MATHEMATICS FOR THE TALENTED ONES AS WELL AS THE OTHERS[1]

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Abstract

The results of a broadly-conceived research carried out in the Czech Republic in 2006 have shown that Math and Science do not belong to favorite subjects at secondary schools. The problem does not reside in Math itself, but in the personal reflection of the experiences from Math teaching. Mathematical competitions, unusual problem solving, games, projects and other motivational activities might provide an opportunity to change the attitude toward Math as a school subject. The confidence of what has just been said has brought the scientific team of the Palacký University in Olomouc to solving a grant of the Czech Ministry of Education, Youth and Sports. Within the grant the team has been seeking and verifying new methods and forms of competitions focusing on creative activities of youth in Math. The research activities can be divided into three areas:

New forms of work with mathematically gifted pupils (Jaroslav Švrček) aimed at preparing and implementing mathematical competitions in the Czech secondary schools environment. International mathematical competitions such as "Mathematical Duel" or "Middle European Mathematical Olympiad" present the chances to compare mathematical knowledge of talented pupils of Middle-European countries. "The Tournament of Towns" is even more widely conceived. Students solve the same problems (two times in the year) in their mother tongues of the cities of participating countries (Praha, Brno, Olomouc and Bílovec are the only towns in the Czech Republic – participation will be offered to all Czech cities next year).

The experience from organizing and didactic use of problems from the internationally-coordinated competition "Mathematical Kangaroo" is used for preparing, creating and advertising its new modification called "Sciences Kangaroo" (Josef Molnár). The nature of the competition as well as the target group (i.e. the "ordinary" elementary and secondary school pupils) remains the same. The tasks fall into the area of Natural Sciences, Physics, Chemistry, Biology, Geography, and Ecology. Their motivational features are stressed as this makes seeing the subject matter more interesting.

The third part, called "Playful Mathematics" (Bohumil Novák), focuses on preparing, creating and supporting a research in the educational efficiency of various mathematical activities in the elementary school environment: class, individual or staged competitions, games or integrated school projects which – following the constructivist approaches – aim at getting to know Mathematics. Reflection of the participants' view is very important in this respect – the participants are welcome to subsequently give their comments on both the content and the form of the event.

Key words: *teaching mathematics, mathematical competitions, games, projects, motivational activities.*

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Introduction and Motivation

The on-going curricular reform creates a good environment for the overall development of pupils' and students' personality in the Czech Republic. However, the results of a widely oriented research scheduled for 2006 suggested that Mathematics and natural sciences in general are not among the popular secondary school subjects. It is not Mathematics itself that is the problem; it is pupils' / students' reflection of experience from learning Mathematics. In this respect, teachers play the key role as well as the way they can make pupils / students see Mathematics. There are a number of ways of changing the attitude towards Mathematics as a school subject: mathematical competitions, solving non-standard tasks, games, projects or other motivational activities.

The assumptions mentioned above have become a starting point of the Ministry of Education, Youth and Sports of the Czech Republic grant scheduled for 2006-2008. The aim of the research project, which is being realized by the scientific team of the Faculties of Science and Education, the Palacký University in Olomouc, is to look for and verify new methods and forms of creative scientific competitions among the youth. By means of the newly-discovered competitions we want to affect the motivation for scientific activity in Mathematics as well as to make Mathematics more popular.

Research Activities

The research activities fall within three areas:

The first area was developed as follows. Until the year 2006 mathematically gifted students of Czech secondary schools had had only two opportunities to take part in national mathematical competitions in the Czech Republic. These were the Mathematical Olympiads and starting from the year 1995 also the well-known Kangaroo. The solvers of this grant wanted to explore and test some world-known activities in Czech conditions.

Since the second half of the year 2006 we have been trying to put into effect (and prove its functionality) the "Tournament of Towns" in the Czech Republic, one of prestigious competitions among pupils of cities of the whole world. In autumn 2006 two Czech cities took part in this competition (Praha – as a big city, and Bílovec – as a small city). In the spring part of the 28th Tournament of Towns this competition was also tested in two other cities (Olomouc and Přerov) both from the Moravia region. In this competition altogether more than 350 pupils of Czech secondary schools of mentioned cities took part. The formulation of all used problems was translated from the original (Russian) version into Czech and the pupils could write theirs full solutions in Czech language. On the grounds of our new experience with these competitions, solvers responsible for this grant created and proposed a more suitable version of this competition in the Czech Republic. Starting from the next year there will probably be the competition among pupils of all the regions in the Czech Republic. The Czech regions are better comparable (by results in this competition) with big cities of the world.

All problems are available on the official website of Tournament of Towns (Turnir gorodov). The Czech version of problems (with solutions) is available on the special website of the Palacký University in Olomouc.

The organization of an international mathematics competition „Mathematical Duel"(or in short „Duel") is being verified within the research. Students of these three secondary have taken part in this competition: Gymnázium Mikulá e Koperníka in Bílovec (CZE), I. LO in Chorzów (POL) and Bundesrealgymnasium in Graz (AUT). The last year of this competition, which is concentrated on the work with mathematically gifted pupils, took place in March 2007. The competition always proceeds in three age categories A, B, and C, as an individual and also team competition (Geretschläger, Švrček, 2005). We present (with consent) a collection of some problems from the last Mathematical Duel (Chorzów, March, 8th, 2007).

PROBLEMS
OF EDUCATION
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Category A (age 18 – 19 years)

Individual competition

1. Suppose the quadratic equation $x^2 - 2007x + b = 0$ with a real parameter b has two positive integer roots. Determine the maximum value of b.

2. Determine all positive real numbers x, y and z for which the following system of equations

$$\begin{aligned} x(x+y) + z(x-y) &= 65, \\ y(y+z) + x(y-z) &= 296, \\ z(z+x) + y(z-x) &= 104. \end{aligned}$$

is fulfilled.

3. Let M be the centre of the unit circle and A, B, C and P points of this circle such that ABM and BCM are equilateral triangles, MP is perpendicular to AM and P lies on the opposite half-plane divided by the line AM with respect to B and C. Prove that the equality

$$|CP| \cdot (|AC| - 1) = \sqrt{2}$$

holds.

4. Let ABC be an acute-angled triangle. Further let D be foot of the altitude from C to the side AB and P, Q be feet of perpendiculars from D on AC, BC, respectively. Let X be the second point of intersection (beside P) of PQ with the circumcircle of the triangle ADP, and similarly the second point of intersection (beside Q) of PQ and the circumcircle of the triangle BDQ. The extension of the ray DX meets AC at the point U and the extension of the ray DY meets BC at the point V. Prove that the two triangles with vertices A, D, U and B, D, V are similar.

Total time: 150 min.

Category C (age 14 – 15 years)

Team competition

1. Determine in how many ways the 4 x 4 checkerboard can be covered using 8 dominoes of the size 2 x 1.

2. A real function $f(x) = x^2 + ax + b$ with $a \neq 0 \neq b$ is given such that

$$f(a) = 51, \quad f(b) = b^3 + b^2 (a + 1) \quad \text{and} \quad f(1) < 0$$

hold. Determine the values of a and b.

3. A hexagon $ABCDEF$ is inscribed in a circle with radius 3 and diameter AD. Let four of its sides AB, CD, DE and FA be a length of 2. Determine the area and circumference of the hexagon.

Total time: 90 min.

The last new activity, in which there were interesting members of the solvers group of this partial task, is the preparation of the 1st Middle European Mathematical Olympiad (MEMO). This competition is scheduled for the end of September 2007 in Austrian Eisenstadt (first of all on the basis of the activity of Austrian colleagues). The national teams of Austria, Poland, the Czech Re-

public, Slovakia, Slovenia, Croatia and Switzerland took part in the first year of this competition. The Czech Republic will be the main organizer of the 2nd MEMO, which will be held at the Palacký University in Olomouc, on September, 4th – 10th, 2008. After this competition finishes, it will be reflected in our research.

As to the second area, the experience from organizing and didactic use of problems from the internationally-coordinated competition "Mathematical Kangaroo" is used for preparing, creating and advertising its new modification called "Sciences Kangaroo" (*Josef Molnár*). The nature of the competition as well as the target group (i.e. the "ordinary" elementary and secondary school pupils) remains the same. The tasks fall into the area of Natural Sciences, Physics, Chemistry, Biology, Geography, Ecology etc. Their motivational features are stressed as this makes seeing the subject matter more interesting.

The first confirmation year of the contest named the "Scientific Kangaroo" took place in the Czech Republic on April 25, 2007. There were two categories: *Cadet* (8th- and 9th-year students of elementary schools, 14-15 years of age) and *Junior* (1st- and 2nd-year students of grammar/secondary schools, 16-17 years of age). In each of the two categories there were 24 problems and the competitors had 45 minutes to solve them.

For an illustration, let us present a few problems used in the competition:

Cadet Category 2007

2. The most features close to a human can be found in case of

(A) a gibbon (B) a gorilla (C) a chimpanzee (D) a macaque (E) an orangutan

4. Which of the following does **not** belong among metals

(A) mercury (B) magnesium (C) zinc (D) silicon (E) tin (stannum)

5. How many sides does a "dodecahedron" have?

(A) 4 (B) 6 (C) 10 (D) 12 (E) 20

6. Water has the highest density at the temperature of

(A) 10 ºC (B) 4 ºC (C) 0 ºC (D) -4 ºC (E) -10 ºC

7. In what language does the word "kangourou" mean kangaroo?

(A) English (B) Deutsch (C) French (D) Polish (E) Estonian

8. A fisherman caught a carp. The carp's tail weighted 1 pound, the head weighted as much as the tail together with a half of the carp's body and the body weighted as much as the head together with the tail. So the whole carp weighted

(A) 6 pounds (B) 8 pounds (C) 10 pounds (D) 12 pounds (E) 14 pounds

10. Choose the most precise answer to the question: "How many bones and muscles are there in a human body?"

(A) a human body has 200 bones and 600 muscles (B) a human body has 600 bones and 200 muscles (C) a human body has 300 bones and 300 muscles (D) a human body has 250 bones and 250 muscles (E) a human body has 250 bones and 400 muscles

13. Christopher Columbus discovered America in

(A) 1592 (B) 1515 (C) 1352 (D) 1392 (E) 1492

14. Martin, whose eyes are 150 cm above the ground, was determining the height of a poplar in front of the school using the reflection of the tree in a puddle. He found out that the puddle was 20 m away from the poplar and when he was standing 3 m from the puddle, he could see the reflection of the treetop in the puddle. The height of the poplar is about

(A) 15 m (B) 20 m (C) 10 m (D) 6 m (E) 22 m

15. Which of the following does not belong among the methods of mixture components separation?

(A) re-crystallization (B) extraction (C) distillation (D) titration (E) chromatography

19. Which of the following belongs to the vegetable that we consume from Solanaceae family?

(A) chives, leek (B) cucumber, patty pan squash (C) broccoli, cauliflower (D) paprika, tomato (E) parsley, celery

20. An inventor of the steam engine James Watt teamed up with a rich Birmingham factory owner Boulton. In order to attract new customers, it was important to express how much horse power the mine owners can save due to their invention. They measured that in one second a strong horse could pull up 75 liters of water from the depth of 1 meter. Thus a unit of power 1 horsepower (hp) came into being. The power of how many Watts is equivalent to 10 horsepowers? (the acceleration of gravity $g = 10$ mxs^{-2}, density of water $\rho = 1000$ kgxm^{-3}).

(A) 750 W (B) 7500 W (C) 1500 W (D) 15000 W (E) 5000 W

21. It is for sure that the nucleus of an atom, which has the nucleon number of 12, consists of:

(A) 12 neutrons (B) altogether 12 protons and neutrons (C) altogether 12 neutrons and electrons (D) 12 electrons (E) 12 protons

23. What is the amount of water produced by burning 10 g of a gas mixture of oxygen and hydrogen in the ideal ratio?

(A) 100g (B) 1kg (C) 0,1g (D) 10g (E) 1g

Junior Category 2007

1. On a one meter long strip of paper we draw marks that divide the strip into 4 parts of equal length. Then we draw other marks that divide it into 3 parts of equal length. Next we cut the strip at every place where there is a (any) mark. How many different lengths do the small stripes have now?

(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

2. Which of the following organelles do plants and animals cells have in common?

(A) chloroplasts (B) cell wall formed by cellulose (C) vacuole (D) mitochondria (E) leucoplasts

13. On March 31, 2007 we remembered the 280th anniversary related to the death of one of the most famous physicists of all times (it was possible to remember the anniversary on March 20, 2007 too as in that time the Julian calendar was valid instead of our today's Gregorian calendar). The famous physicist is

(A) Albert Einstein (B) Thomas Alva Edison (C) Galileo Galilei (D) Isaac Newton (E) André-Marie Ampère

14. A construction kit only contains cuboid pieces of the dimensions 2 x 3 x 1. What is the least amount of these pieces that we need to build up a cube?

(A) 6 (B) 12 (C) 36 (D) 216 (E) 288

17. Which of the following matters stop cell division?

(A) carcinogens (B) cytostatics (C) antihistamine (D) antibiotics (E) antiseptics

18. How many liters of hot water having the temperature of 80 ºC is it necessary to pour to a bathtub where there are 80 liters of water with the temperature of 20 ºC so that in result, we have the bathtub with water that has the temperature of 40 ºC?

(A) 20 liters (B) 30 liters (C) 40 liters (D) 50 liters (E) 60 liters

23. Amount of substance is given by

(A) the product of mass and molar mass (B) the product of mass and molar concentration

(C) quotient of mass and molar mass (D) quotient of molar mass and mass

(E) the product of molar mass and the square of the mass

Correct answers:

Cadet

2 C, 4 D, 5 D, 6 B, 7 C, 8 B, 10 A, 13 E, 14 C, 15 D, 19 D, 20 B, 21 B, 23 D

Junior

1 B, 2 D, 13 D, 14 C, 17 B, 18 C, 23 C

Within the research the answers of the questionnaire survey are being processed. The respondents were the teachers that have realized the competition in their schools. The results and conclusion will be presented.

The main aim is to popularize science fields among the youth, to stimulate and support the interest of pupils and students in these fields, to show their attractiveness and usefulness. A similar goal is set within other activities of teachers and students at the Faculty of Science, the Palacký University in Olomouc such as, for instance, The Fair of Chemistry, Physics and Mathematics which is held at the Olomouc city hall every year (Hátle, 2006), the Run with the Kangaroo which consists of a cross-country running (1.5 – 10 km), jumping in duffel bags and solving problems from Combinatorics, Geometry, Logic and other mathematical, physical and other fields. Another example of such activities is the Summer school of Chemistry, Physics and Mathematics meant for talented students entering the last year of grammar schools with an international participation, internet correspondence Math, Physics and Chemistry competitions, etc.

Verification of the effect of these activities on pupils and students as well as probing the opinions of teachers and other pedagogical workers, who are involved in the activities mentioned above, is a part of the development and research in new forms of care for talents. The individual forms of working with talents will be modified and further developed on the basis of the survey results carried out by psychologists.

The interest in supporting the scientific fields can also be proved by materials processed by practitioners and academics from the Great Britain, Austria, Slovakia and the Czech Republic within a Socrates-Comenius project entitled "PROMOTE MSc" (Nezvalová, Molnár, 2006). Another example is the world association of national organizers of scientific competitions with the focus on the Scientific Kangaroo which is currently being formed in Georgia.

The third area, called "Playful Mathematics" (*Bohumil Novák*), focuses on preparing, creating and supporting research into the educational efficiency of various mathematical activities in the elementary school environment: class, individual or staged competitions, games or integrated school projects, which – following the constructivist approaches – aim at getting to know mathematics. The events are prepared in order to give pupils (even the less mathematically talented ones) a chance to acquire new mathematical experience and especially to let them get to know mathematics as something else than a boring subject – as an environment for personality development, interesting experimenting and discoveries. The teachers could see it as a chance to change their ways of teaching as well as forms and methods they use – all in order to be able to teach mathematics in a creative and interesting way. Reflection of the participants' view is very important in this respect – the participants are welcome to subsequently give their comments on both the content and the form of the event.

The above-mentioned activities reflect the main aims of the curricular reform. Mathematics for tomorrow's young children is to become the environment for developing student's personality. The idea of humanization, which stresses the idea of a school as a service to children and help in their

development, the important part of which are the affective components of learning (*Crowl et al*., 1997), is of primary importance. Developing children's personality is understood as their education in the broader sense of the word. Pupils are no longer objects of indoctrinating but as subject of their own learning. **"**Die Lernenden werden nicht mehr als Objekte der Belehrung, sondern als Subjekte ihres Lernens aufgefasst". (*Wittman*, 1997). A teacher in school is to develop his/her pupils' know-how, their ability to reason as well as encourage their creative thinking. (*Polya*, 1966). This makes space for the change in attitude to teaching mathematics as well as space for changing the forms and methods of teaching so that teachers could teach mathematics in a creative and interesting way and could help in creating a new climate in class.

At the same time this is a challenge for parents – they have an opportunity to get to know at least something of what their children know. They can learn about the nature of the constructivist oriented teaching of mathematics and the way it is based on the subject integration ("open classes"). They also have an opportunity to find out what experience and stimuli can their children find in the lesson. The natural precondition, however, is their coming to the class and sharing the experience with their children (*Kafoussi*, 2006).

Prospective teachers of mathematics – students of the Faculty of Education, Palacký University in Olomouc, took part in preparing and carrying out the events as well. Co-operating with experienced teachers they prepared various unconventional activities aimed at motivating pupils for mathematics, at developing favorable perception of mathematics as a school subject within the subject integration framework, or making pupils learn mathematics in a funny and easy way. Preparing such activities is another tool in making mathematics more popular.

Mathematical activities, performed during the research can be sorted as follows:

- games (e.g. sudoku, crosswords, board games, computer games, brain teasers)
- activities connected with everyday experiences (projects),
- unusual mathematical problems (e.g. "Kangaroo problems").

Conclusion

If pupils are to learn something, they must want to learn. Motivation plays a key role in the pedagogical practice as it gives subjective sense to the learning activities of pupils. It has been repeatedly proved that solving unconventional tasks, playing games, doing projects or other motivational activities can help to change the attitude to mathematics as a school subject. The activities, however, can shape the pupil only on condition that the teacher knows their personality and influences the learning environment in such a way that pupils themselves can organize their work and take active part in the teaching – learning process. This enables the pupils to see learning mathematics as solving interesting tasks.

A broadly organized research into popularity of respective school subjects and attitudes towards them (among pupils aged 11-18, 2006, Czech Republic) concluded that mathematics is not a popular subject and that pupils lack the ability to apply the obtained mathematical knowledge in real life situations (*Grecmanová, Dopita* 2007). This influences the situation at universities and universities of technology as well as even these schools lack good students with positive attitude to mathematics. It is not mathematics that is the problem, it is the personal reflection of experience obtained when being taught mathematics, i.e. the mathematics that the teachers show to pupils / students.

The partial project outcome have been presented at scientific conferences and published in mathematical as well as pedagogical journals. They can be also found at www.souteze.upol.cz

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