.

i.e., (v, w)(x, y, y) = 0. (9)

Now from (5), (9), (6) we get ((x, y, y)v, w) = (x, y, y)(v, w) + ((x, y, y), w)v = 0.

Lemma 2: *Let R be a simple weakly standard ring, then V = {v ∈ R / (v, R = 0 = (vR, R)} is an ideal of R.*

Proof: If we put w = v in (6) then ((v, x, y),z) = 0. From this it follows that (vx. y, z) -(v. xy, z) = 0. Then (vx. y, z) = 0, by the definition of V. Thus vx ∈ V. So V is a right ideal of R. Since (v, R) = 0, (v, x) = 0, i.e, vx = xv ∈ V. So V is a left ideal of R. Hence V is an ideal of R.

Theorem 1: *If R is a simple weakly standard ring, then R is either a (-1, 1) ring or a commutative ring*.

Proof: From (6) and lemma 1, (x, y, y) is in V. Since V is an ideal of R and R is simple, we have either V = 0 or V = R. If V = 0, then R is right alternative, i.e, (x, y, y) = 0.By linearization of this yields (x, y, z) + (x, z, y) = 0.So (1) is satisfied in R. Since we have already prove that (2) is satisfied in R, now it follows that R is a (-1, 1) ring. If V = R, then R is a commutative ring.

REFERENCES

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