THE ROLE AND ASSESSMENT OF TEXTBOOKS IN MATHEMATICS EDUCATION

Soňa Čeretková, Ondrej Šedivý
Constantine the Philosopher University in Nitra, Slovakia
E-mail: sceretkova@ukf.sk; osedivy@ukf.sk
Josef Molnár, Dalibor Petr
Palacký University in Olomouc, Czech Republic
E-mail: molnar@inf.upol.cz; petrdal@yahoo.com

"We distinguish two categories of textbooks – bad textbooks and even worse ones."

"On every page of a randomly-chosen mathematical text, you can find at least one mistake."

(heard in a backstage...)

Abstract

Authors of the article are experienced authors of mathematics textbooks for upper primary schools (Slovaks) and lower and upper primary schools (Czechs). Both textbooks are used in everyday school practice and are very popular in their countries. The article presents general ideas and authors experience about:

- *school curriculum and standards and their influence on maths textbooks content,*
- *competencies of pupils and their teachers in mathematics and the place of competencies in textbooks,*
- *problem solving and maths textbooks*
- *concept maps in mathematics and mathematics textbooks,*
- *main principles and examples of maths textbooks assessment,*
- *textbook research,*
- *requirements set upon mathematics textbooks,*
- *different conceptions of creation maths textbooks.*

Some ideas how to prepare math textbooks and suplement materials innovation using ICT and internet is also mentioned in the article.

Key words: *concept maps, didactic functions, mathematics textbooks, problem solving, reasoning, textbook assessment, textbook research.*

School Curriculum and Standards and Their Influence on Math Textbooks Content

The content of education and the related questions draw the focused attention of the general didactics practitioners, subject didactics practitioners as well as other specialists.

A well-outlined content of education, didactic methods used in mathematics teaching and a coherent organization of education in mathematics form a process, in which a pupil acquires pieces of

knowledge and various activities, builds up the knowledge and skills and develops his/her skills and interests. The education content is specified in a curriculum, textbooks and education standards.

Selection of the curriculum and specifying the education content remain a demanding and contemporary problem. This problem has been fully solved neither in the Czech Republic, nor in Slovakia nor in any other country.

The education content is usually elaborated in textbooks which are also referred to as the school books. These comprise the didactic processing of the subject matter (curriculum) delimited by the school study plans (school curriculum). Thus the school books (textbooks) present a fundamental didactic tool used in the pedagogical-educational process. Besides the educational function, textbooks also have other functions:

- motivational function (a well-written textbook stimulates a learning person's interest, he or she is happy to reach out to such a textbook),
- communication function (develops the extent of vocabulary including technical terms),
- regulatory function (the curriculum is divided into parts respecting the logical sequence),
- application function (it comprises ideas of using the subject matter in practice, it states the examples from the real life),
- integration function (a textbook does not restrict itself to just its subject but it accepts interdisciplinary relations, which leads to a more complex cognition process),
- innovative function (it presents newer knowledge of science, economy or technique),
- control and correction function (a learning person uses the text, exercises and problems to check him-/herself, he/she discovers what he/she did or did not understand, he/she revises the subject matter).

Writing a textbook or a school book is difficult because it needs to respect the age specificities of a pupil (e.g. the central developmental line of human abilities by Piaget), the aims and roles of a given subject (with respect to a given country) as well as the generally-accepted aims of education.

Textbooks and Competences

The National Council of Teachers of Mathematics USA has elaborated a self-contained concept of mathematics teaching and in the standards it states, besides other issues, the competences that a pupil should acquire through this teaching:

- *Problem Solving*
  It concerns the instructions describing how to enable the pupils the following: build new mathematical knowledge through problem solving, to solve problems that arise in mathematics as well as in other contexts, to apply and adjust a strategy adequate to a given solution to a problem, to observe the process of solving mathematical problems and mentally realize this process.
- *Reasoning and Proof*
  It concerns the instructions describing how to make the following possible to pupils: to discover that reasoning and verifying (proving) are aspects characteristic for mathematics, to make and investigate mathematical hypotheses, develop and evaluate mathematical reasoning and proving, choose and apply an appropriate way of reasoning and verification method.
- *Communication*
  This contains instructions that describe how to make the following possible for pupils: to organize and stabilize their mathematical thinking through communication, to acquaint the teacher and other classmates with the particular thoughts in a clear and

PROBLEMS
OF EDUCATION
IN THE 21st CENTURY
Volume 6, 2008

comprehensive way, to analyze and evaluate mathematical ideas together with others, to use language for an accurate expression of mathematical thoughts.

- *Connections*
  It concerns the instructions describing how to enable the following to the pupils: to investigate and use the relations between mathematical terms, to understand how mathematical terms are the related to one another, to use mathematics in non-math contexts.
- *Representation*
  This contains instructions that describe how to make the following possible for pupils: to create and use a presentation that records the connections among mathematical terms, to choose, adjust and apply an appropriate presentation that solves a given problem, to use a suitable presentation that models and interprets a physical, social or mathematical phenomenon.

The European Education Committee has set the following key competences:

- Communication in a native (mother) language
- Communication in a foreign language
- Mathematical literacy and competences in the field of science
- Competences in the field of information and communication technology
- Learning to learn
- Interpersonal social and civil technologies
- Entrepreneurial competences

We shall not comment on each of the above competences, let us have a closer look at the two of them: mathematical literacy and competences in the field of science and learning to learn.

Mathematical literacy resides in the skills of performing addition, subtraction, multiplication and division by heart as well as in writing. It also includes the ability to use these operations to solve problems in everyday life. The emphasis is put on the solution strategy rather than on the result itself, on the realization of a given activity rather than on pupil's (theoretical) knowledge. In case of Science, it concerns pieces of knowledge and methodologies that can be used to explain the phenomena in the world around us. Technology represents the applications of the knowledge as a means through which a person influences the surrounding where he/she lives.

Learning to learn is inevitable for organization and ordering the learning stand alone (either on somebody's own or in a group), for gaining, processing, evaluating and integrating new pieces of knowledge and for the ability to apply these competences in various situations and contexts including learning itself and problem solving at home, in an educational process, at work and in a society.

There are other competences in the field of information and communication technologies that are related to the above-described ones. These competences involve the ability to use multimedia technologies and use them for searching, saving, creating, presenting, sorting out and exchange of information.

Mathematical literacy has the following requirements on elementary school students:

- thorough knowledge of arithmetic operations and the ability to use them in every-day situations (addition, subtraction, division, percentages, ratios, measures units and weights),
- Thorough knowledge of mathematical terms, basics of geometry and algebra,
- Pupils should be able to apply the knowledge on the family budget (the balance of incomes and expenditures, the ability to plan for the future), when doing the shopping (price comparison, measures, weights, the value of money) and it the area of traveling (the time-distance relationship, price and currency),

- Understand the symbolic and formal mathematical language and its relation to the regular (natural) language,
- Use mathematical symbols and formulas,
- Use the mathematical units, understand different ways of expressing the mathematical objects, phenomena and situations, choose the appropriate ways of mathematical formulations,
- Follow and evaluate an argumentation, be able to disclose the main ideas of the argumentation (esp. proofs),
- Be able to use the mathematical ways of thinking
- Be able to abstract away from something, to generalize, to model mathematically, to apply the existing models onto closely-related problems,
- Communicate in mathematics using the mother language,
- Use tools and means (e.g. information technologies),
- Know what kind of questions math is able to give the answer to,
- Understand the mathematical proofs,
- Think critically.

The requirements set on the mathematics textbooks follow from these theoretical basics.

By using the mathematics textbooks, pupils should be able to develop the following skills:

- work with mathematical terms,
- discover things and work creatively,
- think logically,
- be able to prove,
- solve problems,
- work with data and information,
- learn,
- work in groups (teams),
- communicate,
- use tools.

Mathematics textbooks should help develop inside pupils the following:
non-negative attitude toward mathematics,

- interdisciplinary links and relations to science subjects and technical disciplines,
- tolerance toward other countries, people and their spiritual values,
- respect to traditions and understanding the continuity of the past and present,
- positive attitude toward arts and all forms of cultural manifestations,
- the need to protect nature and our environment,
- natural drive to actively develop and protect our own health as well as health of other people,
- positive attitude to life.

Problem Solving and Textbooks

The fundamental activity of education resides in tasks and problems. The tasks and their solutions are the creative content of mathematics teaching at all types of schools and they are used in all stages of math teaching.

PROBLEMS
OF EDUCATION
IN THE 21st CENTURY
Volume 6, 2008

Therefore the selection of tasks for mathematics textbook must respect all their functions, i.e. educational, pedagogical and developing.

Problems with sample solutions shown in the textbook must influence the understanding and acquiring mathematical curriculum and, at the same time, they have to function as the means of teaching the methods of solving mathematical problems. Teaching mathematics is a cognitive process for a pupil ensured through problems and tasks. Hence, by means of problems we teach mathematics and at the same time we teach the solution methods to mathematical problems.

A textbook has to contain a sufficient number of standard as well as non-standard problems. The problems shown in the textbook should create sufficient conditions for a pupil to gain experience. Textbooks should show the solution to the problems. Problem solving takes place in certain stages:

- understanding the problem – i.e. orientation in a problem and its comprehension,
- strategic-operational stage – a thought-demanding stage – it concerns solving a math problem,
- synthetic-verification stage – verification of hypotheses – methods.

A textbook must provide a basis for the above as well.

Let us have a closer look at some questions related to problem solving. Problem solving should be the basic pillar of school mathematics. A problem comprises three components:

1. The initial situation in which we describe or provide information or data.
2. The aim we want to reach.
3. The way from the initial situation to the aim, this way can be evident (clear) or feasible for a solver.

The following three cases can take place:

Problems with the closed way and closed aim, i.e. exercises or routine problems: the initial situation is precisely described, the way to the aim is known, the aim is uniquely given.

Problems with the opened way and closed aim, i.e. tasks or non-routine problems: the initial situation is precisely described, the way of reaching the aim is not known (i.e. the way is opened), the aim is strictly given (i.e. the aim is closed).

Problems with the opened way and opened aim, i.e. the initial situation is precisely described, the way of reaching the aim is not known (i.e. the way is opened), the aim is not strictly/uniquely given or is not given at all (i.e. the aim is opened).

The last of these problems is called mathematical investigation.

We have stated some of the theoretical remarks that should be included to mathematics textbooks, problems in these textbooks should provide the conditions for using the strategies mentioned above. Alternatively, we can present a solution to a problem through which we show the individual strategies of investigation and discoveries.

Concept Maps in Mathematics and Mathematics Textbooks

Concept maps seem to be an appropriate tool for the verification of logical sequencing of terms in a textbook, although we do not come across using these maps very often in literature concerning the theory of textbooks evaluation. Among advantages of using concept maps we can especially name their clearness and good arrangement. The arrows in the following figure show the sequence when acquainting pupils with the new terms.

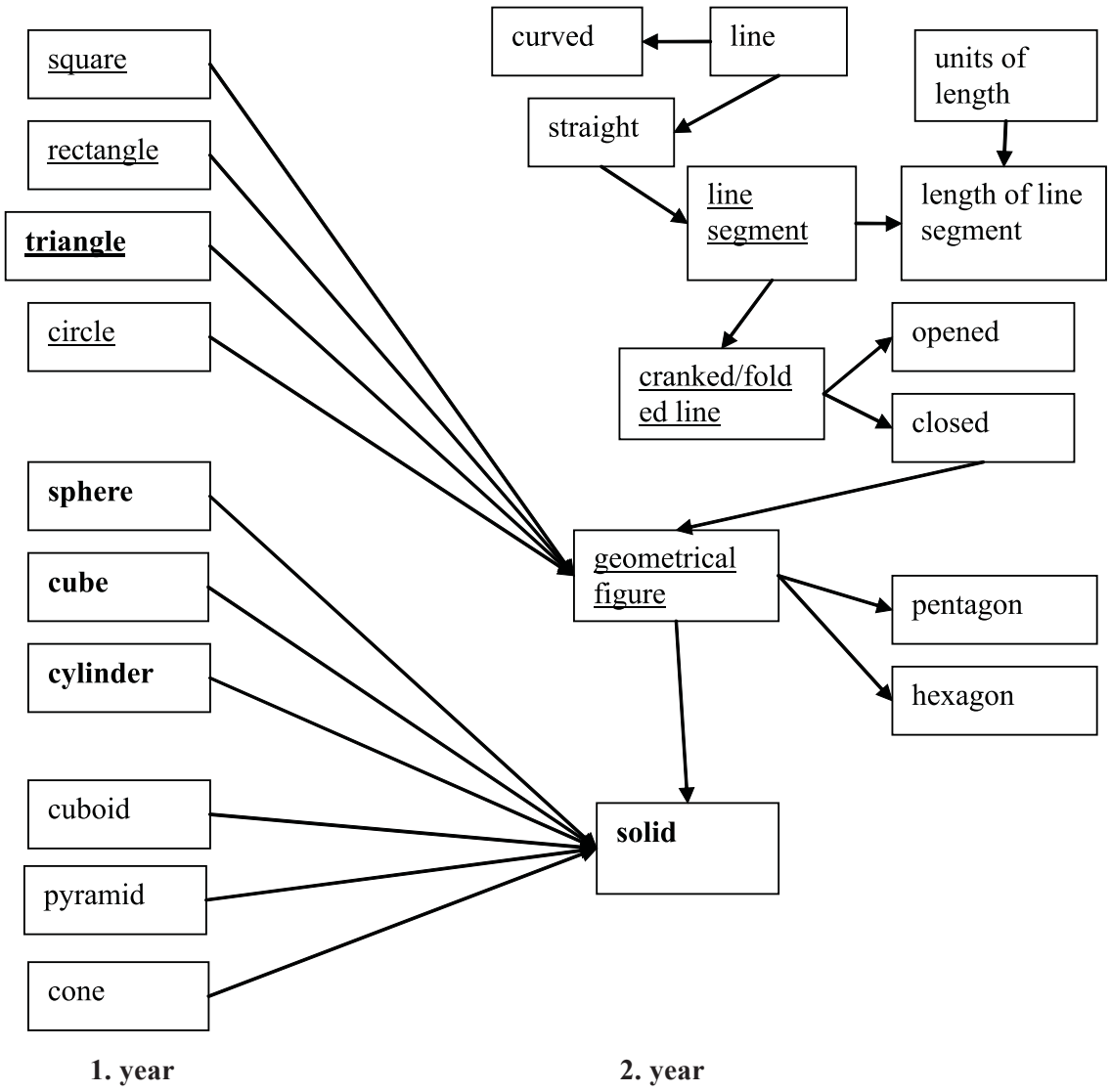

Figure 1. Concept Map: Geometrical figures in textbooks for 1st and 2nd years.

In logical construction of terminology structure of geometry curriculum in textbooks for 1st and 2nd year-students we have managed to spot the absence of the terms a *quadrilateral* and a *prism* which belong to the 3rd –year curriculum as well as the deeper interconnection of knowledge pieces about geometry figures, plane and space.

Textbook Assessment

The criteria stated below are usually accepted. Their use is shown on the example of the textbooks from publishing house PRODOS (Molnár et al. 1998, 1999, 2000, 2001). These criteria were exposed to the analysis in a work by Slavíčková (2003).

Assessment of the Economic Factors: the price of one copy of a textbook compared to the price of textbooks of other subjects used in a given class (year).

Textbook jacket/cover: the quality of the material used, coloring, ability to draw a learner's attention, originality, title dominance.

Textbook Binding: durability when using it, the number of pages, weight of the textbook.

PROBLEMS
OF EDUCATION
IN THE 21st CENTURY
Volume 6, 2008

Lines and Paragraphs: structuring the text in textbooks, using paragraphs and an adequate spacing, text flow.

Orientation Apparatus of Textbooks: in the table below there is shown the presence (+) or absence (-) of individual components of the orientation apparatus in textbooks written by Molnár, J. et al. in 1998, 1999, 2000, 2001:

Table 1. Presence/absence of the components constituting the orientation apparatus in textbooks.

Components of the Orientation Part of the Textbook	Mathematics for the year:			
	6.	7.	8.	9.
foreword	-	-	-	-
contents	+	+	+	+
headers	+	+	+	+
structuring into chapters and subchapters	+	+	+	+
graphic symbols	+	+	+	+
indexes	-	-	-	-
references	+	+	+	+
bibliography	-	-	-	-
pronunciation rules	-	-	-	-

Schemes and graphs, illustrations, tables: in mathematics, among illustrations we can classify graphs, diagrams and sketches. Total number of pictures and the average number of illustrations per page in textbooks written by Molnár, J. et al. in 1998, 1999, 2000, 2001, are shown in the following table:

Table 2. Illustrations in textbooks.

Textbook	number of pages	number of illustrations	number of illustrations per page
Mathematics 6	134	343	2,6
Mathematics 7	155	420	2,7
Mathematics 8	154	388	2,5
Mathematics 9	121	195	1,6

Verbal Assessment of Esthetic Side of Textbook: assessing whether or not textbooks belong to the same series at first sight. They have many pictures, in which we orientate ourselves easily. It is apparent at first sight, what a pupil has to learn, what he/she necessarily has to understand as this information is highlighted by a yellow background. Using the page margins, e.g. historical notes, interests, warnings, interesting problems. From these notes, pupils gain the knowledge from other subjects as well (interdisciplinary relations). In addition, there are textbooks with the commentary for teachers placed right in these margins.

Information Part of the Textbook: the analysis of the text part based on, for example, the

method by Wahla (1983). In my view, all examined textbooks use the text means adequately. None of the means conceivable in mathematics is missing here.

Text Comprehension: if the didactic text is to be comprehensible to pupils and if they are to learn something from it, the authors must adjust the length of the sentences to the age and skills of the pupils. According to Průcha (1989), the average length of sentences in elementary school textbooks should not exceed 13 words.

Problems/Tasks Analysis: frequency analysis of learning problems (tasks), i.e. how many problems and exercises (exercises with the sample solutions presented in the text) can be found in textbooks. There is also an analysis of problems meant for practicing and revising the subject matter.

In a linguistic analysis, we focus on whether the problems are formulated as imperative sentences or questions or we have problems with yes/no questions.

Another question is how many problems should be included in one lesson. However, answering this question requires the knowledge of the number of math lessons as well as the total real number of lesson taught during a regular school year.

Analysis of the Text Extent in Textbooks: we should find out the extent of textbooks as to the number of pages that should be gone over in one lesson. On average, in one lesson the class should go over one page. In mathematics that is not little to do. We should take into consideration the difference between mathematical text and the one in other non-mathematical textbooks. In addition, there are many problems/tasks assigned (on one page) and let us not forget that pupils deal with workbooks too.

Analysis of the Text Difficulty in Textbooks: in order to evaluate the difficulty level of a given text, it is possible to use for instance the C. H. Björnsson method (Pluskal, 1996). The difficulty level measured using this method is given as the value of an index, where L_m is the average length of a sentence (in words) measured in the sample of 200 sentences (so-called "the syntactic factor"), and L_0 is the average length of words that have more than 6 letters in the sample of 2,000 words (so-called "the lexical factor").

Textbook Research

Johansson (2003) lists the subject of the mathematics textbooks research referring to the study (Pepin and Haggarty, 2001) as follows:

Studies concerning the **content and structure** of textbooks:

a. Mathematical plan of the textbook – how the knowledge pieces of mathematics, mathematical thoughts, terms, relations are presented in the textbook, how well the scientific nature of mathematics and the substance of mathematical knowledge are put in the textbook.
b. Pedagogical plan of the textbook – the way of helping a pupil with a study that has the mathematical content, it also concerns the methods and the language of the text in the textbook.
c. Sociological context of textbooks
d. Cultural traditions represented by the textbooks.

Questions concerning further research into the theory of mathematics teaching with the focus on mathematics textbooks are related to the analyses of existing textbooks in different countries taken from the different points of view on textbooks. For example, a textbook and curriculum, a textbook and problems (exercises), a textbook and a lesson, a textbook and a teacher, a textbook and pupil's activities, a textbook and mathematical software, a textbook and interdisciplinary relations, the assessment of textbooks according to different criteria etc.. Each of the mentioned areas renders wide possibilities of investigation. For example, in the area of **a textbook and a lesson**, which is preferred as the next subject of research in Johansson (2003), it is suggested to

PROBLEMS
OF EDUCATION
IN THE 21st CENTURY
Volume 6, 2008

investigate *in what way and why is the textbook being used*.

Studies dealing with using the textbooks by teachers and pupils are concerned with the following questions:

e. Are the textbooks being used by teachers and pupils?
f. Competences of the textbooks?
g. Who uses the textbooks (teacher/pupil) and who decides upon using them?
h. How are the textbooks being used and who decides upon the way of using them? What value do the textbooks present for teachers?
i. Teacher as a mediator of the text.
j. Traditions in the lessons and their possible impact on the work in the class.

When creating mathematics textbooks, questions of the competences of a teacher and pupil in relation to textbooks get into the primary role, the same holds for the new trends in creating the study texts in the form of so-called teaching units (Johansson, 2003, p. 78). These trends can soon significantly affect not only the textbook creation or the role of textbooks but also the textbook policy in a given country.

Brief Summary of Requirements Set Upon Mathematics Textbooks

- The explanatory text should be drawn up such that pupils are able to discover and find out on their own the phenomena, procedures and results, and to use different strategies of problem solving.
- The curriculum should be well-arranged, divided into parts and subparts that form a firm logical chain, in which one link is followed by another one. Each part of the textbooks should start by a revision and summary of the preceding matter.
- The individual theme units or possibly their parts should start by a motivational example problem or a picture.
- It is necessary to encourage students to get involved into the problems being taught.
- A textbook should contain an appropriate amount of graphical material, i.e. illustrations, sketches, diagrams, graphs, photos that complement the explanations, show mathematical pieces of knowledge in familiar situations and practical applications.
- The terms introduced in the lesson should be repeated more often in the text and problems (or exercises). The language should be adequate – one should not use sentences that are too long, important facts should be highlighted.
- Textbooks should contain sufficient amount of problems and exercises at different difficulty levels. There should also be problems ensuring interdisciplinary relations.
- Revision summarizing the studied matter should cover the entire curriculum presented in the textbook.
- Esthetically, the textbook should positively motivate a pupil and it should be attractive for him (her). A reasonable number of pictures should help a pupil orientate easily in the text.
- Last but not least, pupils should find even a mathematics textbook attractive due to the overall appearance, paper used for it, font type used for the texts, binding and a front cover. Pupils should regard a math textbook be their companion when learning mathematics.

In conclusion, it is necessary to point out that a mathematics textbook has to meet the fundamental didactic principles, i.e.

- being scientific and adequate
- being of systematic nature
- accessibility

- consciousness and activity requirements
- clearness
- permanence

Conclusions and Discussion

Creation of a textbook and its introduction into everyday school life is the long-term process. There are several interesting and actual concepts of textbooks series; one very stimulated comes from Austria (Malle, Ramharter, Ulovec, Kandl, 2004). The textbook has three orientations: didactics, practical, and computer. The last one points out the idea of using computers during those maths lessons and activities where it really makes sense. Very useful is also an internet communicative platform for teachers in practice to communicate with authors within the homepage of the book (http://www.mat.univie.ac.at/mv).

There are many research questions concern to mathematics textbook research in the future. Not only the primary focus into: *which, when, why, how much* or simply *how* textbooks are used, but the focus should be pointed to teachers´ work in the mathematics classroom and textbooks using or to textbook language and contexts of problems posed in the textbook as well as to teaching and learning activities posed or provoked by mathematics textbook authors.

Acknowledgements

Funded by projects *KEGA 3/4038/06* and *ESF OP RLZ CZ.04.1.03/3.2.15.2.0263* .

References

Čeretková, S., Šedivý, O. (2005). *Aktuálne problémy teórie vyučovania matematiky.* Nitra: FPV UKF.

Čeretková, S. (2004). Úloha učebnice vo vyučovaní matematiky (Reflexia medzinárodného výskumu). In: *Úloha učebnice vo vyučovaní matematiky.* Nitra: FPV UKF.

Grevholm, B. (2002). Teachers´Work in the Mathematics Classroom and Their Education – What is the Connection?. In: *Challenges in Mathematics Education.* Proceedings of MADIF3. Linköping: Matematiska institutionen, Linköpings Universitet.

Johansson, M. (2003). *Textbooks in Mathematics Education*. Luleå: Department of Mathematics, Luleå University of Technology.

Malle, G., Ramharter, E., Ulovec, A., Kandl, S. (2004). Understanding Mathematics –

A Modern Textbook Concept. In: *Úloha učebnice vo vyučovaní matematiky.* Nitra: FPV UKF.

Molnár, J. et al. (1998). *Matematika 6.* Olomouc: PRODOS.

Molnár, J. et al. (1999). *Matematika 7.* Olomouc: PRODOS.

Molnár, J. et al. (2000). *Matematika 8.* Olomouc: PRODOS.

Molnár, J. et al. (2001). *Matematika 9.* Olomouc: PRODOS.

Molnár, J. (2004). K hodnocení učebnic matematiky primární školy. In: *Cesty (k) poznávání v matematice primární školy*. Olomouc: UP.

Molnár, J. (2007). *Učebnice matematiky a klíčové kompetence.* Olomouc: UP.

Molnár, J. (2007). O klíčových kompetencích ve vyučování matematice. In: *Zborník z medzinárodnej konferencie „Veda – vzdelávanie – prax“*. Nitra: UKF.

Pluskal, M. (1996). *Teorie tvorby učebnic a metody jejich hodnocení.* Olomouc: UP.

Průcha, J. (1989). *Teorie, tvorba a hodnocení učebnic*. Praha: ÚÚVPP.

PROBLEMS
OF EDUCATION
IN THE 21st CENTURY
Volume 6, 2008

Slavíčková, K. (2003). *Hodnocení učebnic matematiky.* Olomouc: UP.

Šedivý, O. (2004). Teoretické východiská k tvorbe učebníc matematiky s akcentom na humanizáciu vyučovania. In: *Úloha učebnice vo vyučovaní matematiky.* Nitra: FPV UKF.

Wahla, A. (1983). *Strukturní složky učebnic geografie.* Praha: SPN.

Adviced by Andreas Ulovec,
University of Vienna, Austria

Soňa Čeretková Ph.D., associated professor, Department of Mathematics, Faculty of Natural Sciences, Constantine the Philosopher University in Nitra, Slovakia.
Tr. A. Hlinku 1, 949 74 Nitra, Slovakia.
Phone: +421 37 6408555.
E-mail: sceretkova@ukf.sk
Website: http://www.en.ukf.sk/

Ondrej Šedivý Professor, Head of Department of mathematics, Faculty of Natural Sciences, Constantinte the Philosopher University in Nitra, Slovakia.
Tr. A. Hlinku 1, 949 74 Nitra, Slovakia
Phone: + 421 37 6408 698.
E-mail: osedivy@ukf.sk
Website: http://www.en.ukf.sk/

Josef Molnár Associated professor, Vice-dean, Department of algebra and geometry, Faculty of Natural Sciences, Palacky University in Olomouc, Czech Republic.
Tr. Svobody 26, 771 46 Olomouc, Czech Republic.
Phone: +420 585 634 170.
E-mail: molnar@prfnw.upol.cz
Website: http://www.upol.cz/en/

Dalibor Petr MSc. in Economics, Department of Foreign Languages, Faculty of Natural Sciences, Palacky University Olomouc, Czech Republic.
Tr. Svobody 26, 771 46 Olomouc, Czech Republic.
Phone: +420 585 634 998.
E-mail: petrdali@prfnw.upol.cz
Website: http://www.upol.cz/en/