

## FUZZY VALUED EVIDENCE THEORY

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### ABSTRACT

In Dempster Shafer (DS) Theory, basic probability assignment plays a key role. All other measures can be defined in the terms of BPA. This assignment, as originally defined, can take only one value in the interval [0,1]. However in actual practice the BPA is usually provided by experts subjectivity. Experts cannot precisely give the value. We have to assign a number on their linguistic expression and there is some round off which can cause the error. To avoid this error, one can make use of fuzzy sets. The original theory does not provide any means to handle fuzzy valued based assignment. The purpose of this paper is to extend definitions of all basic measures in DS theory so that the theory can be applied to fuzzy situations.

We shall first introduce the concepts of generalized summation and multiplication, the purpose of which is to ensure that all operations involved in the theory are closed in unit interval [0,1]. Then we provide the definition of BPA., belief function, plausibility function in terms of fuzzy valued summation and multiplication. Then we will propose how to combine two piece of evidence associated with corresponding fuzzy valued basic probability assignment. We will show that proposed theory is more general than interval valued evidence theory and we derive the original theory from the proposed one by adding constraint. Finally we provide a numerical example to illustrate the approach.

**KEYWORDS:** Fuzzy Numbers, Dempster Shafer Theory

### INTRODUCTION

#### GENERALIZED SUMMATION AND MULTIPLICATION

In order that operation is useful this operation must be closed in the domain of the interest the closeness of binary operation can be defined as follows. Let D be the domain of interest and (\*) is the binary operation on D. If  $A(*)B \in D$  for all A, B in D. Then the binary operation (\*) is closed in domain D.

Example Let  $D = F[0,1]$  = fuzzy power set of [0,1].

Then for any  $A, B \in D$

$$A = \int_{u \in [0,1]} \mu_A(u) / u, B = \int_{v \in [0,1]} \mu_B(v) / v$$

We define the addition with the help of alpha cuts as follows

$$\text{if } A_\alpha = (a_\alpha^1, a_\alpha^2), B_\alpha = (b_\alpha^1, b_\alpha^2),$$

$$A_\alpha + B_\alpha = [U(a_\alpha^1, b_\alpha^1), U(a_\alpha^2, b_\alpha^2)]$$

$$A_\alpha + B_\alpha = [\min(1, a_\alpha^1 + b_\alpha^1), \min(1, a_\alpha^2 + b_\alpha^2)]$$

(1)

U is t-norm. Yager's definition with parameter  $w=1$  is used here.

And

$$\begin{aligned} \text{if } A_\alpha &= (a_\alpha^1, a_\alpha^2), B_\alpha = (b_\alpha^1, b_\alpha^2), \\ A_\alpha \times B_\alpha &= [I(a_\alpha^1, b_\alpha^1), I(a_\alpha^2, b_\alpha^2)] \\ A_\alpha \times B_\alpha &= [\max(0, a_\alpha^1 + b_\alpha^1 - 1), \max(0, a_\alpha^2 + b_\alpha^2 - 1)] \end{aligned} \quad (2)$$

Where I is t-conorm. Yager's definition with parameter  $w=1$ .

We concentrate on + and  $\times$  as two basic operations are involved in DS theory and they are summation and multiplication.

### FUZZY VALUED BASIC PROBABILITY ASSIGNMENT

Let be universe of frame of the discernment. We define the following function.

$$M : 2^\theta \rightarrow F[0,1]$$

with

$$M(\phi) = \tilde{0}, \tilde{0} = \{0 / x \forall x \in [0,1]\}$$

In original DS Theory, the closeness of operation is assured by following constraint.

$$\sum_{A \subset \theta} m(A) = 1$$

In interval valued Dempster Shafer theory the same constraint cannot be defined. Fuzzy valued evidence theory being extension of interval valued DS theory, the same constraint cannot be defined. There is no need of defining summation of all the basic probability assignment equal to 1. We can show that such a constraint becomes meaningless in fuzzy valued situation. Let

$$M(A_i) = \int_{x \in [0,1]} \mu_{A_i}(x) / x$$

$$i = 1, 2, \dots, n$$

where

$$A_i \subset \theta$$

$$\text{then } (M(A_i))_\alpha = [a_\alpha^i, b_\alpha^i], \forall \alpha \in [0,1]$$

$$\text{with } a_\alpha^i \leq b_\alpha^i$$

If we put the constraint on M that

$$\sum_{i=1}^n M(A_i) = \tilde{1}$$

where

$$\mu_{\tilde{1}}(x) = \begin{cases} 1 & \dots \dots \dots x = 1 \\ 0 & \dots \dots \dots x \neq 1 \end{cases}$$

Then it would be equivalent to

$$\sum_{\alpha \in [0,1]} \sum_{i=1}^n [M(A_i)]_{\alpha} = \tilde{1}$$

$$\Rightarrow \sum_{\alpha \in [0,1]} \sum_{i=1}^n [a_i^{\alpha}, b_i^{\alpha}] = [1,1]$$

$$\Rightarrow [ \sum_{\alpha \in [0,1]} \sum_{i=1}^n [a_i^{\alpha}, \sum_{\alpha \in [0,1]} \sum_{i=1}^n b_i^{\alpha}] ] = [1,1]$$

with

$$a_i^{\alpha} \leq b_i^{\alpha} \forall i = 1, 2, \dots, n$$

and  $\alpha \in [0,1]$

This equality holds only if

$$a_i^i = b_i^i \forall i = 1, 2, \dots, n$$

$$a_{\alpha}^i = b_{\alpha}^i = 0 \forall \alpha \in [0,1)$$

In other words, the summation constraint used in the original theory is suitable only for point valued or single number valued situations. Thus we can conclude that fuzzy valued basic probability assignment by adding the constraint

$$\sum_{A \subset \theta} M(A) = \tilde{1}$$

Analogous to original theory, we define the focal element A, as subset of universe where  $M(A) \neq \tilde{0}$

**CASE STUDY REVISED**

Let us consider the example studied in fuzzy and evidence reasoning by Lee. A simplified version of pneumonia diagnosis in which there are only three possible organism causing pneumonia {pneumococcus, legionella, klebsiella}

Suppose the doctor is asked to give his opinion on the cause of pneumonia for particular patient. Due to lack of complete knowledge the doctor may provide his opinion as the chance that disease is caused by pneumococcus is around 50% legionella is around 30%, klebsiella is around 20% chance that the disease is caused by any of these three organism

Obviously there is uncertainty in doctors opinion. The original DS theory does not provide any means to model such uncertainty. Neither interval valued approach can handle this uncertainty. We shall extend DS theory so that it can treat this type of uncertainty.

Fuzzy valued basic probability assignment

Let  $\theta$  be the universe or frame of discernment. We define the following function

$$\tilde{M}: 2^{\theta} \rightarrow F[0,1] \tag{3}$$

With  $M(\phi) = 0, O = \{0/x, \forall x \in [0,1]\}$  (4)

as an fuzzy valued basic probability assignment over  $\theta$  where  $2^\theta$  presents power set of  $\theta$  while  $F[0,1]$  denotes type 1 fuzzy set on  $J$ , where  $J \subset [0,1]$

$J$  is called as likelihood interval. In order to distinguish the fuzzy basic probability assignment from the original one (m), we denote it by  $M$ .

In above example we denote the universe  $\theta = \{P, L, K\}$

The doctors opinion can be described by fuzzy basic probability assignment as follows

$$\tilde{M}(\{P\}) = (0.4, 0.5, 0.6) \text{ Triangular fuzzy number around } 0.5$$

$$\tilde{M}(\{L, K\}) = (0.2, 0.3, 0.4) \text{ Triangular fuzzy number around } 0.3$$

$$\tilde{M}(\{P, L, K\}) = (0.1, 0.2, 0.3) \text{ Triangular fuzzy number around } 0.2$$

Comparing definition represented by equation (1) and (2)

With the basic probability assignment definition in DS theory, this summation constraint is not needed in the interval valued basic probability assignment. Similarly, we propose the constraint as following way.

The nonnegative assignment for every focal element may be fuzzy number (i.e. fuzzy set which is convex and normal).

## FUZZY VALUED BELIEF FUNCTION

The quantity  $\tilde{M}(A)$  actually measures the belief that one commits exactly to  $A$ , not the total belief that one commits to  $A$ . In order to obtain the measure total belief committed to  $A$ , we must add all the basic probability assignments that are subsets of  $A$ .

$$\text{Bel}(A) = \sum_{B \subset A} \tilde{M}(B) \quad (5)$$

where summation represents sum of fuzzy sets.

**Definition:** A function  $\text{Bel} : 2^\theta \rightarrow F[0,1]$  is called fuzzy valued belief function over  $\theta$  if it is defined by equation (3) over fuzzy valued basic probability assignment  $M$  defined by the equation (1) and (2).

From the above example we have

$$\begin{aligned} \text{bel}(\{P, L\}) &= \tilde{M}(\{P\}) = (0.4, 0.5, 0.6) \\ \text{Bel}(\{P, L, K\}) &= \tilde{M}(\{P\}) + \tilde{M}(\{L, K\}) + \tilde{M}(\{P, L, K\}) \\ &= (0.4, 0.5, 0.6) + (0.2, 0.3, 0.4) + (0.1, 0.2, 0.3) \\ &= \begin{cases} \frac{10x-7}{3} & \dots\dots 0.7 \leq x \leq 1 \\ 0 & \dots\dots \text{otherwise} \end{cases} \end{aligned}$$

**FUZZY VALUED PLAUSIBILITY FUNCTION**

Another important measure in the evidence theory is plausibility function that measures the degree to which one finds an event plausible. We define the fuzzy valued plausibility function

$$P\tilde{I} : 2^\theta \rightarrow F[0,1]$$

As follows 
$$P\tilde{I}(A) = \sum_{A \cap B \neq \phi} \tilde{M}(B)$$

Where summation denotes sum of fuzzy sets as defined above.  $P\tilde{I}(A)$  can be interpreted as total probability that A might occur  $P\tilde{I}(A)$  is fuzzy valued as the summation of fuzzy values  $\tilde{M}(B)$  is again a fuzzy number.

There are more than forty methods to compare two fuzzy numbers. In original DS theory

$$Bel(A) \leq P\tilde{I}(A) \dots \dots \dots \forall A \subset \theta$$

On the same standards we will show that

$$Be\tilde{I}(A) \leq P\tilde{I}(A) \dots \dots \dots \forall A \subset \theta$$

In the following example,

$$\begin{aligned} p\tilde{I}(\{P, L\}) &= \sum_{A \cap \{P, L\} \neq \phi} \tilde{M}(A) \\ &= \tilde{M}(\{P\}) + \tilde{M}(\{P, L\}) + \tilde{M}(\{P, L, K\}) \\ &= \begin{cases} \frac{10x - 7}{3} \dots \dots 0.7 \leq x \leq 1 \\ 0 \dots \dots \dots \text{otherwise.} \end{cases} \end{aligned}$$

**EVIDENCE COMBINATION**

Suppose we have two pieces of evidence associated with the fuzzy valued basic probability assignment  $M_1$  and  $M_2$ . The combined fuzzy valued basic probability assignment can be obtained by use of following formula

$$M(C) = \begin{cases} \sum_{A \cap B = C} M_1(A) \times M_2(B) \dots \dots \dots \text{if } C \neq \phi \\ 0 \dots \dots \dots \text{if } C = \phi \end{cases}$$

Where summation denotes sum of fuzzy numbers and cross denotes multiplication of fuzzy numbers.

It should be noted that no normalization is required as we don't have constraint in the definition of fuzzy valued basic probability assignment.

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