

3D VISUALIZATION OF A CASE-BASED DISTANCE MODEL

Péter Volf, Zoltán Kovács, István Szalkai

University of Pannonia, Veszprém, Hungary

Email: volfpeter@gtk.uni-pannon.hu, kovacs@gtk.uni-pannon.hu,
szalkai@almos.uni-pannon.hu

Abstract

Regarding to the conveyance of materials the assignment of stock-keeping units (SKUs) is one of the most important tasks in a warehouse. The units in the interest of their easier handling can be sorted into groups. To classifying SKUs the most frequently used method is the ABC analysis. The traditional approach implements a simplified scheme, which is mostly based on annual dollar usage. Classifying units according to one characteristic can lead to rushed result, because the assignment of stock-keeping units might be influenced by other factors, like number of orders, weight or lead time. In recent years a number of decision models have been developed in order to make it possible to consider more than one criterion in the same time. In general, in the case of representing a model the introduction of the algorithm is more preferable than the visualization of data. However the visualization of results can relieve the comprehension of the given problem and the way, how the model works, even in relation to sorting problems. The aim of this research is to introduce a plotting method which visualizes the results coming from the Case-based distance model developed by Chen, Kilgour and Hipel. This method strengthens the classification by providing separating nonlinear surfaces.

Key words: case-based distance model, inventory management, visualization.

Introduction

Many obstacles can emerge during decision making and support. It can frequently occur that besides the large amount of data the decision has to be made by considering more than one factors or characteristics. For instance, the sorting problem in inventory management where thousands of stock-keeping-units have to be sorted into groups based on their characteristics. Concerning this problem numerous methods have been developed to simplify, analyze and relieve the multicriterial decision making situation in recent years. Visualization techniques can also help the decision support procedure by making the sort of problem more understandable.

The traditional ABC analysis is the most frequently used approach to classify SKUs. The traditional approach implements a general, simplified scheme, which is mostly based on annual dollar usage. Besides its simplicity one of its advantages is the ease of representation of the results in the Pareto-chart (Harry, Mann, De Hodgins, Hulbert & Lacke, 2010), which is based on the famous Pareto-observation (Marshall, 2007). The most important lack of this method is that it cannot take more than one criterion into account at the same time.

In recent years a number of multi-criteria decision models have been developed in order to compensate this defect of the traditional approach using methodologies such as the weighted linear optimization (Ramanathan, 2006), the genetic algorithm (Guvemir & Erel, 1998), the analytic hierarchy process (AHP) (Partovi & Burton, 1993), the fuzzy set theory (Chu, Liang & Liao, 2008), the fuzzy-AHP (Cakir & Canbolat, 2008), etc.

On one hand these models can serve as management tools for solving complex decision

situations (Vetschera, Chen, Hipel, & Kilgour, 2010), on the other hand the expansion of the number of dimensions might cause confusion in interpretation of the results. Not understanding or misunderstanding the decision context is one of the main problems (Ma, 2011) that can lead to mistakes. In general, the visualization of data can relieve the comprehension of the given problem and its environment surrounding it.

Previously Tufte (1983) emphasized the importance of graphical representation analyzing the spread of cholera epidemic. He stated that the graphical analysis was more efficient than the calculation. Meyer (1991) examined the role of visual data in the concept of organizational research and suggested research questions, where visual data can be more meaningful to verbal one. Condon, Golden & Wasil (2003) proposed a methodology based on Sammon map for AHP. They analyzed the group decision procedure and relative judgment of the members. They claimed that using graphical assistant can raise the objectivity of collective decision making. Adler & Raveh (2008) introduced a graphical method for data envelopment analysis (DEA) using Co-plot, which can help to identify efficient units and outliers. Ma (2011) made up a screening methodology supported by graphical visualization based on the concept of multidimensional scaling (MDS).

On the field of inventory management concerning the analyses of SKUs one of the main problems is that the large number of units can lead to impracticable data set that can make the employ of the given model for decision makers (DMs) more complicated and can hinder the appropriate interpretation of the results. In this sense the Case-based distance model developed by Chen, Kilgour & Hipel (2008) excels from other models considering the structure of the model, because it involves an untapped opportunity in the view of interpretation of results. Namely the process of determining the parameters of surfaces separating each group of SKUs is built in the model and visualizing these surfaces can help to discover the magnitude of groups and the relationships between them.

The main purpose of the research is to give a graphical support to Multicriterial ABC Analysis based on Case-based distance model. Hence first of all the process of the initial dataset generation will be discussed. It is followed by an introduction into the visualization of the model, which is needed to explain the main findings of the research. A 2000-point illustrative example serves as demonstration of the plotting method step by step.

Methodology of Research

The main aim of the visualization is to demonstrate, how the Case-based distance model works and to present its results. The necessary dataset was generated on random basis in a controlled model. In this sense the control model means that the intervals were created in order to represent the most analysed criteria like unit value, weight and lead time. On the other hand the three criteria are reasonable because of the representation of the model in three dimensions.

Data Analysis

The main goal of the Case-based distance model is to sort the stock keeping units into groups based on characteristics of the units as the set Q of criteria (Chen, Kilgour & Hipel, 2008). Let T be the set of the units (A^i) being analyzed, where $A^i \in T_g \subset T \mid g = A, B, C$ based on $q_j \in Q \mid j = 1, 2, \dots, m$.

The minimum (c_j^{\min}) and the maximum (c_j^{\max}) values taken up in positions A^- and

A^+ form an interval (Chen, Kilgour & Hipel, 2008). The Euclidean distances taken from these points determine the position of the given unit on criterion q_j , i.e. the distance from the extreme values of the interval.

Let the set $Z_{repg} \subseteq T_g \mid g = A, B, C$ be a subset of T_g that involves the units which principally represent the set T_g and $z_{repg}^r \in T_g \mid g = A, B, C$ is one of these units, where

$$c_j^{\min}(A^-) \leq c_j(z_{repg}^r) \leq c_j^{\max}(A^+) \quad (\text{Chen, Kilgour \& Hipel, 2008}).$$

The Case-based distance model is constructed for n dimensions, which adopted on three criteria becomes capable to be plotted as a *three* dimensional system of co-ordinates defined by the three elements of Q , where respectively radius vector \underline{q}_1 represents the x , \underline{q}_2 the y and \underline{q}_3 the z axis. In the system of co-ordinates the radius vectors can be

given by the co-ordinates of their end, so if the co-ordinates of a P point are q_1^P, q_2^P and q_3^P , then the radius vector heading towards the P point is $\underline{q}_0(q_1^P; q_2^P; q_3^P)$.

In this sense we find that the coordinates of radius vector \underline{q}_j belonging to $z_{repg}^r \in T_g$ on criterion q_j :

- If the upper limit of the interval is the point of comparison:

$$\begin{cases} \underline{q}_{r1}(c_1^{\max}(A^+) - c_1(z_{repg}^r), 0; 0); \\ \underline{q}_{r2}(0; c_2^{\max}(A^+) - c_2(z_{repg}^r), 0); \\ \underline{q}_{r3}(0; 0; c_3^{\max}(A^+) - c_3(z_{repg}^r)). \end{cases}$$

- If the lower limit of the interval is the point of comparison:

$$\begin{cases} \underline{q}_{r1}(c_1(z_{repg}^r) - c_1^{\min}(A^-), 0; 0); \\ \underline{q}_{r2}(0; c_2(z_{repg}^r) - c_2^{\min}(A^-), 0); \\ \underline{q}_{r3}(0; 0; c_3(z_{repg}^r) - c_3^{\min}(A^-)). \end{cases}$$

After launching the normalization factor d_j^{\max} the distance between the lower and upper bounds of the intervals changes to unit, i.e. (Chen, Kilgour & Hipel, 2008)

1. Equation

$$d_j^{\max}(A^+; A^-) = \frac{(c_j^{\max}(A^+) - c_j^{\min}(A^-))^2}{d_j^{\max}} = 1 \mid j = 1, 2, 3$$

Since the distance between the lower and upper bound of interval is equal to the absolute value of radius vector on criterion j , therefore the Euclidean distances taken from the points of comparison can be identified as the length of the radius vectors belonging to each unit. So the

distance of $z_{repg}^r \in T_g$ from the upper limit of the interval on the second criterion is:

$$\begin{aligned} d_2^+(A^+, z_{repg}^r) &= \frac{(c_2^{\max}(A^+) - c_2(z_{repg}^r))^2}{d_2^{\max}} = \frac{(q_{r2}^+(0; c_2^{\max}(A^+) - c_2(z_{repg}^r), 0))^2}{(q_2^{\max}(0; c_2^{\max}(A^+) - c_2^{\min}(A^-), 0))^2} \\ &= \frac{(\sqrt{0^2 + (c_2^{\max}(A^+) - c_2(z_{repg}^r))^2 + 0^2})^2}{q_2^{\max}} = \frac{(c_2^{\max}(A^+) - c_2(z_{repg}^r))^2}{q_2^{\max}}, \end{aligned}$$

where

2. Equation

$$d_2^{\max} = (c_2^{\max} - c_2^{\min})^2 = (c_2^{\max}(A^+) - c_2^{\min}(A^-))^2 = (q_2^{\max}(0; c_2^{\max}(A^+) - c_2^{\min}(A^-), 0))^2 = (q_2^{\max})^2 = q_2^{\max}$$

Based on Chen, Kilgour & Hipel (2008) the weighted aggregation formula of distances are:

3. Equation

$$D^+(z_{repg}^r) = D^+(A_j^+; z_{repg}^r) = \sum_{j \in Q} w_j^+ \cdot d_j(z_{repg}^r) = \sum_{j \in Q} w_j^+ \cdot \frac{(q_j^+(x(z_{repg}^r), y(z_{repg}^r), z(z_{repg}^r)))^2}{q_j^{\max}},$$

4. Equation

$$D^-(z_{repg}^r) = D^-(z_{repg}^r, A_j^-) = \sum_{j \in Q} w_j^- \cdot d_j(z_{repg}^r) = \sum_{j \in Q} w_j^- \cdot \frac{(q_j^-(x(z_{repg}^r), y(z_{repg}^r), z(z_{repg}^r)))^2}{q_j^{\max}}$$

respectively.

Note, that depending on the point of comparison we get two different sorting problems. Based on this fact the visualization has to be separated likewise:

- If $c_j^{\min}(A^-)$ is the point of comparison, then we talk about *minimum transformation* in the minimum (distorted) space;
- If $c_j^{\max}(A^+)$ is the point of comparison, then we talk about *maximum transformation* in the maximum (distorted) space.

In this sense the attribute “distortion” means that the values created by the value function $d_j(z_{repp}^r)$ from the original values are linear and nonlinear distorted and this value transformation affects the sets and the shape of their separating surfaces, too.

The main aim is to plot the surfaces (ellipsoids) bounding the sets in the original space. To achieve this goal the linear and nonlinear transformations together with their effects have been discovered and are introduced it in the following section. The softwares, Lingo and MATLAB were used to calculate the necessary model parameters and to plot the surfaces.

Linear and nonlinear transformations and their effects on the visualization

Space 0: Let $P^i \in R^3$ be the point belonging to a SKU with the co-ordinates $P^i = (p_1^i, p_2^i, p_3^i)$, where $i = 1, 2, \dots, D$ and

$$\begin{cases} m_1 \leq p_1^i \leq M_1, \\ m_2 \leq p_2^i \leq M_2, \\ m_3 \leq p_3^i \leq M_3. \end{cases}$$

where (m_1, m_2, m_3) and (M_1, M_2, M_3) are the minimum and maximum values of the original coordinates, i.e. $c^{\min}(c_1^{\min}; c_2^{\min}; c_3^{\min})$ and $c^{\max}(c_1^{\max}; c_2^{\max}; c_3^{\max})$.

Space 1: Linear transformation: $E = (\varepsilon_1, \varepsilon_2, \varepsilon_3) = T(P)$, where

$$\varepsilon_1 = \frac{p_1 - m_1}{M_1 - m_1}, \quad \varepsilon_2 = \frac{p_2 - m_2}{M_2 - m_2}, \quad \varepsilon_3 = \frac{p_3 - m_3}{M_3 - m_3},$$

i.e. in the case of $i = 1, 2, \dots, D$

5. Equation

$$\varepsilon_1^i = \frac{p_1^i - m_1}{M_1 - m_1}, \varepsilon_2^i = \frac{p_2^i - m_2}{M_2 - m_2}, \varepsilon_3^i = \frac{p_3^i - m_3}{M_3 - m_3},$$

consequently the linear distorted co-ordinates are:

$$E^i = (\varepsilon_1^i, \varepsilon_2^i, \varepsilon_3^i) = T(P^i).$$

As a result we get a unit cube placed in „o” origin.

Space 2: Minimum world (nonlinear transformation): $\xi = (\xi_1, \xi_2, \xi_3) = \Xi(E)$

Let

6. Equation

$$\begin{cases} \xi_1^i = (\varepsilon_1^i)^2 = \frac{(p_1^i - m_1)^2}{(M_1 - m_1)^2}, \\ \xi_2^i = (\varepsilon_2^i)^2 = \frac{(p_2^i - m_2)^2}{(M_2 - m_2)^2}, \\ \xi_3^i = (\varepsilon_3^i)^2 = \frac{(p_3^i - m_3)^2}{(M_3 - m_3)^2}, \end{cases}$$

so in the case of $i = 1, 2, \dots, D$ $\xi^i = (\xi_1^i, \xi_2^i, \xi_3^i) = \Xi(T(P^i))$.

Note, that for $i = 1, 2, \dots, D$ $0 \leq \xi_1^i \leq 1, 0 \leq \xi_2^i \leq 1, 0 \leq \xi_3^i \leq 1$,

that is the transformed points $\xi^i = d_j^-(z_{repg}^r)$ are in the *same* unit cube.

Space 3: Maximum world (nonlinear transformation): $\eta = (\eta_1, \eta_2, \eta_3) = H(E)$

Let

7. Equation

$$\begin{cases} \eta_1^i = (1 - \varepsilon_1^i)^2 = \frac{(M_1 - p_1^i)^2}{(M_1 - m_1)^2}, \\ \eta_2^i = (1 - \varepsilon_2^i)^2 = \frac{(M_2 - p_2^i)^2}{(M_2 - m_2)^2}, \\ \eta_3^i = (1 - \varepsilon_3^i)^2 = \frac{(M_3 - p_3^i)^2}{(M_3 - m_3)^2}, \end{cases}$$

so in the case of $i = 1, 2, \dots, D$ $\eta^i = (\eta_1^i, \eta_2^i, \eta_3^i) = H(T(P^i))$.

Note, that in the case of $i = 1, 2, \dots, D$ $0 \leq \eta_1^i \leq 1, 0 \leq \eta_2^i \leq 1, 0 \leq \eta_3^i \leq 1$,

that is the transformed $\eta^i = d_j^+(z_{repj}^r) | i = r$ are also in the unit cube.

It is easy to state that the relation between Space 2 and 3 is:

8. Equation

$$\sqrt{\xi_1} + \sqrt{\eta_1} = 1, \sqrt{\xi_2} + \sqrt{\eta_2} = 1, \sqrt{\xi_3} + \sqrt{\eta_3} = 1.$$

Depending on which space we are analyzing the position of the points P^i the parameters searched are the followings (Chen, Kilgour & Hipel, 2008):

- Space 2:** $\xi^i = (\xi_1^i, \xi_2^i, \xi_3^i) = \Xi(T(P^i))$
- weight vector: $w^- = (w_1^-, w_2^-, w_3^-)$;
 - radii: R_B^- and R_C^- .

These parameters are given by the optimization model A^-MCABC expressed by the introduced transformation (Ξ) based on (Chen, Kilgour & Hipel, 2008):

9. Equation

$$\min ERR = \sum_{r=1}^{n_A} (\alpha_C^r)^2 + \sum_{r=1}^{n_B} [(\alpha_B^r)^2 + (\beta_B^r)^2] + \sum_{r=1}^{n_C} (\beta_A^r)^2,$$

where the conditions are:

9b. Equation

$$\sum_{j=1}^3 w_j^- \cdot \xi_j^r + \alpha_C^r \leq R_C^- | r = 1; 2; \dots; n_C$$

9c. Equation

$$\sum_{j=1}^3 w_j^- \cdot \xi_j^r + \alpha_B^r \leq R_B^- | r = 1; 2; \dots; n_B$$

9d. Equation

$$\sum_{j=1}^3 w_j^- \cdot \xi_j^r + \beta_B^r \leq R_C^- | r = 1; 2; \dots; n_B$$

9e. Equation

$$\sum_{j=1}^3 w_j^- \cdot \xi_j^r + \beta_A^r \leq R_B^- | r = 1; 2; \dots; n_A$$

where

9f. Equation

$$0 < R_C^- < 1; 0 < R_B^- < 1; R_C^- < R_B^-;$$

$$-1 \leq \alpha_B^r \leq 0, \quad 0 \leq \beta_A^r \leq 1,$$

$$-1 \leq \alpha_C^r \leq 0, \quad 0 \leq \beta_B^r \leq 1,$$

$$w_j^- > 0; \sum_{j \in Q} w_j^- = 1.$$

Space 3: $\eta^i = (\eta_1^i, \eta_2^i, \eta_3^i) = H(T(P^i))$

- weight vector: $w^+ = (w_1^+, w_2^+, w_3^+)$;
- radii: R_A^+ and R_B^+ .

Parameters are given in the case of A^+MCABC by the following model expressed by the introduced transformation (H) based on (Chen, Kilgour & Hipel, 2008):

10. Equation

$$\min ERR = \sum_{r=1}^{n_A} (\alpha_A^r)^2 + \sum_{r=1}^{n_B} [(\alpha_B^r)^2 + (\beta_B^r)^2] + \sum_{r=1}^{n_C} (\beta_C^r)^2,$$

where the conditions are:

10b. equation

$$\sum_{j=1}^3 w_j^+ \cdot \eta_j^r + \alpha_A^r \leq R_A^+ \mid r = 1; 2; \dots; n_A$$

10c. Equation

$$\sum_{j=1}^3 w_j^+ \cdot \eta_j^r + \alpha_B^r \leq R_B^+ \mid r = 1; 2; \dots; n_B$$

10d. Equation

$$\sum_{j=1}^3 w_j^+ \cdot \eta_j^r + \beta_B^r \leq R_A^+ \mid r = 1; 2; \dots; n_B$$

10e. Equation

$$\sum_{j=1}^3 w_j^+ \cdot \eta_j^r + \beta_C^r \leq R_B^+ \mid r = 1, 2, \dots, n_C$$

where

10f. Equation

$$\begin{aligned} 0 < R_A^+ < 1; \quad 0 < R_B^+ < 1; \quad R_A^+ < R_B^+; \\ -1 \leq \alpha_A^r \leq 0, \quad 0 \leq \beta_B^r \leq 1, \\ -1 \leq \alpha_B^r \leq 0, \quad 0 \leq \beta_C^r \leq 1, \\ w_j^+ > 0; \quad \sum_{j \in Q} w_j^+ = 1. \end{aligned}$$

Plotting method of linear and nonlinear separating surfaces

The nonlinear transformation implied by the value function $d_j(z_{repg}^r)$ on $z_{repg}^r = P_{repg}^i \mid i = r$ transformed the ellipsoids separating the set g into planes. In this way the planes separating the sets in the distorted spaces can be given by the following formula:

Space 2: $\xi^i = (\xi_1^i, \xi_2^i, \xi_3^i) = \Xi(T(P^i))$

11. Equation

$$\xi_3 = \frac{1}{w_3^-} (R_g^- - w_1^- \xi_1 - w_2^- \xi_2),$$

where $g = B^-, C^-$.

Space 3: $\eta^i = (\eta_1^i, \eta_2^i, \eta_3^i) = H(T(P^i))$

12. Equation

$$\eta_3 = \frac{1}{w_3^+} (R_g^+ - w_1^+ \eta_1 - w_2^+ \eta_2),$$

where $g = A^+, B^+$.

Note that due the transformations above the assumed relation between Space 2 and 3 cannot be assessed in distorted spaces. The DMs have to return back to the original space in order to determine the interaction of the two classifications in the same space.

To get the three groups of SKUs and the ellipsoids separating them in the original space, executing the inverse transformation is needed. Since $\xi^i = (\xi_1^i, \xi_2^i, \xi_3^i) = \Xi(T(P^i))$ and $\eta^i = (\eta_1^i, \eta_2^i, \eta_3^i) = H(T(P^i))$ are monotone transformations of P^i we can determine the ellipsoids separating the sets in the following way:

Space 2: $\xi^i = (\xi_1^i, \xi_2^i, \xi_3^i) = \Xi(T(P^i)) \rightarrow \Xi(T(P^i))^{-1} = (p_1^i, p_2^i, p_3^i) = P^i$

The equation of planes dividing the sets in Space 2 is:

$$\xi_3 = \frac{1}{w_3^-} (R_g^- - w_1^- \xi_1 - w_2^- \xi_2),$$

which can be reformed using the 6. equation into following formula:

$$\frac{(p_3 - m_3)^2}{(M_3 - m_3)^2} = \frac{1}{w_3^-} \left(R_g^- - w_1^- \frac{(p_1 - m_1)^2}{(M_1 - m_1)^2} - w_2^- \frac{(p_2 - m_2)^2}{(M_2 - m_2)^2} \right),$$

expressing p_3 can we find the equation of the ellipsoids:

13. Equation

$$p_3 = m_3 + \sqrt{\frac{(M_3 - m_3)^2}{w_3^-} \left(R_g^- - w_1^- \frac{(p_1 - m_1)^2}{(M_1 - m_1)^2} - w_2^- \frac{(p_2 - m_2)^2}{(M_2 - m_2)^2} \right)},$$

where $g = B^-, C^-$.

Space 3: $\eta^i = (\eta_1^i, \eta_2^i, \eta_3^i) = H(T(P^i)) \rightarrow H(T(P^i))^{-1} = (p_1^i, p_2^i, p_3^i) = P^i$

The equation of planes dividing the sets in Space 3 is:

$$\eta_3 = \frac{1}{w_3^+} (R_g^+ - w_1^+ \eta_1 - w_2^+ \eta_2),$$

which can be reformed using the 7. equation into following formula:

$$\frac{(M_3 - p_3)^2}{(M_3 - m_3)^2} = \frac{1}{w_3^+} \left(R_g^+ - w_1^+ \frac{(M_1 - p_1)^2}{(M_1 - m_1)^2} - w_2^+ \frac{(M_2 - p_2)^2}{(M_2 - m_2)^2} \right),$$

expressing p_3 can we find the equation of the ellipsoids:

14. Equation

$$p_3 = M_3 - \sqrt{\frac{(M_3 - m_3)^2}{w_3^+} \left(R_g^+ - w_1^+ \frac{(M_1 - p_1)^2}{(M_1 - m_1)^2} - w_2^+ \frac{(M_2 - p_2)^2}{(M_2 - m_2)^2} \right)},$$

where $g = A^+, B^+$.

Note, that in this case the centre of ellipsoids is the upper limit of the available values, i.e. the *maximum value*.

Since the position of the points P^i representing the units in n dimensional space is determined by the same criteria, therefore the sets are able to be plotted in the same space, where their sections can be identified as the *nine sets before the reclassification*.

The linear and nonlinear transformations employed by the value function $d_j(z_{reg}^r)$ kept the relations between sets, that is why applying the 14. equation the reformed shape of the planes separating the sets in Space 2 is

15. Equation

$$\xi_3 = 1 - \left(1 - \sqrt{\frac{R_g^-}{w_3^+} - \frac{w_1^+}{w_3^+} (1 - \sqrt{1 - \xi_1})^2 - \frac{w_2^+}{w_3^+} (1 - \sqrt{1 - \xi_2})^2} \right)^2$$

4/4 surface („eggs”), where $g = B^-, C^-$.

Respectively the reformed shape of the planes from the Space 3 can be given in Space 2 by the following formula:

16. Equation

$$\eta_3 = 1 - \left(1 - \sqrt{\frac{R_g^+}{w_3^-} - \frac{w_1^-}{w_3^-} (1 - \sqrt{1 - \eta_1})^2 - \frac{w_2^-}{w_3^-} (1 - \sqrt{1 - \eta_2})^2} \right)^2$$

This surface is 4/4 surface („eggs”) likewise, where $g = A^+, B^+$.

Results of Research

The results show that limitation of the n dimensional model into *three* dimensions made it possible to visualize the model in operation. The mathematical background of plotting method with the requisite formulas has been briefly introduced above. The following illustrative example supports to demonstrate the main findings of research concerning the visualization of Case-based distance model developed by Chen, Kilgour & Hipel (2008) step-by-step in figures.

The illustrative example consists of 2000 units. Each unit has three parameters represented on the three axes of a three dimensional coordinate-system, unit value (x), ranges from 1,002 to 17,385; weight (y) ranges from 9, 59 to 99,111 and lead time (z) ranging from 0,014 to 4,999 assuming that the lead time is a continuous variable, which indicates the average of the regular lead times and in this sense it is not needed to be discrete. The intervals of unit value and weight were generated based on normal distribution, in the case of lead time a shorter interval was determined based on uniform distribution.

Figure 1 illustrates the 2000-point dataset in three dimensional coordinate-system. The distribution of the unit value consists of three normal distributions with different parameters. Hence it is separated into three parts in order to make the movement of the dataset in the model to be traceable.

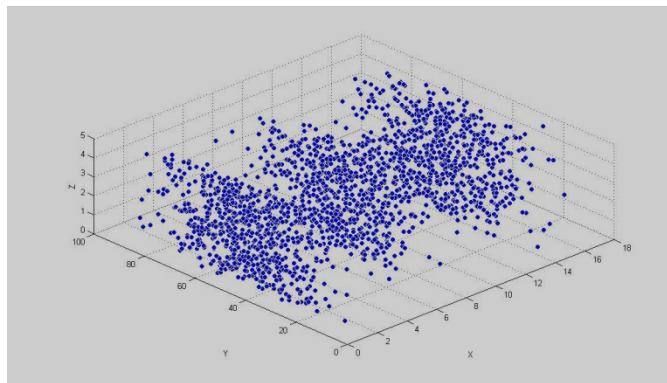


Figure 1: Visualization of points $P^i = (p_1^i, p_2^i, p_3^i) | i = 1, 2, \dots, 2000$ in Space 0 (original co-ordinates).

Applying the parameters of the results of A^-MCABC and A^+MCABC each group can be represented in three dimensional coordinate-system. The model parameters are the followings:

Space 2 (Minimum world): $\xi^i = (\xi_1^i, \xi_2^i, \xi_3^i) = \Xi(T(P^i))$

- weight vector: $w^- = (0,1840; 0,7241; 0,0919)$;
- radii: $R_B^- = 0, 2956$ and $R_C^- = 0, 1508$.

Space 3 (Maximum world): $\eta^i = (\eta_1^i, \eta_2^i, \eta_3^i) = H(T(P^i))$

- weight vector: $w^+ = (0,1990; 0,7420; 0,0590)$;
- radii: $R_A^+ = 0, 2348$ and $R_B^+ = 0, 4119$.

Note, that both in the minimum and maximum worlds the centre of the comparison is the “o” origin, but the order of sets is converse (see Figure 2). The converse order of these sets has been caused by the effect of linear and nonlinear transformations (introduced briefly in 6-7 equations).

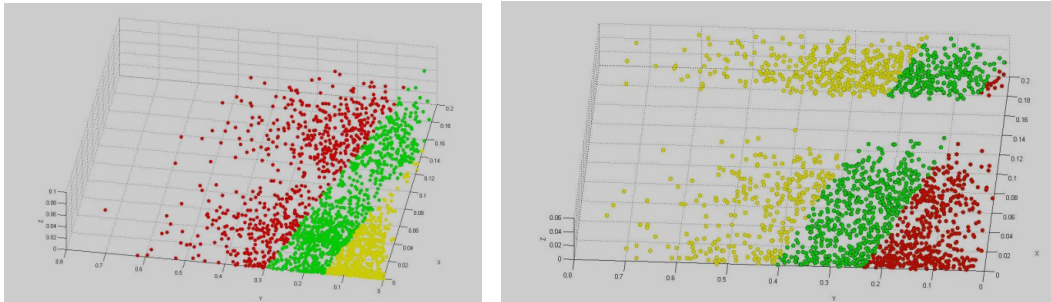


Figure 2: Visualization of groups in Space 2 and 3 (marking: Black – group A, Grey – group B, Light grey – group C).

Substituting the coordinates and the parameters above into equations 11 and 12 makes it possible to plot the surfaces separating each group from the others (see Figure 3). In both distorted spaces the separating surfaces quested for are planes.

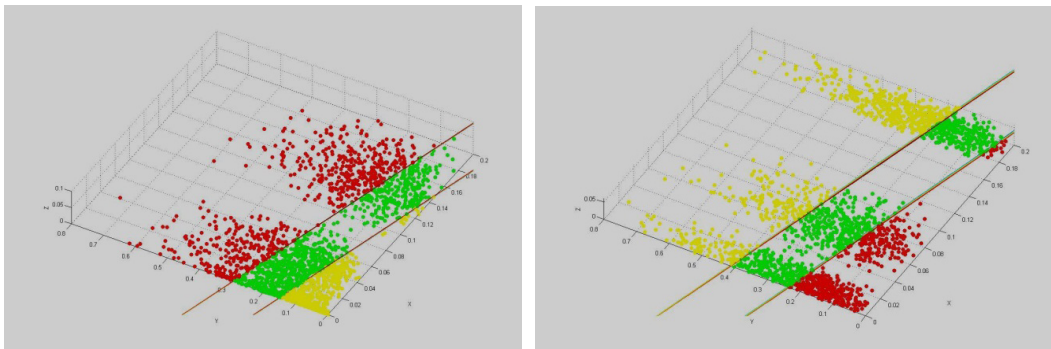


Figure 3: Visualization of groups and planes separating them in Space 2 and Space 3.

The A, B and C groups can be plotted in the original spaces after executing the inverse transformation indicated as $\Xi(T(P^i))^{-1} = (p_1^i, p_2^i, p_3^i) = P^i$ and $H(T(P^i))^{-1} = (p_1^i, p_2^i, p_3^i) = P^i$. The bounding surfaces belonging to each group can be represented by substituting these reformed coordinates with changeless model parameters into equations 13-14 (see Figure 4). These surfaces in the original space are ellipsoids – nonlinear surfaces in accordance with the initial assumptions.

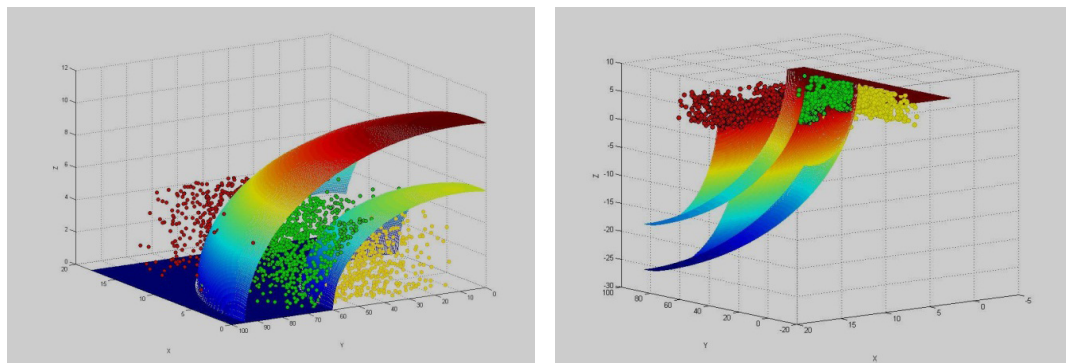


Figure 4: Visualization of groups of SKUs and ellipsoids bounding them based on the minimum and maximum world using the co-ordinates of

$$\chi(\mathbb{T}(P^i))^{-1} = (p_1^i, p_2^i, p_3^i) = P^i \text{ and } \mathbb{H}(\mathbb{T}(P^i))^{-1} = (p_1^i, p_2^i, p_3^i) = P^i$$

Due to the inverse transformation the results of the two classifications can be represented in the same original space (see Figure 5).

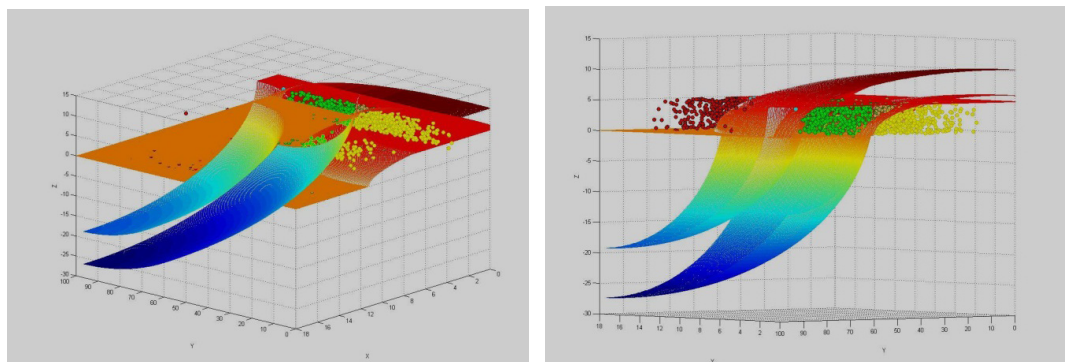


Figure 5: Visualization of classification based on minimum and maximum worlds in one space (Space 0) – bottom-view and side-view.

Since the implied linear and nonlinear transformations are monotonic, that is why the representation of the results of classifications keeps the relations between the groups in distorted spaces as well. The minimum and maximum worlds can be completed by using equations 15-16 respectively (see Figure 6).

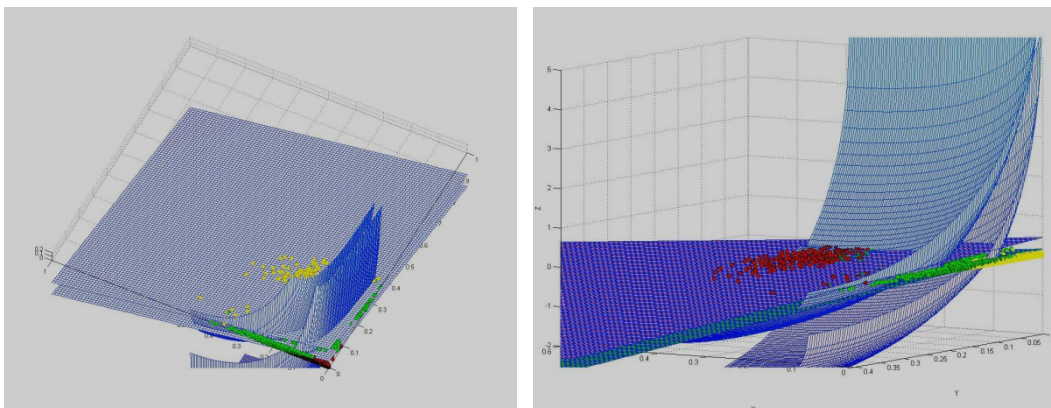


Figure 6: The reformed shape of the planes from Space 2 in Space 3 and from Space 3 in Space 2.

Figure 6 shows that the shape of the surfaces in one world is oval (“egg”) in the other world intercepting the planes.

Discussion

The adage “A picture is worth a thousand words” refers to the idea that a complex problem can be modeled in the way that managers can overview easily. The models like presented above can support every stages of decision making. Based on this managers will be able to make faster the process to get to the right decision. It increases the responsiveness of organizations.

Visualization is extremely important in the case of Multicriterial decisions, when more factors are supposed to be taken into consideration at the same time. The plotting method proposed in this paper is a managerial tool to demonstrate, how the magnitude of groups varies caused by changing in the selected criteria. Knowing the linear and nonlinear transformations and the formulas of each surfaces the effect of criteria selection to the classification or the effect of switching from one criterion to another keeping the other criteria stable can be studied, which is one of the main contribution of the plotting method.

In this sense the visualization can support not only the DMs but also the developers of the model to discover the opportunities in the model and to avoid the occurring problems. The visualization highlighted, what happens to the original data during the model that can make the explanation of results much easier.

Discovering the movement and the distortion of the original dataset in each spaces could lead to the observation of the real relation between the two classification approaches (minimum and maximum worlds). Using formulas (equation 13-14) gained from the model allowed to produce Figure (4) which illustrates the position of the categories in the way as original model developers (Chen, Kilgour & Hipel, 2008) assumed. Furthermore, only the representation of the surfaces (ellipsoids) in the original space could help to determine the relationship firstly between the distorted spaces and secondly between the groups.

On the other hand the visualization of the groups and surfaces in a three dimensional coordinate-system revealed that the rule of reclassification proposed by Flores & Whybark (1987) and used as a basis of reclassification in Case-based distance model (Chen, Kilgour & Hipel, 2008) is not unambiguos. Plotting the categories of SKUs from the two different approaches in one space still does not result in the final classification. The method of regrouping of the nine groups has to be reconsidered and can form a subject of further research.

In recent years a number of models for Multi-criteria ABC analysis have been developed in order to make it possible to take more than one criterion into account at the same time. But in general, in the case of representing a model the introduction of the algorithm is more preferable than the visualization of data. However the visualization of results can relieve the comprehension of the given problem and the way how the model works even in relation to sorting problems based on Case-based distance model.

The difficulty with interpretation of the results of classification is that the original data have been transformed by the value function employed in the model which affected the position of the groups and the shape of the surfaces bounding them. During the analysis of the mathematical background we contribute to the Case-based distance model with a graphical extension which is a proposal to the representation of the final classification of the groups of SKUs in one space.

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Advised by Zsolt T. Kosztyan, University of Pannonia, Hungary

Received: *September 15, 2011*

Accepted: *October 30, 2011*

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| Péter Volf | Ph. D. Candidate, University of Pannonia, H-8200 Egyetem Street 10, Veszprém, Hungary. E-mail: volfpeter@gtk.uni-pannon.hu Website: http://wiki.gtk.uni-pannon.hu/mediawiki_hu/index.php/Volf_P%C3%A9ter_adatlapja |
| Zoltán Kovács | Professor, University of Pannonia, H-8200 Egyetem Street 10, Veszprém, Hungary. E-mail: kovacs@gtk.uni-pannon.hu Website: http://wiki.gtk.uni-pannon.hu/mediawiki_hu/index.php/Dr._Kov%C3%A1cs_Zolt%C3%A1n_adatlapja |
| István Szalkai | Assistant Professor, University of Pannonia, H-8200 Egyetem Street 10, Veszprém, Hungary. E-mail: szalkai@almos.uni-pannon.hu Website: http://math.uni-pannon.hu/~szalkai/ |