



Model and Solve the Bi-Criteria Multi Source Flexible Multistage Logistics Network

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Abstract

Flexible Multistage Logistics Network (fMLN) is an extension of the traditional multistage logistics network whereby a customer can procure goods directly from plants or distribution centers needless of retailers. This research intends to formulate the bi-criteria multi source single product fMLN model and discover methods to solve it. Here, total logistics cost and total product delivery time should be minimized simultaneously. By far, fMLN problems have been dealt with in single source form, meaning each customer could only be served by only one source. Because this issue is NP-hard, meta-heuristic techniques such as Genetic Algorithm (GA) have been used to solve the problem. However, under realistic settings, fMNL is multi-source, meaning each customer may be served by a number of facilities simultaneously. Because a multi-source fMNL problem is more complex than the single source in terms of both options as well as constraints, GA will also require enhancement. The proposed solution of this research is representing the chromosome in a new state, capable of improvising the constraints of the problem by a considerable ratio and with the defined crossover and mutation to solve the general bi-criteria multi-source fMNL. The obtained result using enhanced GA will show that it is dramatically improved comparing with using standard GA in order to having lower cost and time.

Key Words

Bi-criteria multi source Flexible Multistage Logistics Network (fMLN), Genetic Algorithms, Multi-objective optimization, PARETO solution.

I. INTRODUCTION

Gen et al. [11] claimed that although the traditional multistage logistics network model and its application had made a big success in theory and business practices, the traditional structure of logistics network is unable to fit very well with the fast changing competitive environments and meet the diversified customer demands. Therefore, Gen et al. [11] introduced three new delivery modes in which the goods can move from plants to retailer directly not via distribution centers, or sometimes the customer can get the goods from plant or from distribution center directly not via retailer. The authors called this new logistics network as the flexible Multistage Logistics Network (fMLN) as it is shown by Figure 1.

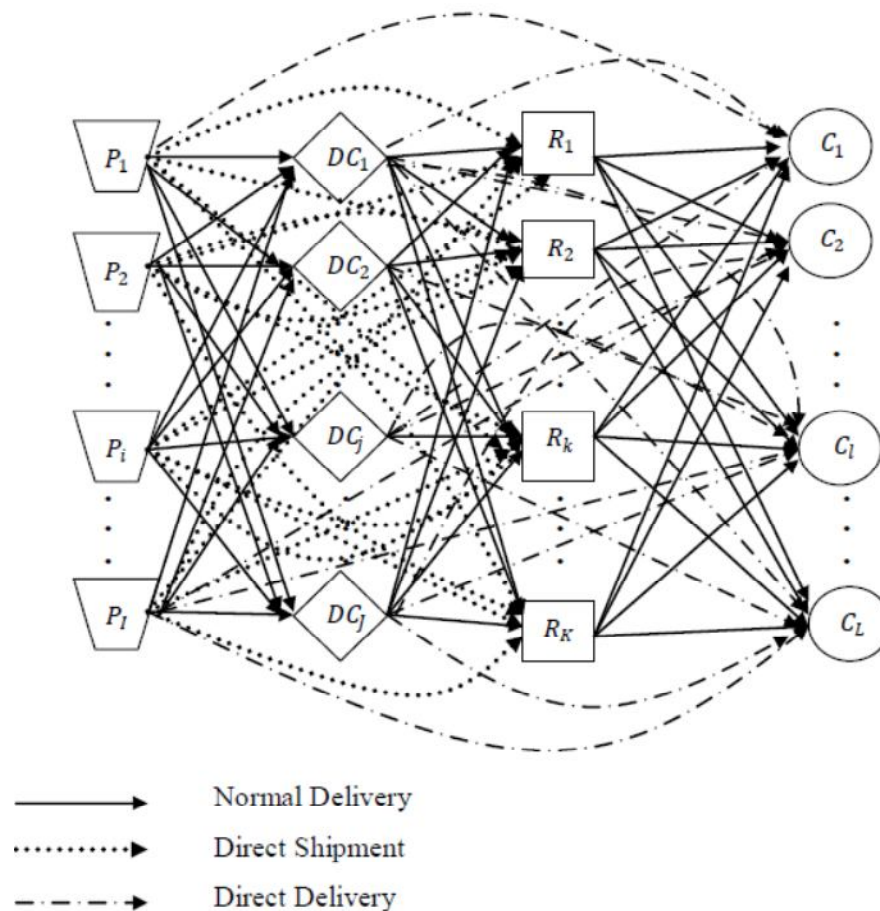


FIGURE 1: THE STRUCTURE OF FLEXIBLE MULTISTAGE LOGISTICS NETWORK (fMLN) MODELS [11].

Gen et al. [11] indicated that the bi-criteria linear logistics model (or bi-criteria transportation problem: bTP) is a special case of multi-objective logistics model since the feasible region can be depicted with a two dimensional criteria space. The following two objectives are considered:

minimizing the total logistics cost and minimizing the total delivery time.

Rajabalipour et al, [16] considered two- stage logistic networks comprised of potential suppliers, distributing centers (DCs) and also actual consumers at the first level. Each consumer has pre-specified demand of single item product for a period of time (e.g. season, year and, etc.) and the network could be flexible with potential (probably expensive) direct shipments only from the supplier to the consumers.

Recently, GAs has been successfully applied to logistics network models. Michalewicz et al. [15] and Viagnaux and Michalewicz [19] are among the first who discussed the use of GA for solving linear and nonlinear transportation problems. In their study, while matrix representation was used to construct a chromosome, the matrix-based crossover and mutation had been developed. Another (GA) approach for solving solid TP was given by Li et al. [13]. They used the three dimensional matrix to represent the candidate solution to the problem. Syarif and Gen et al. [18] considered production/distribution problem modeled using tsTP and proposed a hybrid genetic algorithm. Gen et al. [10] developed a priority-based Genetic Algorithm (priGA) with new decoding and encoding procedures considering the characteristic of tsTP. Altiparmak et al. [3] extended priGA to solve a single-product multi-stage logistics design problem. The objectives are minimization of the total cost of supply chain, maximization of customer services that can be rendered to customers in terms of acceptable delivery time (coverage), and maximization of capacity utilization balance for DCs (i.e. equity on utilization ratios). Furthermore, Lin et al. [14] proposed a hybrid genetic algorithm to solve the location-allocation model's problem of logistic network, and Altiparmak et al. [2] also apply the priGA to solve a single-source, multi-product multi-stage logistics design problem. As an extended multi-stage logistics network model, Lee et al. [12] apply the priGA to solve a multi-stage reverse logistics network problem (mrLNP), minimizing the total costs to reverse logistics shipping cost and fixed cost of opening the disassembly centers and processing centers. Gen and Syarif [18] proposed a new approach called spanning tree-based hybrid genetic algorithm (hst- GA) to solve the multi-time period production/distribution and inventory problem (mt-PDI). Costa et al. [5] presented an innovative encoding–decoding procedure embedded within a genetic algorithm (GA) to minimize the total logistic cost resulting from the transportation of goods and the location and opening of the facilities in a single product three-stage supply chain network.

For any optimization problem, there is an optimization criterion (i.e. evaluation function) to be minimized or maximized. The evaluation function represents a measure of the quality of the developed solution. Searching the space of all possible solution is a challenging task. An additional constraint on the domain of search for the parameters makes the problem quite difficult. The constraints might affect the performance of the evolutionary process since some of the produced solutions (i.e individuals) may be unfeasible. Unfeasible solution represents a waste of computation effort. In fact, it was reported that no general methodology to handle constraints exist although several methods were introduced. Rejecting unfeasible individuals, penalizing unfeasible individuals or moving these individuals to the feasible domain are among the many

methods proposed [17].

There are some approaches to handle the constraints optimization problems such as death penalty, static penalties, dynamic penalties, GENOCOP system, Behavioral memory and etceteras [21]. For some similar problems to fMLN with high constraints some researchers tried to add some heuristic rules to GA to satisfy the problem constraints and obtain a good solution. Yaohua and Chi [20] proposed a random search based on heuristic rules and a dynamic rule selection method based on GA to solve large size single-stage batch scheduling problem and Alim and Ivanov [1] proposed some heuristic rules embedded GA to solve In-Core Fuel Management Optimization Problem. Craenen et al. [6] compared three different heuristics based Evolutionary Algorithm (EA) on the same problems and suggested the best one to solve the constraints optimization problems.

The general objectives of this paper are to formulate the bi-criteria multi-source single product flexible Multistage Logistics Network (fMLN) problem and to discuss the algorithms that we have developed to solve it.

II. MATERIALS AND METHODS

Bi-Criteria Multi Source Single Product fMLN Model

In general, a bi-criteria multi source flexible multistage logistics network (fMLN) problem is to establish the optimum product amount shipped from plants to the customers and the best product delivery routes to fulfill the customer's order with the optimum product delivery time in all network phase that reduce the total logistics network costs. The mathematical model of the bi-criteria (fMLN) is developed with the following assumptions:

1. Single product and single time period (week, month, season, year or etc) case of a logistics network optimization problem is considered.
2. There are a maximum of four levels: plants, DCs, retailers and customers.
3. There are three delivery modes: normal delivery, direct shipment and direct delivery
4. Every customer, retailer and distribution center (DC) can be served by multi facilities. There is no preference for retailers, DCs and customers to provide orders, consequently fulfilling orders through multi sources at once.
5. Customer demands are known in advance.
6. Customers will get products at the same price, no matter where he/she gets them; it means that the customers have no special preferences.

The following notations are used to formulate the model:

Notation:

Indices:

- i : index of plant ($i = 1, 2, 3, \dots, I$)
- j : index of DC ($j = 1, 2, 3, \dots, J$)
- k : index of retailer ($k = 1, 2, 3, \dots, K$)
- l : index of customer ($l = 1, 2, 3, \dots, L$)

Parameters:

- I number of plants
- J number of DCs
- K number of retailers
- L number of customers
- P_i Plant i
- DC_j DC $_j$
- R_k Retailer k
- C_l Customer l
- B_i Output of plant i
- d_l Demand of customer l
- C_{1ij} Unit shipping cost of product from P_i to DC_j
- C_{2jk} Unit shipping cost of product from DC_j to R_k
- C_{3kl} Unit shipping cost of product from R_k to C_l
- C_{4il} Unit shipping cost of product from P_i to C_l
- C_{5jl} Unit shipping cost of product from DC_j to C_l
- C_{6ik} Unit shipping cost of product from P_i to R_k
- T_{1ij} Shipping time per lot of product from P_i to DC_j
- T_{2jk} Shipping time per lot of product from DC_j to R_k
- T_{3kl} Shipping time per lot of product from R_k to C_l
- T_{4il} Shipping time per lot of product from P_i to C_l
- T_{5jl} Shipping time per lot of product from DC_j to C_l
- T_{6ik} Shipping time per lot of product from P_i to R_k
- u_j^D Upper bound of the capacity of DC_j
- u_k^R Upper bound of the capacity of R_k
- f_j^F Fixed part of the open cost of DC_j
- C_j^{1v} Variant part of the open cost (lease cost) of DC_j
- q_j^1 Throughout of DC_j
- $q_j^1 = \sum_{i=1}^I X_{1ij}, \forall j$
- f_j Open cost of DC_j
- $f_j = f_j^F + C_j^{1v} q_j^1, \forall j$
- g_k^F Fixed part of the open cost of R_k
- C_k^{2v} Variant part of the open cost (lease cost) of R_k

$$q_k^2 \quad \text{Throughout of } R_k$$

$$q_k^2 = \sum_{l=1}^L X_{3kl}, \forall k$$

$$g_k \quad \text{Open cost of } R_k$$

$$g_k = g_k^F + C_k^{2V} q_k^2, \forall k$$

Decision Variables:

$$X_{1ij} \quad \text{Product amount shipped from } P_i \text{ to } DC_j$$

$$X_{2jk} \quad \text{Product amount shipped from } DC_j \text{ to } R_k$$

$$X_{3kl} \quad \text{Product amount shipped from } R_k \text{ to } C_l$$

$$X_{4il} \quad \text{Product amount shipped from } P_i \text{ to } C_l$$

$$X_{5jl} \quad \text{Product amount shipped from } DC_j \text{ to } C_l$$

$$X_{6ik} \quad \text{Product amount shipped from } P_i \text{ to } R_k$$

$$y_j^1 = \begin{cases} 1, & \text{if } DC_j \text{ is open} \\ 0, & \text{otherwise} \end{cases}$$

$$y_k^2 = \begin{cases} 1, & \text{if } R_k \text{ is open} \\ 0, & \text{otherwise} \end{cases}$$

$$W_{1ij} = \begin{cases} 1, & \text{if } X_{1ij} > 0 \\ 0, & \text{otherwise} \end{cases}$$

$$W_{2jk} = \begin{cases} 1, & \text{if } X_{2jk} > 0 \\ 0, & \text{otherwise} \end{cases}$$

$$W_{3kl} = \begin{cases} 1, & \text{if } X_{3kl} > 0 \\ 0, & \text{otherwise} \end{cases}$$

$$W_{4il} = \begin{cases} 1, & \text{if } X_{4il} > 0 \\ 0, & \text{otherwise} \end{cases}$$

$$W_{5jl} = \begin{cases} 1, & \text{if } X_{5jl} > 0 \\ 0, & \text{otherwise} \end{cases}$$

$$W_{6ik} = \begin{cases} 1, & \text{if } X_{6ik} > 0 \\ 0, & \text{otherwise} \end{cases}$$

The two objective functions are to minimize the total logistic cost Z_1 and total product delivery time Z_2 . The mathematical model for the fMLN problem is given as follows:

$$\text{Min } Z_1 = \sum_{i=1}^I \sum_{j=1}^J C_{1ij} X_{1ij} + \sum_{j=1}^J \sum_{k=1}^K C_{2jk} X_{2jk} + \sum_{k=1}^K \sum_{l=1}^L C_{3kl} X_{3kl} + \sum_{i=1}^I \sum_{l=1}^L C_{4il} X_{4il} + \sum_{j=1}^J \sum_{l=1}^L C_{5jl} X_{5jl} + \sum_{i=1}^I \sum_{k=1}^K C_{6ik} X_{6ik} \quad (1)$$

$$\text{Min } Z_2 = \sum_{i=1}^I \sum_{j=1}^J T_{1ij} W_{1ij} + \sum_{j=1}^J \sum_{k=1}^K T_{2jk} W_{2jk} + \sum_{k=1}^K \sum_{l=1}^L T_{3kl} W_{3kl} + \sum_{i=1}^I \sum_{l=1}^L T_{4il} W_{4il} + \sum_{j=1}^J \sum_{l=1}^L T_{5jl} W_{5jl} + \sum_{i=1}^I \sum_{k=1}^K T_{6ik} W_{6ik} \quad (2)$$

Subject to:

$$\sum_{j=1}^J X_{1ij} + \sum_{l=1}^L X_{4il} + \sum_{k=1}^K X_{6ik} \leq b_i, \quad \forall i \quad (3)$$

$$\sum_{i=1}^I X_{1ij} = \sum_{k=1}^K X_{2jk} + \sum_{l=1}^L X_{5jl}, \quad \forall j \quad (4)$$

$$\sum_{j=1}^J X_{2jk} + \sum_{i=1}^I X_{6ik} = \sum_{l=1}^L X_{3kl}, \quad \forall k \quad (5)$$

$$\sum_{i=1}^I X_{4il} + \sum_{j=1}^J X_{5jl} + \sum_{k=1}^K X_{3kl} \geq d_l, \quad \forall l \quad (6)$$

$$\sum_{i=1}^I X_{1ij} \leq u_j^D y_j^1, \quad \forall j \quad (7)$$

$$\sum_{l=1}^L X_{3kl} \leq u_k^R y_k^2, \quad \forall k \quad (8)$$

$$X_{1ij}, X_{2jk}, X_{3kl}, X_{4il}, X_{5jl}, X_{6ik} \in N_0, \quad \forall i, j, k, l \quad (9)$$

where;

$$N_0 = \{0, 1, 2, 3, \dots\}$$

$$y_j^1, y_k^2 \in \{0, 1\}, \forall j, k \quad (10)$$

$$W_{1ij}, W_{2jk}, W_{3kl}, W_{4il}, W_{5jl}, W_{6ik} \in \{0, 1\}, \forall i, j, k, l \quad (11)$$

Where there are two objective functions that the objective function Eq. 1 means to minimize the total logistic cost and Eq.2 means to minimize the total product delivery time. The constraint in Eq.3 represents the production limit of plants. The constraints in Eq.4 and Eq.5 are due to the flow of conservation principle. The constraint in Eq.6 ensures that the customers' demands will be satisfied. The constraints in Eq.7 and Eq.8 ensure that the upper bound of the capacity of DCs and retailers cannot be surpassed. Eq.9 shows that the decision variables related to product amount are non-negative integer.

Proposed Route Based GA (RB-GA) to Solve Bi-Criteria Multi Source Single Product fMLN Problem

A tree-based representation is known to be one way for representing network problems [2]. There are three ways of encoding tree:

- (1) Vertex-based encoding
- (2) Edge-based encoding
- (3) Edge-and-vertex encoding

Gen et al. [11] used vertex-based encoding to solve single product fMLN problem. Using this chromosome representation, if the total demand to the plant exceeds its supply capacity, the customer is assigned to another plant with sufficient products supply and the lowest transportation price between the plant and the customer. According to the above representation, solution is based on finding the best route for delivering the product to each customer when the network is single source in the last layer. Based on above mentioned solution the customer is not allowed to split the order to be fulfilled from different sources simultaneously. The length of every chromosome here is equal to: $3 \times L$.

It is argued that the solution proposed by Gen et al. [11] is not useful to solve multi source fMLN problem. Here, Edge-and-vertex encoding is used to solve bi-criteria multi source fMLN problem and the new algorithm namely Route Based GA (RB-GA) is developed. Figure 2 represents the chromosome with edge- and vertex encoding. In a normal shipment which the number of plants, DCs, and retailers are I, J and K , representing the number of possible routes for product delivery from plant to each customer, using permutation theory is given by: $I \times J \times K$.

The total number of possible routes in fMLN = the number of routes for normal delivery + the number of routes for direct shipment + the number of routes for direct delivery = $I \times J \times K + I \times K + I + I \times J = I \times (J + 1) \times (K + 1)$

Therefore, the total number of possible routes for each customer named NOR and is given by:

$$NOR = I \times (J + 1) \times (K + 1)$$

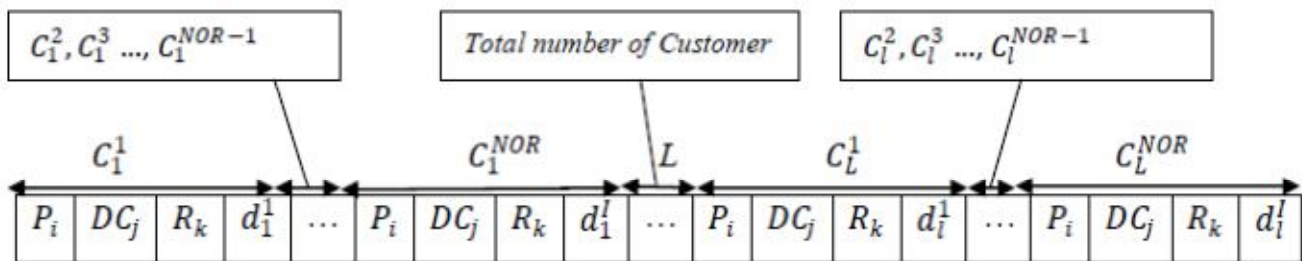


FIGURE 2: EDGE-AND-VERTEX ENCODING

Referring to Figure 2, it is obvious that the demand of customer I (d_i) is distributed to the possible routes which are pertinent to customer I with random amount between 0 and d_i . Therefore;

$$d_i = d_i^1 + d_i^2 + d_i^3 + \dots + d_i^l \text{ where } d_i^1, d_i^2, d_i^3, \dots \text{ and } d_i^l \text{ are generating as follows:}$$

$$d_i^l = \text{randomize}(d_i), \forall l$$

$$d_l^2 = \text{randomize} (d_l - d_l^1) , \forall l$$

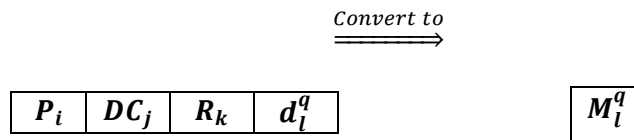
$$d_l^3 = \text{randomize} (d_l - (d_l^1 + d_l^2)) , \forall l$$

$$\vdots$$

$$d_l^{NOR-1} = \text{randomize} (d_l - \sum_{q=1}^{NOR-2} d_l^q) , \forall l$$

$$d_l^{NOR} = d_l - \sum_{q=1}^{NOR-1} d_l^q , \forall l$$

There are several units of the chromosome for every customer (the number of *NOR*) which every unit indicates one possible product delivery route to a customer. Here, to simplify and decrease the number of gene especially for large size problem case, every unit is changed to be one gene of the chromosome. Therefore, every gene shows a possible route for customer *l* with the amount of customer *l*'s demand as follows:



where

M_l^q indicates the q^{th} possible route to fulfill the l^{th} customer order, that is, the amount product shipped to customer l through the q^{th} route, where $q= 0, 1, 2, \dots, NOR$.

Therefore, Figure 2 can be changed to Figure 3 as below:

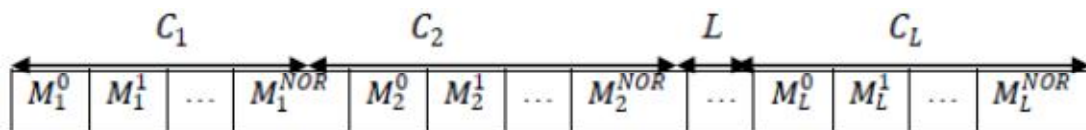


FIGURE 3: PROPOSED RB-GA CHROMOSOME REPRESENTATION

d_l is the l^{th} customer demand and $M_l^0, M_l^1, \dots, M_l^{NOR-1}$ and M_l^{NOR} could be generated as follows:

$$M_l^0 = \text{randomize} (d_l) , \forall l$$

$$M_l^1 = \text{randomize} (d_l - M_l^0)$$

$$M_l^{NOR-1} = \text{randomize} (d_l - \sum_{q=0}^{NOR-2} M_l^q), \forall l$$

$$M_l^{NOR} = d_l - \sum_{q=0}^{NOR-1} M_l^q, \forall l$$

therefore

$$d_l = M_l^0 + M_l^1 + M_l^2 + \dots + M_l^{NOR}$$

In fMLN with J number of DCs and K number of retailers are, the qth route for the Ith customer is defined by the following procedures:

- i) Let w be the quotient of $q \div (K+1)$ and s is the remainder of $(q \div (K+1))$.
- ii) Let q be the quotient of $w \div (J+1)$ and r is the remainder of $(w \div (J+1))$.

The ID of plant involved q^{th} route is: $q+1$, which indicates the first node of the route.

The ID of DC involved q^{th} route is: r , which indicates the second node of the route.

The ID of retailer involved q^{th} route is: s , which indicates the third node of the route.

For further illustration suppose that $I=2$ which is the number of plants, $J=2$ which is the number of DCs and $K=3$ which is the number of retailers. Here, the total number of possible routes for each customer is calculated as: $I \times (J + I) \times (K + 1) = 2 \times (2 + 1) \times (3 + 1) = 24$. Suppose that the 17th route is required, therefore, according to above procedures this route is defining as follows:

$$17 \div (3 + 1) \implies w = 4 \text{ and } s = 1$$

$$4 \div (2 + 1) \implies q = 1 \text{ and } r = 1$$

Therefore;

The ID of plant involved 17th route is $q+1= 2$, the ID of DC involved this route is $r =1$ and the ID of retailer involved this route is $s = 1$, that is:

Plant 2 \rightarrow DC 1 \rightarrow Retailer 1 \rightarrow Customer /

Subsequently the following decision variables could be defined as well:

X_{121} which is product amount shipped from Plant 2 to DC 1.

X_{211} which is product amount shipped from DC 1 to Retailer 1.

X_{31l} which is product amount shipped from Retailer 1 to Customer l

Based on above explanation, all types of decision variables ($X_{1ij}, X_{2jk}, X_{3kl}, X_{4il}, X_{5jl}, X_{6ik}$) and their values can be obtained using the following procedures (Figure 4):

```
Procedure: Define the product amount shipped at every arc of fMLN.
Input: Number of Plants (I), number of DCs (J), number of Retailers
      (K) and the number of Customer
Output: Decision variables and their values
for l = 1 to L
  for i = 1 to I
    for j = 0 to J
      for k = 0 to K

        if j ≠ 0
          we have:  $X_{1ij}$ 
          if k ≠ 0
            we have:  $X_{2jk}$  and  $X_{3kl}$ 
          else
            we have:  $X_{5jl}$ 
          end

        else
          if k ≠ 0
            we have:  $X_{6ik}$  and  $X_{3kl}$ 
          else
            we have:  $X_{4il}$ 
          end
        end
      end
    end
  end
end
end
Output: Decision variables and their values
```

FIGURE 4: PSEUDO-CODE TO DERIVE DECISION VARIABLES FROM POSSIBLE ROUTES

It is obvious that to serve every customer at least one plant is needed. Considering the multi source assumption where each customer can be served by multi facilities the ID of plants must be non zero while the ID of DCs and retailers could be zero since for the flexible logistics model, there is direct shipment or direct delivery. Therefore, the customer would be able to split his/her order to be fulfilled from different facilities. Based on the above mentioned chromosome representation (Figure 4), the total number of genes for every customer is equal to:

$$I \times (J + 1) \times (K + 1)$$

(I is the total number of plants, J is the total number of DC and K is the total number of retailer). Subsequently the total number of genes for every chromosome is calculated as follows:

$$[I \times (J + 1) \times (K + 1)] \times L \quad (L \text{ is the total number of Customers}).$$

Every gene is one set of ID of a plant, ID of DC and ID of retailers with the part of amount of customer demand. The NOR number of gene constitutes one unit for every customer where each unit represents all possible delivery routes to a customer with amount of customer demand for each route from the plant via DC and retailer.

Using this encoding method, an infeasible solution may be generated, which violates the facility capacity constraints, where the penalty method could be useful. As it was mentioned earlier about difficulty for satisfying the two main constraints which are:

$$\sum_{i=1}^I X_{1ij} = \sum_{k=1}^K X_{2jk} + \sum_{l=1}^L X_{5jl} , \quad \forall j$$

$$\sum_{j=1}^J X_{2jk} + \sum_{i=1}^I X_{6ik} = \sum_{l=1}^L X_{3kl} , \quad \forall k$$

Proposed RB-GA could satisfy them simply by embedding to the chromosome representation. The above mentioned constraints depict that the summation of incoming product amount to each facility (DC or Retailer) must be equal to summation of out coming product amount from the same facility.

Using proposed RB-GA; define the all possible routes for product delivery to each customer which every route contains the constant amount of the product at all stage of network. It is obvious that every route which passes each DC or retailer has the same amount of incoming and out coming in that DC or retailer. Therefore the summation of all incoming of all routes are equal to the summation of all out coming of all routes at every DC or retailer. In conclusion, it is true that the above equality constraints are satisfied simply.

By this chromosome representation the best delivery route/s for each customer and the optimum product amount for every stage of the network will be found. Furthermore the decision about open/close facilities will be made.

Crossover for Edge-and-Vertex Encoding (RB-GA):

Figure 5 shows an example of proposed crossover in this research. It randomly selects two cutting points and then exchanges the substrings between the two parents.

Cutting point 1 = Randomize (L)

Cutting point 2 = Randomize (L)

where; L = total number of customers.

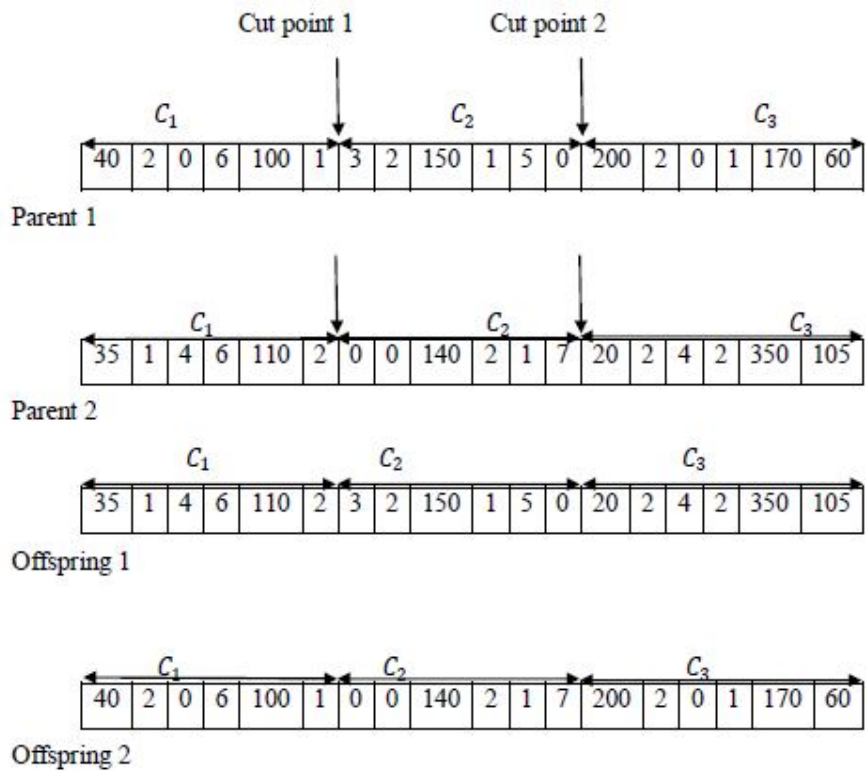


FIGURE 5: AN EXAMPLE OF PROPOSED CROSSOVER

The cross points located at the beginning of every unit, therefore, after crossover, an infeasible solution may not be generated and repairing procedure is not needed.

Mutation for Edge-and-Vertex Encoding (RB-GA)

Here, some offspring will be selected according to mutation rate and the mutation procedure is explained as below:

- 1- Generate randomize $(L-1) + 1$ // I^{th} customer will be found//
- 2- Generate randomize (d_i) denoted as m_n .
- 3- Generate randomize (NOR) denoted as m_u , and NOR is the total possible routes for product delivery to each customer // the number of specific gene of I^{th} customer that must be mutated will be found //
- 4- $m_u \leftarrow m_n$
- 5- Generate randomly the number of route (randomize (NOR)) denoted as A , then set $g_m = RT - A$ // the number of second specific gene of I^{th} customer that must be mutated will be found //
- 6- $g_m \leftarrow m_n - \text{randomize}(m_n)$

It is noted that using the proposed crossover and mutation, the chromosomes still would be able to satisfy the equality constraints.

As it was mentioned before, the bi-criteria fMLN problem is a special case of multi objective optimization. In general, with the multiple criteria/objectives, it is impossible to obtain a distinctively optimal solution for all the proposed models. This means that search techniques are required to search a set of concession solutions first, followed by the part where the decision maker uses the preference relation to rank them.

Genetic Algorithms had already been used to solve multi objective problems. Here, RB-GA is developed to solve bi-criteria multi source single product fMLN problem. In principle, multiple objective optimization problems are very diverse from single objective optimization problems. For single objective case, one tries to obtain the best solution, which is totally superior to all other options. In the case of multiple objectives, a solution that is the best with respect to all objectives does not necessarily exist mainly because of the incommensurability and conflict among objectives. A solution may be the best in one objective but the worst in other objectives. Therefore, more often than not exists a set of solutions for the multiple objective cases which cannot simply be compared with one another. Such type of solutions can be named as non-dominated solutions or Pareto optimal solutions, for which no improvement in any objective function is achievable devoid of sacrificing at least one of the other objective functions.

III. RESULT

As a matter of fact, in bi-criteria fMLN problem there are two objective functions represented as f_1 and f_2 that must be evaluated in the selection step of GA as following:

$$f_1 = \text{Min } Z_1 = \sum_{i=1}^I \sum_{j=1}^J C_{1ij} X_{1ij} + \sum_{j=1}^J \sum_{k=1}^K C_{2jk} X_{2jk} + \sum_{k=1}^K \sum_{l=1}^L C_{3kl} X_{3kl} +$$

$$\sum_{i=1}^I \sum_{l=1}^L C_{4il} X_{4il} + \sum_{j=1}^J \sum_{l=1}^L C_{5jl} X_{5jl} + \sum_{i=1}^I \sum_{k=1}^K C_{6ik} X_{6ik} + \sum_{j=1}^J f_j y_j^1 + \sum_{k=1}^K g_k y_k^2$$

$$f_2 = \text{Min } Z_2 = \sum_{i=1}^I \sum_{j=1}^J T_{1ij} W_{1ij} + \sum_{j=1}^J \sum_{k=1}^K T_{2jk} W_{2jk} + \sum_{k=1}^K \sum_{l=1}^L T_{3kl} W_{3kl} + \sum_{i=1}^I \sum_{l=1}^L T_{4il} W_{4il} + \sum_{j=1}^J \sum_{l=1}^L T_{5jl} W_{5jl} + \sum_{i=1}^I \sum_{k=1}^K T_{6ik} W_{6ik}$$

Where f_1 shows the total logistics cost and f_2 shows the total product delivery time of fMLN. In this case we try to obtain a PARETO optimal solution as it is shown in Figure 6.

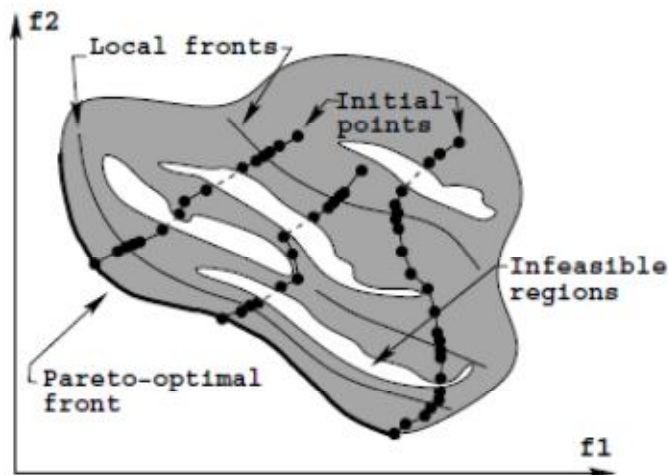


FIGURE 6: AN EXAMPLE OF PARETO OPTIMAL SOLUTION OF BI-CRITERIA PROBLEM [4].

PARETO optimality characterizes the boundary of solutions that can be achieved by trading-off clashing objectives in the most favorable approach. From here, a decision maker (be it a human or an algorithm) is able to finally select the settings that suit best according to his judgment. The notation of optimal in the Pareto logic is sturdily according to the meaning of supremacy and dominance: An element x_1 dominates (is preferred to) an element x_2 if x_1 is better than x_2 in at least one objective function and not worse with reverence to the rest of the objectives. Pareto Optimal definition is an element $x^* \in X$ is Pareto optimal (and hence, part of the optimal set X^*)

if it is not dominated by any other element in the problem space X . When it comes to Pareto optimization, X^* is called the Pareto set or the Pareto Frontier.

In the selection and evaluation part RB-GA there are two dissimilar values of fitness function for each chromosome. One is the delivery cost value (f_1) and the second one is delivery time value (f_2) of each chromosome. In order to attain PARETO solution it require to obtain the optimal front (front 1) that these solution are not dominated by the other feasible solutions. Figure 6 shows the Pseudo-code of fitness evaluation part of proposed algorithms selection part.

```

Procedure: Selection of bi-criteria fMLN
Input: two objective functions ( $f_1$  and  $f_2$ ) and size of them ( $m$ ), old Population ( $N$ )
*/Total number of chromosomes ( Population size + all offspring)/*
Output: new Population
Input: Cost and time of every chromosome

N= Population size + all offspring

S(1 to N)=N+1; */ defining the fronts in problem space based on fitness function,
for instance S(1) is the front of chromosome 1/*
f=1; */ front 1/*

while(max(S)==N+1)
  for all the chromosomes with  $S \geq f$ 
    if (cost of chromosome (i) <= (cost of chromosome (j))
      or
      (time of chromosome (i) <= (time of chromosome (j))
        S(chromosome(i)) ← f
    end
  end
  f=f+1;
end
Sort all chromosomes based on S
Output: new population
  
```

FIGURE 6: PSEUDO-CODE OF FITNESS EVALUATION PART OF PROPOSED ALGORITHMS SELECTION PART

IV. DISCUSSION

Here, the numerical results of using RB-GA are presented, and also standard GA is used to proof that the solutions quality obtained using RB-GA is good. As it was explained earlier, instead of having only one solution, there is a set of solution for every case. The proposed algorithm would be able to obtain a PARETO front based on defined selection part which it had already been explained in Figure 6. The solutions exist in PARETO front are the mentioned solution set that should be considered. Every case result presents the non-dominated solutions in PARETO front including transportation cost and product delivery time. For every customer there is a set option of the best delivery route as long as there is a set solution including optimum total logistics cost and optimum total product delivery time. Three problem cases will be examined using GA and RB-GA as shown in following tables:

TABLE 1: PROBLEM CASE 1 WITH THE OBTAINED PARETO SOLUTION

<i>Problem Case # 1: I=2 , J=2 , K=2 , L=20 , Maximum Generation =200</i>			
<i>GA</i>		<i>RB-GA</i>	
<i>Cost</i>	<i>Time</i>	<i>Cost</i>	<i>Time</i>
3507	391	1211	224
3545	387	1964	127

TABLE 2: PROBLEM CASE 2 WITH THE OBTAINED PARETO SOLUTION

<i>Problem Case # 2: I=3 , J=5 , K=7 , L=100 , Maximum Generation =2000</i>			
<i>GA</i>		<i>RB-GA</i>	
<i>Cost</i>	<i>Time</i>	<i>Cost</i>	<i>Time</i>
169263	17918	7186	1632
168993	17989	14012	1089

TABLE 3: PROBLEM CASE 3 WITH THE OBTAINED PARETO SOLUTION

<i>Problem Case # 3: I= 4 , J= 6 , K= 9 , L= 150 , Maximum Generation =4000</i>			
<i>GA</i>		<i>RB-GA</i>	
<i>Cost</i>	<i>Time</i>	<i>Cost</i>	<i>Time</i>
266145	25932	31965	3395
264688	26022	35330	3197
-	-	35020	3197

The data used in this research was generated by the authors as the problem case. Diverse 3 problem cases have been created by this research to implement the proposed algorithm and compare the obtained solutions. Hardware platforms employed by the researcher were a 2 GHz processor intel core 2 duo with 1GB memory and running windows 7 professional. It is noted that the scope of this research did not include establishing necessary conditions to hardware requirements. The mathematical model of fMLN was translated into program written in Matlab version 7, 2009. The problem cases were kept to use for different proposed algorithm implementation and the obtained results were displayed in a proper manner in Matlab.

V. CONCLUSION

In this paper, the bi-criteria multi source single product fMLN model that considers the transportation cost and time was formulated. Subsequently, the proposed solution RB-GA which has been explained comprehensively was further developed. To solve multi objectives problems, PARETO solution is needed where a set solution was presented instead of one. The definition of PARETO optimality and non-dominated solution was explained in this paper as well. Additionally, the Pseudo-code of proposed selection part of mentioned algorithm was presented. Lastly, the numerical experiment was explained where it is proven that the obtained solutions using RB-GA are more preferable than the obtained solutions using GA.

The research work presented in this thesis has opened a new line of research in which a number of avenues of future work remain to be investigated. Although there was attempt to solve bi-criteria flexible multistage logistics network problem in this research, however there is still possibility to solve some more variants of fMLN problems by changing the current assumptions of fMLN model and it might be needed to propose some new techniques such as combinatorial algorithms or hybrid algorithms to solve certain specific defined problems. Besides, there is still possibility to propose new methods to decrease the running time of algorithm. Therefore, the following research works could be recommended to enhance the contributions of this area:

- 1- To solve the fMLN problems when the product price is not fix for every condition of the network. For instance, two types of product price are different when the customer wants to order one by one or wants to order together.
- 2- To solve multi objectives fMLN problem when there are more than two objectives, such as minimizing the transportation cost, minimizing the inventory cost, minimizing the product delivery time, minimizing the warehousing cost and maximizing the customer satisfaction level simultaneously.
- 3- To develop the algorithm for constraints handling in a better way for the above mentioned problems

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