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DIRICHLET'S PRINCIPLE AS AN ELEMENTARY MATHEMATICAL MODEL IN MATHEMATIC EDUCATION

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Abstract

One important part of nowadays research in theory of mathematics education is focused on searching for some interesting and non-traditional mathematical topics and themes. The topic named the "Dirichlet's Principle" is aimed to promote and exercise mathematical competencies by using quite simple mathematical rules. The rules were discovered and described in the 19th century by Johann Peter Gustav Lejeune Dirichlet (1805-1859). The article describes case studies as the results of qualitative research of mathematics' lessons with the topic of Dirichlet's principle at upper-secondary school and university. The elements of the history of mathematics concerning the topic and Dirichlet's personality are also involved. Definitions, examples of exercises, tasks and problems are given in the article, too. Recommended methods of teaching with the focus on active methods, which support inquiry based learning, are described. Two case studies, one realized at secondary grammar school (17 year-old pupils) and one at university (student teachers) are described as examples of theoretical and methodological background implementation. The context of tasks and problems which are solved by using Dirichlet's principle is set in real life situations. The topic content therefore helps to show the importance of mathematical knowledge for everyday life.
Key words: *case study, elementary mathematical model, Johann Peter Gustav Lejeune Dirichlet, problem solving.*

Introduction

In different countries the principle is given different names. As Grozdev states (2006, s.6), it is sometimes called "the drawers principle", "the pigeon-hole principle". It is also well-known as bearing the name of the German mathematician Johann Peter Gustav Lejeune-Dirichlet. The principle was named after Dirichlet not for being given the credit of inventing it but to honour his use of the principle to solve wide range of different problems from the field of number theory.

On one hand, Dirichlet's principle is undemanding because it is based on a simple, common-sense logic. On the other hand, the principle enables to derive other, more demanding results, what makes it a good stimulus and motivation method for raising the students' interest in mathematics within the education system as well as in other surroundings.

There are several reasons why it could be interesting to involve the problems using Dirichlet's principle in teaching:

1. Simplicity: we mean the simplicity of the tasks and their proofs as the task itself is usually

clear and it is possible for students not only to prove it but also to understand and explain it.

2. Problem solving: the tasks that are solved by the use of Dirichlet's principle teach students to solve problems, develop their mathematical competencies and logical thinking.

3. Variety of problems: the tasks vary in their demands; in their focus on the field of mathematics they are based on. Consequently, every problem can be taken from different context what can be motivating for students.

4. The beauty of mathematics: except for the reasons mentioned above, these problems are usually, short, elegant and understandable, and that is the way they help to reveal the beauty of mathematics.

Lessons' Plan Considerations

All these points were taken into consideration when creating the lessons' plans concerning Dirichlet's principle which could be applied in secondary schools' mathematics teaching. Nowadays it is believed that mathematics education should be reflected in students' mathematics competences to understand, process and apply the acquired knowledge in real life situations. To be able to help in everyday life, mathematics must be accepted by students as a friend and a helper. It is mostly not common for students to look at mathematics from this point of view. This belief opens the space for teachers, new methods and ideas to show and prove students that mathematics can be exciting and that it is more than formulas, equations or worries about correctness of results.

Lessons' Plan for Teaching Dirichlet's Principle at Secondary Schools

Because of the fact that Dirichlet's principle is not an extensive unit in mathematics, we propose it to be integrated into lessons between two larger units. It would mean a "short relax" for students as it would not be a usual topic of lessons and at the same time it could give them the opportunity to improve their abilities and experiences in solving different kinds of problems. We assume that it is enough to plan two or three 45-minute lessons to introduce, explain, practice and evaluate the basic points suitable for secondary school students.

In the suggested lesson plan, the first lesson starts by introducing the idea that the topic of the following lessons will be different from traditional ones. New activities will require more thinking and reasoning though they are based on a certain principle, but many times it is very well hidden. In spite of that, students are expected to cooperate and be fully engaged in the activities because they will be required to evaluate their findings and solutions in the end.

Nowadays almost every secondary school in Slovakia is adequately technically equipped. It opens up more possibilities that might be used in teaching of mathematics. The introduction to the topic of Dirichlet's principle is made by PowerPoint presentation which includes the following steps in exposition.

1. Brainstorming: Students are asked to brainstorm all the names of mathematicians they know. Their list of names is compared to the mathematicians mentioned in the first slide of the presentation. We expect students to be familiar with (at least) the following people: Euclid, Pythagoras, Thales, Newton, Viété.

2. Dirichlet's identification: Students are shown Dirichlet's picture (2nd slide of the presentation) and asked to identify him. If they do not succeed (what is highly probable), they are asked to solve the crossword on the third slide of the presentation whose answer is the name of the mathematician. They are given clues to reveal the mathematical expressions which constitute the lines of the crossword. The clues are based on mathematical knowledge from

different fields of mathematics. All of them should be familiar to students of the particular grade.

3. Dirichlet's biography: Dirichlet's life and work are introduced in brief and catchy points. Elements of history are not commonly taught in mathematics lessons, so they can be seen as another interesting factor of lessons. The information about Dirichlet's biography and contribution was taken from the archive of University of St Andrews, Scotland in the MacTutor History of Mathematics online. Students are also informed that they can look here up information about any mathematician they want.

The following events of Dirichlet's life are mentioned in the presentation:

- Johann Peter Gustav Lejeune Dirichlet is one of the biggest mathematicians of all times.
- He was born on 13th February 1805 (Düren), died 5th May 1859 (Göttingen, Hanover).
- The origin of his name comes from "Le jeune de Richelet" meaning "Young from Richelet" as his family came from the Belgium town of Richelet.
- At the age of 12 he was buying mathematics books and he was an excellent gymnasium student.
- After two years at the Gymnasium in Bonn he started to attend the Jesuit College in Cologne and there he was taught by Ohm.
- When he was 16, he had completed all the necessary school qualifications and went to study at the university in Paris because the standards in German universities were not high.
- He had some of the leading mathematicians as teachers there.
- Since the summer of 1823 Dirichlet worked for General Maximilien Sébastien Foy who was an important figure in the army during the Napoleonic Wars.
- Dirichlet's first paper was to bring him instant fame since it concerned the famous Fermat's Last Theorem. The theorem claimed that for $n > 2$ there are no non-zero integers x, y, z such that $x^n + y^n = z^n$. Dirichlet attacked the theorem for $n = 5$.
- After General Foy's death he returned to Germany where he was supported by Alexander von Humboldt.
- He taught at Breslau, later was appointed a professor at the University of Berlin where he remained from 1828 to 1855.
- After Gauss's death in 1855, he was offered his chair at Göttingen, which he accepted and enjoyed the quieter life there, though not for a very long time.
- Dirichlet's character and teaching qualities are summed up as follows: *excellent teacher, great clarity, modest in his manners, shy and reserved, seldom speaking and not willing to appear in public.*
- Thomas Hirst described Dirichlet at the age of 45 in the following way:

He is a rather tall, lanky-looking man, with moustache and beard about to turn grey with a somewhat harsh voice and rather deaf. He was unwashed, with his cup of coffee and cigar. One of his failings is forgetting time, he pulls his watch out, finds it past three, and runs out without even finishing the sentence.

- Koch summed up Dirichlet's contribution as follows:

. ... important parts of mathematics were influenced by Dirichlet. His proofs characteristically started with surprisingly simple observations, followed by extremely sharp analysis of the remaining problem. With Dirichlet began the golden age of mathematics in Berlin.

4. Dirichlet's principle with the proof: The principle is explained in its weak and strong forms, only the simplest form is proved. A teacher can ask students about possible applications

of the principle. The basic rule, a weak form of the principle, says:

(1) *If n + 1 objects are distributed into n groups, then at least two objects are in at least one and the same group.*

We can prove it by contradiction: Let there be a group with not more than one object. Then there are no more than *n* objects, what is a contradiction.

The principle can be generalized as follows (Grozdev, 2006, s.6):

(2) *If m objects are distributed into n groups and m>n, then at least two of the objects are in one and the same group*.

In its strong form, the principle is presented in the following statement:

(3) *Let q_1, q_2, ..., q_n be natural numbers. If there are $q_1 + q_2 + ... + q_n + 1$ objects distributed into n groups, then either the first group contains at least q_1 + 1 objects, or the second group contains at least q_2 + 1 objects, or the n^{th} group will contain at least q_n+1 objects.*

Particularly, in the presentation it is shown as the consequence of this statement:

(4) *If m objects are distributed into n groups and m > nk, where k is a natural number, then at least k+1 objects fall into one of the groups.*

5. Examples with solutions – from the simplest to the most difficult: Different kinds of problems with Dirichlet's principle are presented. At first they can be fully explained, later, depending on students' understanding, students can be asked to participate in solving them. According to the pace of the activities and students' abilities, they can be asked to solve one of the problems in the presentation by working on it in pairs. Another option is to give one problem as homework assignment if the presentation has not been covered by the length of the lesson. The following examples were used:

a) *If there are 4 flowers to be put in 3 vases, at least 2 flowers will be in the same vase.*

b*) In each group consisting of 13 people, there are at least 2 people who were born in the same month*.

Solution: There are 13 people and just 12 months of the year. All the people can not have their birthday in a different month, so there must be at least 2 people who were born in the same month. (This can be done as a test in the classroom as well.)

c) *There are 10 socks which are mixed in the drawer and it is dark in the room. How man socks do we have to take out of the drawer to be sure that we have a matching pair?*
Solution: If we choose 10 or less socks, it is possible that all will be the same. But if we take any 11 of them, there must be at least one matching pair.

d) *In each group of 22 people there are at least 4 people who were born in the same day of the week*.
Solution: To solve this problem it is possible to use the form of Dirichlet's principle in its strong form (4), but also to use the notion of ceiling and floor functions that can be defined as follows: $\lfloor x \rfloor = \max\{m \in Z; m \le x\}$ and $\lceil x \rceil = \min\{n \in Z; n \ge x\}$
That is why:

$$\left\lceil \frac{m}{k} \right\rceil = \left\lceil \frac{22}{7} \right\rceil = 4$$

e) *There are at least two people living in Great Britain whose three initials are the same.*
Solution: There are about 55 million people living in Great Britain.
Total number of possible initials' combinations is $(26)^3$ =17 576.

PROBLEMS
OF EDUCATION
IN THE 21st CENTURY
Volume 31, 2011

$$\left\lceil \frac{55000000}{17576} \right\rceil = 3130$$

So there will be at least 3130 people with the same three initials of their names.

f) *There were n people at the party. Prove that among them there are at least two people who know the same number of people at the party. (The relationship is mutual – if Jane knows Jim, then Jim knows Jane.)*
Solution: All people are divided into „pigeonholes"according to the number of people they know. We find out that there are as many people as pigeonholes – *n* because each person can know 0 or *n-1* people. But this does not prove anything. We need to explore the extreme cases. If somebody knows 0 people, then he cannot know all the people. That means that there are only *n-1* pigeonholes and *n* people. By applying Dirichlet's principle, the statement is proved.

g) *Ludo game plan is formed by a circle with 36 squares. What is the minimum number of men we need to be able to throw out any man by any other man in random arrangement and throw of a dice?*
Solution: Let's have, at first, 18 men placed on the plan so that occupied and empty squares alternate. Now, if we throw a one (or a three or a five), then it is not possible to throw out any man. So 18 is not enough.
We will prove that 19 men are enough. Let's spot all the men randomly on the plan, and suppose we throw a number *k.* We assume that the men are spotted so that none of them can throw out any other man. Let's put the first man *k* squares forward. This square must be empty. Move the second, the third, etc. It cannot happen that two men would be moved to the same square. So we have 19 squares where the men are standing and 19 squares that must be empty. In total, we have found 38 different squares but there are only 36. And that is a contradiction.

h) *There are 15 subjects at school. Each student has 4 of them. Prove that if the school has 18 students, then 2 students have 2 subjects in common.*
Solution: Let the pairs of subjects (A, B) to be the „pigeonholes". For each student there are $C_2^4 = 6$ subjects (pigeons) and $C_2^{15} = 105$ pigeonholes. In total, there are 18 x 6 = 108 pigeons, so two of them must be in the same pigeonhole.

The second lesson starts with checking the homework if it was set. In that case a teacher can get the first feedback on the topic, according to the results and solutions of the homework. Because of the activities supposed to be done during the second lesson, students are divided into groups of three or four students. Groups get an assignment consisting of ten different problems dealing with Dirichlet's principle. They are asked to solve them in the time limit of 30 to 40 minutes. At the end of the lesson, groups compare their solutions. This can be done in different ways. It is important to bear in mind that students should be required to explain their solutions as it is the topic that is necessarily connected with understanding the idea itself.

An alternative course of the lesson might be organized in the following way. Students are divided into pairs or groups of three to four. Each group chooses one problem written on the separate sheet of paper. Students should work on the given task and write or draw their solutions on posters. When all posters are finished, one member of each group presents the solution of their problem and explains it to be understandable to the rest of the class.

The third lesson is optional. It can be used to develop students' possible interest in the

topic or to complete the unfinished problems from the previous lessons. If students seem to enjoy the topic, they can be offered another set of problems to be solved or they can search for some on the Internet or try to invent new problems in the context they like.
The contents of the second and third lessons can be combined in any order but it is important to give students the opportunity to explain, understand and think about problems.

Case Studies

Two case studies were carried out with two different groups of students. What both case studies have in common are the objectives. The researchers were focused on finding out if it is possible to implement Dirichlet's principle into practice in upper-secondary school and to study its attractiveness among students, their ability to work with it, understand it and use it for solving problems. The case studies differ in the age of participating students and in the course of the second lesson.

The case studies were realized in natural and authentic environment. The methods of involved observation and documents' analysis were used. The aim of the research was to introduce non-traditional approach and topic into mathematics' education on upper-secondary school mathematics lessons and at university pre-service teachers' education. The researcher's role was fulfilled by preparing teaching materials, lessons' plan, organizing the teaching, reflection of own teaching practice, collecting students' works and their evaluation, and interpreting the research results in the case study.

Case Study 1

First case study was realized with third-grade students of secondary grammar school in Levice, Slovakia. As it was mentioned above, students at secondary schools are not familiar with the topic. Furthermore, it is obvious that solving problems with Dirichlet's principle requires certain sense and talent for mathematics. That is the reason why the class was chosen based on students' mathematical competences and their interest in mathematics. It is not a mathematical class but has few students with deeper interest in mathematics.

The case study was realized according to the proposed lesson plan. The only enrichment were the use of interactive board for completing the crossword which enabled students to actively take part in the exposition, and additional explanation of floor and ceiling functions, which was necessary for understanding of some problems. The first alternative was chosen for the second lesson. After both lessons, students were given a questionnaire where they were asked to express their opinion on different aspects of the past lessons: teaching methods, the topic and its comprehensibility and applicability, and the atmosphere during lessons. Students were asked to answer using the scale from 1 to 5 where 1 means strong agreement, 5 means strong disagreement. There was also one open question to let students express any opinions and feelings they want to share.

The teacher who taught both lessons was consulted before, between and after the lessons. She was very enthusiastic about all the problems and activities.

As it was expected, students knew some famous mathematicians and they recognized some of the names given on the first slide. But generally, it can be said that students do not associate particular terms, connected to certain mathematical operations, with real people from history. Similarly, they did not know the person in the picture, so they used the crossword to find it out. The first forty-five minute-lesson was covered by the presentation. Immediately after the lesson, the teacher said that the lesson had been very well accepted by students who had responded to the questions in an appropriate and correct way. Moreover, they were very quickly able to solve demonstrated tasks. One problem was set for homework.

The study was completed during the second lesson in four days' time. Students were divided in groups. The division was not arbitrary. It was based on teacher's knowledge of her students and each group consisted of four members. One student in every group was the more talented one in order to help the other members in problem solving in case of necessity. According to this criterion, the class was divided into seven groups that were solving ten problems in 40 minutes. All their solutions were written on separate sheets papers, collected and analyzed for the purpose of the case study.

The following problems were chosen for problem solving:

1. There are 4 pairs of black socks and 5 pairs of red socks in the drawer. It is dark in the room. How many socks do we have to take out to be sure that we have:
   a) a black pair? b) a red pair? c) a pair of the same colour?
2. There are 29 students in the class. Prove that at least 7 students will get the same mark in the test.
3. Decide how many US citizens we have to choose to be sure that there at least 100 citizens of the same American state.
4. There is a million pine trees in the forest. No pine has more than 600 000 needles. Prove that there are two pines with the same number of needles.
5. The school canteen has 95 tables and 465 seats. Can we be sure that there is a table with at least 6 seats?
6. There are 30 students in the class. In the test, one student made 12 mistakes, the others made less than 12. Prove that at least 3 students made the same number of mistakes.
7. Seven performances are played in the theatre club in one season. 5 actresses are always cast in three of these plays. Prove that there is a play where at least 3 women play.
8. The teacher begins each year with 3 jokes. He hasn't repeated any triplet of jokes in 12 years' time. What is the minimum number of jokes he has to know for this to happen?
9. If you draw 5 points on an orange's surface, then there is a way how to cut the orange into two halves, so that four of these points will be lying on the same half of the orange (we assume that the point lying on the borderline belongs to both halves).
10. Make up a problem with Dirichlet's principle in any context.

Analysis of Case Study 1

When we compare the solutions of groups, we can say that one group was able to solve all the problems, one group nine problems, one group seven problems and four groups solved six problems.

First two tasks were solved very easily by all groups. The third task was solved by 5 groups, one group did not start to solve it and one group did not come to the correct solution. The fourth, fifth and sixth tasks were solved similarly, and in one case number six was interpreted in a wrong way. Three groups started to solve the tasks number seven and eight. The other groups either did not have time to do it or skipped them because of another task that might have been easier for them. All their solutions were correct. The task number nine was solved by one group that added a picture as an explanation as well. Alternative examples were made up by two groups. Their proposals are:

- At least how many states of the EU must be chosen to be sure that there will be at least 5 women from the same state?
- There are several rows of 6 desks in the classroom. The class consists of 35 students including 19 girls. Show that at least 2 girls sit together. (students sit by two).
- At least how many numbers must be chosen from numbers 10, 3, 15, 1, 8 in order to have these two whose sum is a prime number.

The third lesson did not follow because Dirichlet's principle as a method of solving problems was understood. Moreover, the groups solved different numbers of problems from the common list, so if we wanted to continue the topic, we would have to prepare a new set of problems. We believe that they should be more demanding, and that is why they might require additional explanation.

After the lessons students were given the questionnaire. They were asked to evaluate not only their interest in the topic and methods but also how demanding the problems were.

The results of the questionnaire show that the majority of students liked the activities as well as the topic of the lessons. Twenty-one students agreed with the statement that they had enjoyed the lesson, just three of them disagreed. Twenty-four students confirmed that they had learned something new, while three disagreed. Similarly, the majority of nineteen claimed that they had been doing interesting things during the lessons, three of them disagreed and five students did not know to answer.

The question whether we should know more about people from history of mathematics was answered positively by 13 students, negatively by 2 and 12 of them were not able to answer.

Fifteen students believe that they would remember Dirichlet's principle, 9 of them did not know and three students thought they wouldn't remember it after two lessons.

To summarize their answers, we can say that most students were interested in the topic and activities. At the same time, there were mostly three students who were quite negative towards all the points mentioned. We can say that they might be those few students who are not really keen on mathematics and they appreciate easier tasks with customary solving methods. This corresponds to the teacher's observations from regular mathematics' lessons.

The second part of the questionnaire shows that twenty students understood the examples and problems; twelve students thought the problems were not difficult, eight of them did not know and seven students believed it was a demanding task.

Furthermore, seventeen students agreed with the statement that they like dealing with unusual tasks, three of them did not know and seven students did not agree.

Final question evaluated by the scale was about students' opinion on group work. Twenty-three students supported the idea that they like working in groups, three of them did not know and one student did not agree.

In the last part of the questionnaire students were free to express any comments. Seventeen students commented the lessons. The following comments represent the most common opinions:

- „I have learnt something new and interesting."
- "Problems were miscellaneous, some of them were easier, others more difficult. But all the solutions on the lesson were understandable."
- "It was something different, I enjoyed it because of the topic but the methods were fine as well."
- "We had two interesting lesson with non-traditional teaching methods which I enjoyed much more than traditional lessons."
- "To keep it in our heads we would have to deal with it in more detail. But, in fact, I think it wasn't so easy."
- "I enjoyed the lessons because the problems were interesting and non-traditional. Some of them were more difficult, some could be understood very quickly."
- "It was a good change in mathematics."
- "…another view on mathematics."
- "Well, I have learned something new though I doubt I will use it, but I still think that it's good to know more.
- "A good relax from compulsory learning."

PROBLEMS
OF EDUCATION
IN THE 21st CENTURY
Volume 31, 2011

On the other hand, the teacher has appreciated this material, too. Her impressions after teaching were highly positive and she was obviously satisfied and enthusiastic because of activities done within the lessons. She confirmed that it is motivating, useful and enriching material and she would not change any procedures and activities if she was to teach it again. She found it activating for all students but especially suitable for talented students.

Case Study 2

The second case study was carried out with prospective teachers of mathematics at Constantine the Philosopher University in Nitra, Slovakia. They were expected to have heard about Dirichlet's principle before. Their interest in mathematics was clear as they were studying mathematics. During the case study they were also instructed to think about the methodological point of view.

The case study took place at two 90 –minutes long seminars with two groups of students. At the beginning, the case study followed the lesson plan for the first lesson covered by the PowerPoint presentation. Students knew a lot of names of mathematicians and during the brainstorming they had also mentioned the name of Dirichlet but they were not able to recognize him in the picture. The clues of the crossword were easily revealed. One group could remember well the principle they were going to deal with, the other group was more silent and did not show the knowledge they had.

In the second part, both seminar groups followed the second alternative of the lesson. Students worked in groups of two or three. They had around twenty minutes to analyze the problem they got. The tasks were not difficult; some of them were similar to those that the secondary school students were asked to solve. All students could consult their task with the teacher who was giving them some hints and suggestions how to come to solutions. Some of the tasks seemed to be quite demanding for students. Students wrote and drew their explanations on posters which were displayed on the visible place about 15 minutes before the end of the lesson. Afterwards, one person from each group presented their solutions to the rest of the class. Everyone was free to ask any additional questions. All students were able to justify their answers.

After the lesson, few students were interviewed concerning their opinions and attitudes about the past lesson. They said that the interesting part of the lesson was drawing on the posters and introducing the topic via placing Dirichlet in history and then coming to the point. The problems themselves were a difficult part because it was their task to find the method for solving them. They also believed that such tasks can be more engaging for students at schools because it is mathematics that is not only about numbers and formulas but it also develops imagination and offers freedom in methods of problem solving.

Conclusions

As it was described in this paper, the aim of was to find out if it is possible to implement Dirichlet's principle into teaching at secondary schools. According to the above described case studies we can say that it seems to be a suitable alternative to be included in mathematics teaching. Students were motivated to learn and deal with the principle, though there was a small number of students who were not interested because it was not so stereotyped and straightforward problem as they usually deal with. Solving the problems with Dirichlet's principle requires a real interest in the principle itself as well as in solving non-traditional problems. Moreover, students appreciated group work and the use of ICT though in its simplest form. They probably do not use it regularly, if ever, on mathematics lessons.

Based on the case studies we can support the idea that students would prefer to have more innovations implemented in mathematics teaching. New topics and teaching methods could make them much more motivated and interested and they will be aware of the fact that mathematics can be worth spending time with. The presented lesson plan for Dirichlet's principle is just one example of how it could be done and how different methods and topics might be used to promote attractiveness of the subject.

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References

Bogomolny, A. (1996-2001). *Pigeonhole Principle.* In: Interactive Mathematics Miscellany and Puzzles. [online]. [Cited 10th February 2011]. Available from: http://www.cut-the-knot.org/do_you_know/pigeon.shtml.

Chetwynd, A. & Diggele, P. (1995). *Discrete Mathematics.* Edward Arnold, a division of Hodder Headline PLC, London, p. 86-89.

Gardiner, A. (1987). *Discovering mathematics the Art of Investigation*. OUP.

Grozdev, S. (2006). *Dirichlet Principle*. In: Identification, Motivation and Support of Mathematical Talents in European Schools, p. 6-16. MATH.EU Project.

Medek, V. (1984). *Matematická terminológia*, tretie doplnené vydanie. SPN Bratislava.

Larson, L. C. (1990). *Metódy riešenia matematických problémov.* Alfa, Bratislava.

Lukáč, S. (2002). Implementácia IKT do vyučovania matematiky. 2002. In: *Zborník príspevkov z 3. celoštátnej konferencie Infovek* [online]. [cited on 31st March 2011]. Available from: http://www.infovek.sk/archivwebu/konferencia/2002/zbornik/kp-lukac.php.

O´Connor, J. J., Robertson, E. F. (2000). Johann Peter Gustav Lejeune Dirichlet. In: *The MacTutor History of Mathematics archive* [online]. Scotland: University of St Andrews. [cited on 5th January 2011]. Available from: http://www-history.mcs.st-and.ac.uk/Biographies/Dirichlet.html.

Stanovský, D. (2000). Dirichletuv princip. In: *Matematický korešpondenční seminar.* KAM MFF UK [online] [cited on 5th January 2011]. Available from: http://mks.mff.cuni.cz/library/DirichletuvPrincipDS/DirichletuvPrincipDS.pdf.

Dirichletov princíp v krátkosti. (2006). In: *Učme radi. Databáza aktivít – kniţnica inšpirácii*. [online]. Projekt ESF. [Cited on 10th February 2011]. Availvable from: http://www.ucmeradi.sk/plain.php/dirichlet.pdf?fetch=100&file=1141&hfile=5f2d7b3941fa8500c0964fca9e354bbb&full=1&lang=1&ident=1&da_show=465.

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