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MODERNIZING MATHEMATICS EDUCATION IN SLOVENIA: A TEACHER FRIENDLY APPROACH

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Abstract

Nowadays, we are facing a large number of varied educational projects which aim to direct and modernize mathematics education. Many institutions (from university bodies and institutes to individual secondary and elementary schools, networks of schools and private enterprises) make an appearance on the project market and contribute their ideas. Such quantity can cause confusion among teachers. Encouraged by the article, a mathematician's lament, by Paul Lockhard, we present some simple principles for classroom work that, in our opinion, would enhance the efficiency of mathematics classes in the long term. Thus we try to help mathematics teachers build a strategy for a fruitful approach to the ideas, recommendations and advertisements on the educational market. Showing the respect towards the quality mathematics teachers and avoiding discouraging and confusing them are some of the leading ideas that we should pursue in our attempt to improve mathematics education. At the end, we also offer some recommendations about the teacher training system.
Key words: *mathematics education, pedagogical approach to teachers, ICT, key aspects of teaching mathematics*.

Introduction

New technologies can challenge the teachers to use technical advantages to enhance the efficiency of mathematical education; however, they can also cause an atmosphere which confuses and discourages an experienced mathematical teacher. In this section, we sketch psychological background that was one of the motives for the following considerations.

A number of indicators in Slovene schools suggest to the mathematics teacher that new technologies introduce tectonic changes into education and the need of completely different approaches in teaching mathematics is self-evident. Some of these indicators are the following: extensive investments in technology, formation of development teams and working groups for modernization of teaching on school level, numerous updating seminars and various educational projects that either involve teachers as collaborators or introduce their project results as educational recommendations.

In the atmosphere of tectonic changes, quality teachers are put at the hardest test. On the one hand, they are invited to cooperate in order to try to understand and support the changes; on the other hand, experienced teachers with a carefully prepared and proven successful system would not give up their own methods easily. Being aware of a number of past school reforms that proved ineffective, their healthy doubt is, in fact, reasonable and justified. If now the changes are inadequately prepared and new approaches announced in an unbalanced and improper way, this may cause either confusion or rejection among teachers.

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Moreover, as a result of the atmosphere described, the *good–bad* duality is being transformed into the *new–old* one. This simplified atmosphere gradually infected politicians, school headmasters, as well as many parents. Instead of good performance, often a new-fashion performance is praised. Thus the teachers feel that their expertise, which had been built through the years of careful preparation, suddenly loses real essence. This, of course, has a discouraging effect.

It would, therefore, be necessary to develop – besides the standard pedagogical approach towards students – a more carefully planed pedagogical approach towards teachers. Firstly, a more balanced approach toward the *new-old* duality would be needed. Secondly, a more evolutionary approach to change would enable teachers for an easier incorporation of new ideas into their own system. Therefore, instead of a tectonic movement announcement, we propose an approach more in the sense of the following idea: "We know that you are doing well and with maximum effort. We have, however, recently tested such and such idea, which has worked well in practice. This is a detail we suggest you might also try." Such approach shows respect and recognition, and would thus be in accordance with certain key points of Palmer's foundational book:

"*In our rush to reform education, we have forgotten a simple truth: reform will never be achieved by renewing appropriations, restructuring schools, rewriting curricula, and revising texts if we continue to demean and dishearten the human resource called the teacher on whom so much depends.*" *(Palmer, 1998, p. 3).*

As a modest contribution in this direction, we suggest a few starting points derived from Lockhart's article, which would help quality teachers confront their teaching methods on a substantive level with the proposed updating guidelines in a way that would either allow the inclusion of the suggested ideas into their system or their rejection without unnecessary burden.

Modernization of Teaching Mathematics in Slovenia

In recent years, a large number of projects for modernizing the teaching of mathematics took place in Slovenia. Many were coordinated by the National Education Institute of Slovenia (Zavod RS za šolstvo) or implemented by various institutions, from universities, secondary and primary schools to private entities through the application to the calls announced by the Ministry of Education of the Republic of Slovenia and financially supported by different European founds.

In this section, we summarize the key points that these projects promoted. In doing so, we focus not so much on the "academic projects" but on projects that in one way or another reach the teachers in practice. Later, we will present the main points of Lockhard's article. On these grounds we will produce the baseline, in the light of which we will again critically examine the main ideas of these projects.

Having examined the projects aimed at modernizing mathematics teaching in Slovenia in the past 15 years, we found that the main concepts emphasized by these projects were the following:

- Promotion of the use of new technologies in education, especially ICT. Preparation of sample materials, particularly electronic teaching materials and teacher training for their preparation and use in the classroom.
- Promotion of the presentation of mathematics in a wider context. Encouraging the search for cross-curricular links, presentation of examples of the applied value of mathematics in the other sciences and beyond.
- Promotion of alternative teaching methods for teaching mathematics.

A considerable amount of money and energy has been invested in projects of all three types. The question is whether – seen from the perspective of a mathematics teacher – the results are in proportion to this enormous input. Let us take a look at the tree types of projects in more detail.

The size of the projects for the promotion of new technologies can be understood if we know that they include equipping schools with appropriate technology, enhancing the awareness of teachers regarding the benefits, preparing appropriate sample materials and teacher training for proper use. The investments in this field (on the state level, combined with European funds) have been tremendous.

Considering the recent projects for promotion of ICT use in mathematics education in Slovenia, we should in the first place mention the *E-um* project. Its aim was to produce a sample of a pedagogically carefully planed set of e-learning materials, which would be prepared to an extent allowing both independent work by students at home as well as use in the classroom. In the second case, usage requires no additional technical knowledge from the teacher, only some additional teaching skills. During this project, an array of more than 60 experts from different fields, such as mathematics, didactics of mathematics, computer science, etc., and teachers from different school levels, designed more than 1,200 units of e-learning material, covering the full range of primary and secondary school mathematics. The project is ongoing. In the most recent stage, the e-materials have been enriched by dynamically created exercise problems and with applets, which explain certain concepts in the spirit of pantomime. The portal is also constantly being improved in response to user comments. The result of the project is a website (URL http://www.e-um.si/). Some of its didactical foundations are explained in the articles *The challenge of E-learning* (Hvala, Kobal, Zmazek B. & Zmazek V., 2007) and (*E-) Mind thinking with E-um* (Kobal, Zmazek B., 2007), while Repolusk in an article in this volume offers a deeper evaluation of interactive e-learning materials in Slovenia.

E-um is just one in a multitude of projects involving the promotion of ICT and the production of e-learning materials. There are many smaller and less comprehensive projects, each of them with its own key emphases and strengths.

Projects related to the presentation of mathematics in the broader context of science and society has been particularly popular recently. These are organized somewhat inductively, using a bottom-up approach: a broad base of collaborators looks for examples of good practice, which are later collected and treated at higher levels. The basic idea is praiseworthy: overly circumscribed concepts benefit no one – and certainly not mathematics.

Broad objectives have also been the hallmark of various projects for the educational renewal of mathematics teaching. At the declaratory level, these should promote approaches that increase student motivation of as well as the sustainability and usefulness of student knowledge. In this context, there was a period of intense promotion of certain teaching strategies, particularly "cooperative learning". These projects were similar to many other reform attempts, which appeared promising in explanation, showing the weak points of the existing system and pointing out potential improvements; however, these produced few documented lasting effects.

Paul Lockhard's Article

Among the multitude of varied recommendations considered in the first section, there is a danger of the teacher losing the sense of essential elements in teaching mathematics. A series of constant small movements under the influence of external incentives may, therefore, drive a teacher of mathematics to the point where he loses his global orientation.

In his article (Lockhard, 2002), which is available on the web site of the Mathematical Association of America and has never been formally published, the author courageously confronts the state of mathematics teaching in schools of various levels. He is bothered by his own observation that mathematics teaching has been diminished to training for mere execution of routine procedures, while the core of mathematics is increasingly lost. These findings are not

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only local in character, but to a large extent also apply to the teaching of mathematics in Slovenia. Under the influence of this brave, uncompromising, and in parts even radical perspective, we have created our own reflections on this subject, which we present in the following sections.

Two Key Aspects of Teaching

Generally, we believe that the teaching of mathematics, like teaching of other subjects in the school program, should be based on meeting the following two main objectives. The first one is to **provide basic literacy skills** in a specific area and at the appropriate level. Thus, a student in the final year of primary school should know enough grammar of his or her mother tongue to be able to write a simple, error-free letter. Similarly, a student at the same level should have mastered basic calculation operations and understand simple relationships between the different (financial, areal, volume, mass, etc.) units. For a student of physics, basic literacy in mathematics certainly means much more; for example, understanding the concepts of derivation, integral and differential equation. Thus, for each area or group of learners, specific elements of their basic literacy skills could be determined.

Mathematics pedagogy has always encountered problems of this kind. Lively debates about curriculum composition or about ways of making students better understand a particular notion or concept have always been common at all levels.

In addition to providing basic literacy skills, there is another aspect of teaching, which has recently been neglected. This is something that not only creates new local connections in the brain but also dilates the pupils of the eyes and takes away the breath. This is the aspect of **spreading enthusiasm among students by the very essence of a certain activity**, science or art, without quibbling over particularities and specific objectives. This is the ability of a teacher to show student the greatness of certain areas at the macro level, the opportunity to show the view from the top and into the depth of an essence. This involves creating conditions for direct entry into the magic world of particular sciences and arts, and it consists of the **creation of powerful events**, in which each science or art appears in its wonderful light. Here, the curriculum is no longer of such major importance, and the same is valid for the details of the event that create such breakthroughs. It is only important that the breakthrough occurs.

Examples outside mathematics

Prior to deepening the mathematical aspects of the above considerations and aligning them with the realities of the teaching market, let us reinforce it with several simple examples from the world outside mathematics.

- The aim of the youth department of a Mountaineering Section is not only to acquaint young hikers with all possible risks they may encounter in the mountains, but to show them views that make them get out of bed in the early morning and carry a heavy backpack through the dark woods. It is relatively unimportant what particular view caused the leap or whether we succeeded in covering all possible views from a certain (curricular) list.
- The aim of teaching literature is not only to provide detailed knowledge about the authors and their works but the creation of awareness that literature puts our lives under inspection, explains our motives and actions. Literature is an opportunity to learn from the mistakes of others; it sharpens our views and clearly sets our moral positions and our self-criticism. To achieve this aim, a teacher needs to use certain exceptional works of world literature, but it is far less important which ones exactly. A quality teacher can make his own selection and adapt it to the particular environment and the students.
- Similar arguments could be used for teaching music or art. These two subjects served Lockhard to illustrate the crucial point of his article and to draw a lovely parallel

between the teachers of mathematics and those of music and art. In addition, it seems that in the last decades the list of basic literacy skills for musical education in Slovenia has increased, while the last of the above aims is losing importance. In my schooldays, we had enjoyable music lessons with singing, listening to music and enjoying exciting opera plots and anecdotes about composers' lives. Nowadays, much of this has been replaced by tones and half-tones in major or minor musical scales, and the types of valves in wind instruments. It is obvious which of these concepts brings the pupil closer to the essence of music.

Examples within mathematics

Similarly, it is important to find fragments from the world of mathematics where all the beauty of mathematical thought is concentrated, and to use these to bring students closer to the essence of mathematics. Or, in Lockhard's words:

So how do we teach our students to do mathematics? By choosing engaging and natural problems suitable to their tastes, personalities, and level of experience. By giving them time to make discoveries and formulate conjectures. By helping them to refine their arguments and creating an atmosphere of healthy and vibrant mathematical criticism. By being flexible and open to sudden changes in direction to which their curiosity may lead. In short, by having an honest intellectual relationship with our students and our subject. (Lockhard, 2002, p. 10).

We advise teachers to present systematically (at regular intervals of, let us say, two weeks) certain *special events* with which they would try to spread their enthusiasm among their students. These events – let us call them **powerful events** – depend on the teaching subject and the specific topic and may be of many different kinds:

- The basic idea is to locate each beautiful mathematical idea and to take time to underline its beauty with the use of adequate dramatization. To fulfil this goal, we sometimes present a beautiful result even if it was not directly anticipated in the curriculum.
- Sometimes the sense of a powerful event can be created by the fact that students are essentially involved in its realization. At a certain point in a mathematical consideration, we stop and with the appropriate drama leave students to make a decisive step themselves. Even if it does not produce a great result or a spectacular proof, these small flashes may alter a routine proof into an inspiring event.
- Another option is visualization of results using computer software. A typical example is, for instance, visualization of geometry results using dynamic geometry programs, or visualization of calculus results using computer programs for drawing graphs of functions of one or two variables (Fourier analysis or the analysis of the extremes of functions of two variables). The events are even stronger if the students themselves are actively included in their presentation.
- The next idea is the use of stories from the history of mathematics. Before the introduction of Euler Gamma and Beta functions, naturally some incidents from the interesting life story of Leonhard Euler are appropriate. Before the results that are associated with mathematicians from the local environment or their teachers, effective presentation of their background is welcome. (A well known slovene mathematician and mathematical education reformer, F. Močnik, worked with the famous mathematician Cauchy during his stay in Gorizia. This story is regularly used when considering results connected with Cauchy.)
- A good idea is the connection of academic topics with current events or topics familiar to students (like actual movies, books, people).

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- Sometimes the results can be skillfully disguised as entertaining stories. Ordinary formulations can be dry and dull, but enfolded in an appropriate story they acquire a much juicier life. For example, the Thomsen equality could be nicely weaved into a story about a boy who enters an empty spaceship belonging to visitors from outer space, presses one of three keys on the computer keyboard, and of his Grandpa, who seeks to correct his mischief.

These are just some of ideas used at the lectures for students in the mathematics teacher training programs at the University of Maribor. Similar examples of exciting events should be occasionally prepared by every teacher of mathematics at all levels of education.

Inspiring Examples and Drama. Creative Atmosphere: the Cosmic Soup of Which the Stars are Formed

A closer look at the cases in the previous section shows that only some directly correspond to the points given by Paul Lockhard. Indeed, these inspiring events can be divided into two groups: those in which **the full essence of mathematics** (in the spirit of P. Lockhard) **comes to light**, and to those that **generate a creative atmosphere** that allows us to continue with concentrated, intensive work even when the substance is not that spectacular.

Creating a creative atmosphere is also important in the light of providing basic literacy skills. It is my experience that the powerful events described above break up the monotony and fill students with positive expectations, which can be easily followed by an even longer period of demanding and less juicy topics providing basic literacy skills.

To create such strong events, a teacher must have freedom: a necessary condition for spreading enthusiasm is the existence of enthusiasm in the teacher himself, which is subjective. Detailed instructions on this matter are not possible: what is needed is a teacher who can kindle an intellectual spark and who knows how to create a climate in which the students will themselves feel the spark. In this case, the details of the approach as well as of the curriculum are not as important.

Another Glance at the Educational Projects

In the spirit of the previous sections, we suggest to the teachers a painless approach to the projects and recommendations of experts by answering these three questions:

1. Does the project or recommendation contain any ideas that could help to create a new powerful event to bring the student close to the very essence of mathematics?
2. Does the project offer any new ideas for creating a classroom atmosphere that would enable us to help the student through demanding sections of the curriculum even when the substance is not so spectacular?
3. Does the project contain new ideas about explaining to students a certain detail of the curriculum that belongs to the basic literacy skills list at a particular level?

In the light of these three questions, we return to the projects from the second section. Good knowledge of the *E-um* project results allows us to find examples where the answer to any of the above questions would be positive. In the e-lesson about the area of the circle, we found a wonderful illustration of the formula $A = r^2\pi$ using the computer program GeoGebra. The following figure illustrates the core of the idea:

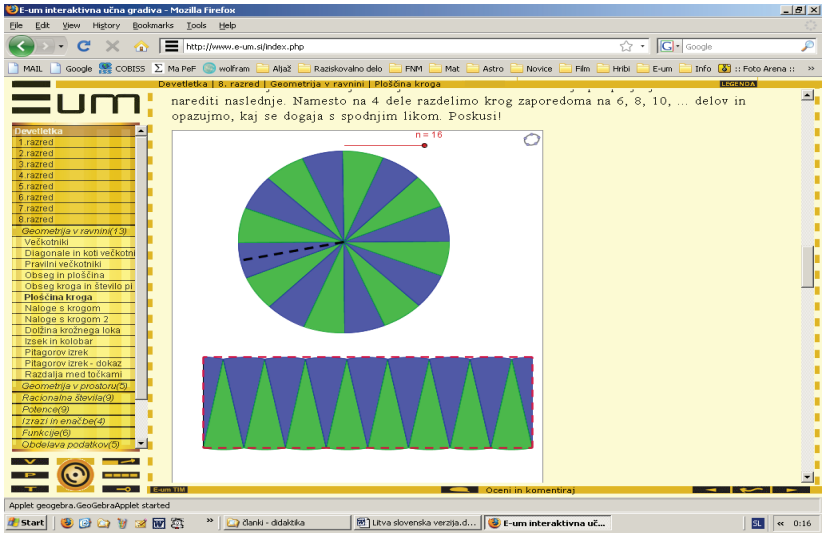

Figure 1. E-um applet illustrating the circle area formula.

Similarly, the e-lesson on the radians brings a lovely illustration of this concept by winding a string on the unit circle. Again, we add a more revealing picture below:

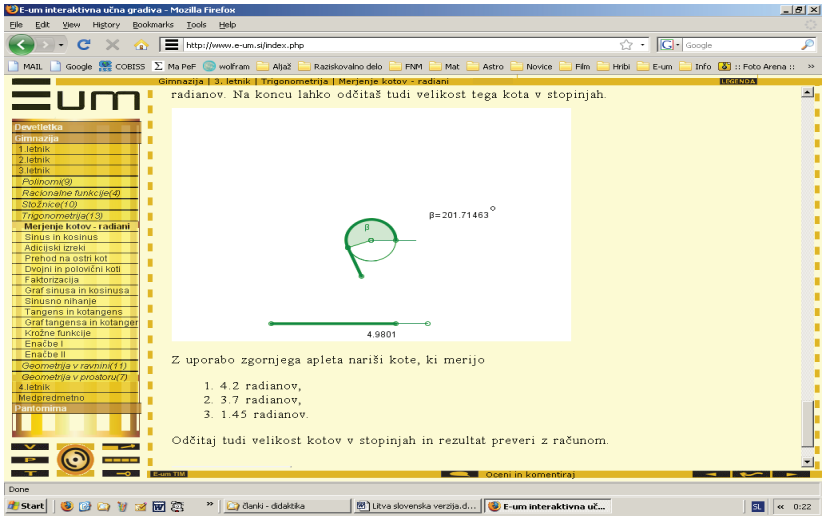

Figure 2. E-um applet illustrating the concept of radians.

In both cases, we are dealing with dynamic applets, which allow the student to vary the parameters using the sliders. We can also find e-lessons which create a pleasant atmosphere to attract students using an open and sincere approach and using the interactive possibilities provided by the technology.

What we primarily want to state here is that one concrete project produced some important and useful results not because it was using new technologies or new approaches, but because the answers to the three questions above were positive. **New media are not by themselves sufficient – a master is needed who knows how to handle them and use them to make a substantially deep performance.** The danger of cheap showmanship in these circumstances is permanent; therefore, the existence of new media is far from being an assurance of a quality approach. The

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duality *new – old* is inappropriate. We should return to the *good–bad* performance duality.

Similarly, we could evaluate recent projects that seek to place mathematics into the context of science and society. Each correctly and carefully presented beautiful example of the use of mathematics can be understood as a strong event suitable for promoting mathematics and science in general. Theoretical principles of persuasion about the need for such examples have no real weight: what counts are practical and fascinating examples. In this case, the project managers left the key part of the task to the teachers: they prepared the theoretical introduction and called in broad base of teachers to find the examples. The teachers will probably face some problems: with the extension of knowledge, the number of appropriate examples rises, but at lower levels the choice is not that wide. All fascinating cases found are therefore welcome. Cases of violent, unnatural contextualisation of mathematics can cause more harm than benefit. Our advice is that the teacher should not be concerned by the theoretical aspects of contextualization but happy with the discovery of every appropriate example that could possibly be incorporated into his teaching.

With respect to the projects for the diversification of teaching ideas, our position is similar: if at some point a didactic idea allows the creation of conditions either to promote mathematics or to foster a creative atmosphere, then it is welcome. In the academic literature we have treasuries of such ideas; just for illustration, we can mention the book *What successful Math Teacher Do* (Posamentier & Jaye, 2006), presenting 79 teaching strategies to increase motivation in teaching math. Among them are many exciting ideas based on the authors' own experience, backed up by advice and research results. A careful study of these ideas should be mandatory reading for every teacher of mathematics. Specifically, we cite this book because we want to emphasize that in principle we do not oppose cooperative learning, which was intensely promoted in a certain period in Slovenia. However, this is only one of 79 ideas from Posamentier's book for fostering a creative atmosphere. A broader approach to the matter is needed, and above all, the ultimate aim should be posed more clearly: the atmosphere in the classroom, which generates powerful events – like the cosmic soup from which the stars are formed.

False Assumptions

According to the above, one could get the mistaken impression that our recommendations are too elitist and that strong events are designed only for the most capable students. This is not true. In the brochure edited by Eremita and Hvala (2008), Katja Glazer Leskovšek, a primary school teacher of mathematics, made the following observations:

"One can feel [*the beauty of mathematics] not only when one grasps a solution to some really difficult mathematical problem but also when one sees a smiling pupil who after an immense number of exercises and after enormous efforts celebrates his first mathematical victory*." *(Eremita & Hvala, 2008, p. 13).*

This example demonstrates that the powerful events are not reserved for the best students alone. Often the superiority of the teacher matters less to able students: their ability and imagination allow them to experience, despite an average teacher performance, the magic of the subject. Using appropriate teaching skills, even less talented students may also be initiated into enjoyment of the glory of mathematics.

Recommendations for Teacher Education

The candidate for a teacher of mathematics must possess a primary enthusiasm, which can subsequently be shared with and passed on to students through the creation of powerful events. In addition, the candidate must be socially mature to understand the teaching profession as a profession of service to the interests of their students and to be capable of creating a pleasant and creative atmosphere for quality work. These requirements are serious and difficult to fulfil. They

remind us of a two-step selection in the application process for the teacher education programs of some universities in Finland. The first step in this selection provides intellectually capable candidates, while in the second step the candidate's pedagogical sense is examined through an appropriately organized system of interviews. This detail is likely to be one of the reasons for the Finnish educational miracle. In this context, access to the program for teacher education in Slovenia is, in my opinion, too open, and the number of candidates unnecessarily large. The result is that the output does not guarantee the qualities we seek. At this stage, I would recommend a more selective approach. To achieve this, the faculty financing system should be adapted: linear financing according to the number of students stimulates quantity and foregrounds an argument that is difficult to overcome even with the argument of providing quality teachers for our school system.

On the other hand, we think that the key points of Lockhard's article and the resulting reflection on the macro approach to teaching mathematics that we have tried to explain in this article should be appropriately presented to the teaching staff at teacher education colleges, to students in teacher training programs and (particularly carefully) to school teachers of different levels.

Conclusion

Based on the Lockhard's article, we have reached the conclusion that, in addition to mathematics pedagogy, which analyzes mathematical concepts and circumstances of their perception at the micro level, it is necessary to foster mathematics pedagogy at the macro level. Such an emphasis would provide that students, with the help of "powerful events", find a way towards the essence of mathematics and become aware of its real nature.

To achieve this goal, we need teachers to be on a certain intellectual level, which allows them to feel sincere enthusiasm for mathematics. Furthermore, teachers must be socially mature and professionally trained to know how to create a proper creative atmosphere in the classroom. Many quality mathematics teachers in Slovenia are already meeting these requirements. Therefore, the approach to promoting the professional growth of these should be more sensitive; it should pay full respect to and provide recognition for their mastery and not confuse or discourage them.

A completely different issue arises around those teachers whose work is not at the level required, usually owing to the lack of some essential features. To avoid this, we would require a more selective approach on entry to the programs for teacher education in Slovenia and the grading of the system of in-service educational programs for teachers, one that would address those who really need it.

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